Moment of Inertia

Purpose

To determine the theoretical value for an object's moment of inertia, then find that object's moment of inertia experimentally and compare the two values. We will know our results are accurate if the two values agree.

Pre-Lab Exercises

Read through this lab handout carefully and prepare a spreadsheet for recording data.

Use your PHSX 215N textbook to find the appropriate formula for the moment of inertia of a solid disk ($R > L$) rotated about its central axis

There is a preview of this lab's equipment in use posted to this lab's Canvas module (although there is no SmartTimer connected to the photogate in the video), which you may find helpful. Note that the video shows the disk being rotated about its central axis as well as its central diameter, but we will only be doing the former.

Introduction

You will determine the moment of inertia I of a solid disk, rotated about its central axis.

First, you will directly calculate the theoretical value $I_{theoretical}$ using the appropriate formula from the pre-lab exercises and the object's mass and physical dimensions.

Then, you will find the experimental value $I_{experimental}$ by applying various torques τ and measuring the object's angular acceleration α. These values are all related by the equation:

$$\tau = I\alpha$$

Eq. 1

By plotting the relationship between the object's torque and angular acceleration, you can determine the moment of inertia $I_{experimental}$ from the slope of the best fit line.

Torques will be applied by hanging masses from the end of a string which winds around the rotation base's aluminum cylinder and then goes over a pulley, such that the mass hanger can descend over the side of the lab bench to the floor, rotating the cylinder (and solid disk when attached) as the string unwinds.

The tension force in the string can be described with the following equations:

$$\Sigma F_y = F_{gy} + F_{Ty}$$

$$ma_y = -mg + F_T$$

$$m(-a) = -mg + F_T$$

$$F_T = m(g - a)$$

Eq. 2

Here, m is the **accelerating mass** (the mass which contributes to the acceleration, found by calculating the difference between the hanging mass and the friction mass as described in the *Procedure* section), a is the linear acceleration of the mass hanger, and g is the acceleration due to gravity which for the purposes of this lab is taken to be $g = 9.81\ m/s^2$.

The torque due to the tension in the string can be described with the following equations:

$$\vec{\tau} = \vec{r} \times \overrightarrow{F_T}$$

$$\tau = rF_T \sin \phi_T$$

$$\tau = rm(g - a) \sin(90°)$$

$$\tau = rm(g - a)$$

Eq. 3

Here, $\vec{r}$ is the radius of the aluminum cylinder around which the string is wrapped and ϕ_T is the angle between the cylinder's radius vector and the string's tension force vector.

Finally, the linear acceleration a of the mass hanger and the angular acceleration α of the aluminum cylinder are related by the equation:

$$a = r\alpha$$

Eq. 4

Procedure

Determining the theoretical value

Take the appropriate measurements of the solid disk. Use the formula you found in the Pre-Lab section to calculate a theoretical value for the moment of inertia $I_{theoretical}$.

Don't forget to record the uncertainties for all measurements and to also calculate $\delta I_{theoretical}$ using the appropriate rule of error propagation.

Determining the experimental value

Use the calipers to measure the diameter d of the aluminum cylinder that the string wraps around.

Place the solid disk onto the base so that it's rotated about its central axis (so the disk is lying flat, parallel to lab bench), then use the adjustable feet on the base and the level to make sure the disk is as level as possible.

Determine the "friction mass" – Because the theory used to find $I_{experimental}$ does not include friction, it will be compensated for in this experiment by finding how much mass over the pulley it takes to overcome kinetic friction and allow the mass to drop at a constant speed. This "friction mass" $m_{friction}$ will be subtracted from the hanging mass $m_{hanging}$ for each set of trials, in order to find the accelerating mass m.

1) Turn on the SmartTimer and set it to measure the pulley's **speed** in rad/s.
2) Attach a few paper clips to the end of the string over the pulley, then give the solid disk a small tap to start it rotating.
3) Repeatedly measure the speed as the paper clips descend.
   - If the speed decreases, add another paper clip, re-wind the string, and repeat this step until the speed is about constant.
   - If the speed increases, remove a paper clip, re-wind the string, and repeat this step until the speed is about constant.
4) When you have found the number of paper clips that allows them to drop at a near constant speed, remove the paper clips from the string and find their combined mass on the electronic scale. This is your friction mass $m_{friction}$.

Now you will perform 6 sets of 5 trials. Each set of trials will have a different total mass hanging from the string.

⚠ *Adjust the SmartTimer settings to measure the pulley's **linear acceleration** before proceeding to the next steps.*

For each set of trials, do the following:

1) Add mass ($20\ g$) to the hanger. Measure and record the hanging mass $m_{hanging}$ for this set of trials.
2) For each of the 5 trials:
   i. Let the hanging mass fall from rest (don't push).
   ii. About 1/3 of the way through the descent, begin to measure the linear acceleration a using the SmartTimer (watch units).
   iii. Re-wind the string onto the cylinder and repeat.
3) Find $\bar{a} \pm \delta\bar{a}$ from your 5 values for a.

Once you have finished the 6 sets of 5 trials (for a total of 30 trials), you can use your collected data with the theory presented in the *Introduction* to construct the relevant quantities.

1) For each set of trials, find the average value for the torque τ and the angular acceleration α. Don't forget to use the accelerating mass m rather than the hanging mass $m_{hanging}$ for these calculations.
2) Use the provided Python plotting template to make a plot of τ vs. α. The graph should include vertical error bars representing your values for $\delta\tau$.
3) Determine the slope of the best fit line and use it to find $I_{experimental}$.

List of all measurements & calculations

THEORETICAL

Measure:

- $D \pm \delta D$
- $M \pm \delta M$

Calculate:

- $I_{theo} \pm \delta I_{theo}$

EXPERIMENTAL

Measure:

- $d \pm \delta d$
- $m_{friction} \pm \delta m_{friction}$
- $m_{hanging} \pm \delta m_{hanging}$
- a (x5)

Calculate:

- $m \pm \delta m$
- $\bar{a} \pm \delta\bar{a}$
- $\tau \pm \delta\tau$
- α
- $I_{exp} \pm \delta I_{exp}$

x6

Final Considerations

You should come to the post-lab quiz prepared to state your final results, show your work, and justify whether or not your results are accurate.

Don't forget to upload the GitHub link for your plot on Canvas before the post-lab quiz.