Cryptography of Hyperledger Indy

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1 Syntax of Hyperledger Indy

The first four steps are similar to the register operation, and the last four steps look like login.¹

- 1. Issuer determines a credential schema S: the type of cryptographic signatures used to sign the credentials, the number l of attributes in a credential, the indices $A_h \subset [1, l] = \{1, 2, ..., l\}$ of hidden attributes, the public key P_k , the non-revocation credential attribute number l_r and non-revocation public key P_r . Then he publishes it on the ledger and announces the attribute semantics.
- 2. Holder retrieves the credential schema from the ledger and sets the hidden attributes.
- 3. Holder requests a credential from issuer. He sends hidden attributes in a blinded form to issuer and agrees on the values of known attributes $A_k \leftarrow [1, l] \backslash A_h$.
- 4. Issuer returns a credential pair (C_p, C_{NR}) to holder. The first credential contains the requested l attributes. The second credential asserts the non-revocation status of the first one. Issuer publishes the non-revoked status of the credential on the ledger.
- 5. Holder approaches verifier. Verifier sends the Proof Request \mathcal{E} to holder. The Proof Request contains the credential schema \mathcal{S}_E and disclosure predicates \mathcal{D} . The predicates for attribute m and value V can be of form m = V, m < V, or m > V. Some attributes may be asserted to be the same: $m_i = m_j$.
- 6. Holder checks that the credential pair he holds satisfies the schema \mathcal{S}_E . He retrieves the non-revocation witness from the ledger.
- 7. Holder creates a proof \mathcal{P} that he has a non-revoked credential satisfying the proof request \mathcal{E} and sends it to verifier.
- 8. Verifier verifies the proof.

¹All content refers to Hyperledger Indy HIPE.

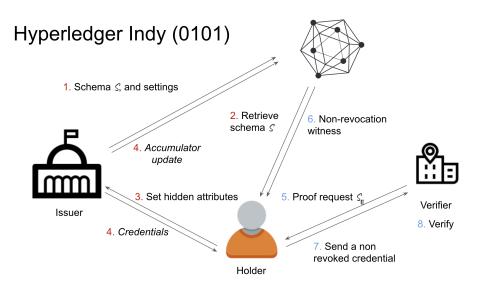


Figure 1: Syntax of Hyperledger Indy

Symbol	Definition
\mathcal{S}	Schema, the empty data form with only fields.
(l, l_r)	Attributes number and non-revocation credential attribute number.
L	The volume of a non-revocation list.
l_a	Message length for all attributes. In Sovrin, $l_a = 256$.
$(\mathcal{A}_k, \mathcal{A}_h)$	The indices of known attributes and hidden attributes respectively.
	By default, $\{1,3\} \subset \mathcal{A}_h$ and $\{2\} \subset \mathcal{A}_k$.
(P_k, P_r)	Public keys of primary credentials and non-revocation credentials resp.
\mathcal{P}_1	Correctness proof of P_k .
(i, \mathcal{H})	The index and identifier of a holder in the issuer's view.
(\mathcal{V},acc)	The indices and accumulator of the current non-revocation list.
(C_P, C_{NR})	The primary credential and the non-revocation credential.

Table 1: Symbol table

2 Environment setup

Issuer generates the key pair (P_k, s_k) and a proof \mathcal{P}_1 through $setup_{PC}(l)$ (Algorithm 1), as well as the non-revocation key pair (P_r, s_r) through $setup_{NR}(l)$ (Algorithm 3); then, he keeps (s_k, s_r) secret and publishes $(\mathcal{S}, \mathcal{A}_h, l_r, P_k, P_r, \mathcal{P}_1)$ to the ledger. Everyone can verify the correctness of P_k (via proof \mathcal{P}_1) through $verify_{P_k}(l, P_k, \mathcal{P}_1)$ (Algorithm 2).

2.1 Primary Credential (CL-Signature)

```
Algorithm 1 setup_{PC}(l)
    p', q' \leftarrow_R \{0, 1\}^{1536}
                                                                                          \triangleright p' and q' are prime; |p'| = |q'| = 1536
    p \leftarrow 2p' + 1; q \leftarrow 2q' + 1; n \leftarrow pq
                                                                                                                                      \triangleright p and q are prime
    t \leftarrow_R \mathbb{Z}_n^*; S \leftarrow t^2 \pmod{n}
    x_z \leftarrow_R \mathbb{Z}^*_{p'q'}, Z \leftarrow S^{x_z} \pmod{n}
    \{x_{r_i} \leftarrow_R \mathbb{Z}_{p'q'}^*, R_i \leftarrow S^{x_{r_i}} \pmod{n}\}_{\forall i \in [1, l]}
P_k \leftarrow (n, S, Z, \{R_i\}_{\forall i \in [1, l]}), s_k \leftarrow (p, q)
    \tilde{x}_z \leftarrow_R \mathbb{Z}^*_{p'q'}, \, \tilde{Z} \leftarrow S^{\tilde{x}_z} \pmod{n}
                                                                                                                > correctness proof from here.
    \{\tilde{x}_{r_i} \leftarrow_R \mathbb{Z}_{p'q'}^*, \, \tilde{R}_i \leftarrow S^{\tilde{x}_{r_i}} \pmod{n}\}_{\forall i \in [1,l]}
    c \leftarrow H_1(Z||\tilde{Z}||\{R_i, R_i\}_{\forall i \in [1, l]})
                                                                                                                  \triangleright H_1 is by default SHA2-256
    \hat{x}_z \leftarrow \tilde{x}_z + c \cdot x_z; \{\hat{x}_{r_i} \leftarrow \tilde{x}_{r_i} + c \cdot x_{r_i}\}_{\forall i \in [1, l]}
    \mathcal{P}_1 \leftarrow (c, \hat{x}_z, \{\hat{x}_{r_i}\}_{\forall i \in [1, l]})
    return (P_k, s_k, \mathcal{P}_1)
```

Algorithm 2 $verify_{P_k}(l, P_k, \mathcal{P}_1)$

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(n, S, Z, \{R_i\}_{\forall i \in [1, l]}) \leftarrow P_k; (c, \hat{x}_z, \{\hat{x}_{r_i}\}_{\forall i \in [1, l]}) \leftarrow \mathcal{P}_1
\tilde{Z} \leftarrow Z^{-c} S^{\hat{x}_z}; \{\tilde{R}_i \leftarrow R_i^{-c} S^{\hat{x}_{r_i}}\}_{\forall i \in [1, l]} \pmod{n}
\mathbf{return} \ c == H_1(Z||\tilde{Z}||\{R_i, \tilde{R}_i\}_{\forall i \in [1, l]})
```

2.2 Non-Revocation Credential

Algorithm 3 $setup_{NR}()$

```
\mathbb{G}_{1} \times \mathbb{G}_{2} \to \mathbb{G}_{T} \quad \triangleright \text{ pick a type-III pairing where } |\mathbb{G}_{1}| = |\mathbb{G}_{2}| = |\mathbb{G}_{T}| = q
g \leftarrow_{R} \mathbb{G}_{1}; \ g' \leftarrow_{R} \mathbb{G}_{2}
h, h_{0}, h_{1}, h_{2}, \tilde{h} \leftarrow_{R} \mathbb{G}_{1}; \ u, \hat{h} \leftarrow_{R} \mathbb{G}_{2}
sk \leftarrow_{R} \mathbb{Z}_{q}^{*}, \ pk \leftarrow g^{sk}; \ x \leftarrow_{R} \mathbb{Z}_{q}^{*}, \ y \leftarrow \hat{h}^{x}
P_{r} \leftarrow (h, h_{0}, h_{1}, h_{2}, \tilde{h}, \hat{h}, u, pk, y), \ s_{r} \leftarrow (sk, x)
\mathbf{return} \ (P_{r}, s_{r})
```

2.3 CKS Accumulator

Issuer creates a new accumulator using $setup_{Acc}(L, P_r)$ (Algorithm 4).

Algorithm 4 $setup_{Acc}(L, P_r)$

```
r \leftarrow_{R} \mathbb{Z}_{q}^{*}; \{g_{i} \leftarrow g^{r^{i}}, g_{i}' \leftarrow g'^{r^{i}}\}_{\forall i \in [1, 2L] \setminus \{L+1\}}
z \leftarrow e(g, g')^{r^{L+1}}; V \leftarrow \emptyset; acc \leftarrow 1
P_{a} \leftarrow (z, \{g_{i}, g_{i}'\}_{\forall i \in [1, 2L] \setminus \{L+1\}}), s_{a} \leftarrow r
\triangleright \text{ issuer publishes } (P_{a}, \mathcal{V}) \text{ on the ledger with identifier } ID_{a} \leftarrow z.
\mathbf{return} \ (P_{a}, s_{a}, \mathcal{V}, acc)
```

3 Credential Issuance

Let i < L and \mathcal{H} be the index and identifier of the holder in the issuer's system, respectively. The holder acquires the schema \mathcal{S} , indices \mathcal{A}_h and public keys (P_k, P_r) from the ledger in addition to a random number n_0 and the identifier \mathcal{H} from the issuer; then he sets the hidden attribute $\{m_i\}_{\forall i \in \mathcal{A}_h}$. The credential issuance process is interactive, which follows:

- 1. The holder computes a temporary result (P_h, s_h) by excuting (Algorithm 5) $issue_1(\mathcal{S}, \mathcal{A}_h, \{m_i\}_{\forall i \in \mathcal{A}_h}, n_0, \mathcal{H}, P_k, P_r)$; then, he keeps s_h private and sends $(P_h, \{m_i\}_{\forall i \in \mathcal{A}_k})$ to the issuer.
- 2. On receiving $(P_h, \{m_i\}_{\forall i \in \mathcal{A}_k})$ from the holder, the issuer firstly verifies P_h through $verify_{P_h}(P_k, P_h)$ (Algorithm 6). If it passes, the issuer fetches the current non-revoked indices \mathcal{V} and accumulator acc on the ledger, and runs $issue_2(i, \mathcal{H}, \mathcal{V}, acc, \{m_i\}_{\forall i \in \mathcal{A}_k}, P_k, s_k, P_r, s_r, P_a, s_a, P_h)$ (Algorithm 7) to generate $(P_{PC}, P_{NR}, \mathcal{V}, acc)$. Finally, the issuer stores the holder's information and index i in issue's local database; then, he updates acc and \mathcal{V} on the ledger and returns (P_{PC}, P_{NR}) to the holder.
- 3. While receiving (P_{PC}, P_{NR}) from the issuer, the holder firstly runs Algorithm 8 to verify P_{NR} . If True $\leftarrow verify_{P_{NR}}(acc, \mathcal{H}, s_h, P_r, P_{NR})$, the holder excutes Algorithm 9 $issue_3(P_k, P_h, s_h, P_{PC}, P_{NR})$ to do more verifications. If all verification pass, the holder keeps the returned credential (C_P, C_{NR}) .

4 Credential Revocation

The revocation process is quite straightforward. The issuer fetches the current non-revoked indices \mathcal{V} and accumulator acc on the ledger. Then, he revokes user with index i via Algorithm 10 and updates (\mathcal{V}', acc') on the ledger after running $(\mathcal{V}', acc') \leftarrow revoke(\mathcal{V}, acc, i)$.

Algorithm 5 $issue_1(\mathcal{S}, \mathcal{A}_h, \{m_i\}_{\forall i \in \mathcal{A}_h}, n_0, \mathcal{H}, P_k, P_r)$

Algorithm 6 $verify_{P_h}(P_k, P_h)$

```
(n, S, Z, \{R_i\}_{\forall i \in [1, l]}) \leftarrow P_k; (U, c, \hat{v}', \{m_i\}_{\forall i \in \mathcal{A}_h}, n_1, U_r) \leftarrow P_h
\tilde{U} \leftarrow U^{-c} S^{\hat{v}'} \prod_{i \in \mathcal{A}_h} S^{\hat{m}_i} R_i^{-c} \pmod{n}
\mathbf{return} \ c == H(U||\tilde{U}||n_0)
```

Algorithm 7 $issue_2(i, \mathcal{H}, \mathcal{V}, acc, \{m_i\}_{\forall i \in \mathcal{A}_k}, P_k, s_k, P_r, s_r, P_a, s_a, P_h)$

```
\overline{m_2 \leftarrow H(i||\mathcal{H})}
                                                                                                        ▶ the primary credential
v'' \leftarrow \{0,1\}^{2723}; e'' \leftarrow \{0,1\}^{596}
                                                                           |v''| = 2723, |e| = 596 \text{ and } e \text{ is prime}
(n, S, Z, \{R_i\}_{\forall i \in [1,l]}) \leftarrow P_k; (U, c, \hat{v}', \{m_i\}_{\forall i \in A_h}, n_1, U_r) \leftarrow P_h
Q \leftarrow Z(US^{v''} \prod_{i \in \mathcal{A}_k} R_i^{m_i})^{-1} \pmod{n}; r \leftarrow_R \mathbb{Z}_{p'q'}^*
A \leftarrow Q^{e^{-1} \pmod{p'q'}}; \hat{A} \leftarrow Q^r \pmod{n}
c' \leftarrow H(Q||A||\hat{A}||n_1); s_e \leftarrow r - c'e^{-1}
P_{PC} \leftarrow (\{m_i\}_{\forall i \in \mathcal{A}_k}, A, e, v'', s_e, c')
s'', c \leftarrow_R \mathbb{Z}_q^*
                                                                                                  ▷ non-revocation credential
(h, h_0, h_1, h_2, h, h, u, pk, y) \leftarrow P_r, (sk, x) \leftarrow s_r
(z, \{g_i, g_i'\}_{\forall i \in [1, 2L] \setminus \{L+1\}}) \leftarrow P_a, r \leftarrow s_a
\sigma \leftarrow (h_0 h_1^{m_2} U_r g_i h_2^{s''})^{(x+c)^{-1}}; \, \sigma_i \leftarrow g'^{(sk+r^i)^{-1}}; \, u_i \leftarrow u^{r^i}
w \leftarrow \prod_{j \in \mathcal{V}} g'_{L+1+i-j}; \mathcal{V} \leftarrow \mathcal{V} \cup \{i\}, acc \leftarrow acc \cdot g'_{L+1-i}
wit_i \leftarrow (\sigma_i, u_i, g_i, w, \mathcal{V})
P_{NR} \leftarrow (I_A, \sigma, c, s'', wit_i, g_i, g_i', i)
return (P_{PC}, P_{NR}, \mathcal{V}, acc)
```

Algorithm 8 $verify_{P_{NR}}(acc, \mathcal{H}, s_h, P_r, P_{NR})$

```
\begin{array}{l} (h,h_0,h_1,h_2,\hat{h},\hat{h},u,pk,y) \leftarrow P_r; \ (I_A,\sigma,c,s'',wit_i,g_i,g_i',i) \leftarrow P_{NR} \\ (\sigma_i,u_i,g_i,w,\mathcal{V}) \leftarrow wit_i; \ (v',s') \leftarrow s_h \\ s \leftarrow s'+s''; \ m_2 \leftarrow H(i||\mathcal{H}) \\ \text{if} \ e(g_i,acc)(e(g,w))^{-1} \neq z \ \text{then} \\ \text{return False} \\ \text{else if} \ e(pk\cdot g_i,\sigma_i) \neq e(g,g') \ \text{then} \\ \text{return False} \\ \text{else if} \ e(\sigma,y\cdot\hat{h}^c) \neq e(h_0h_1^{m_2}h_2^s\cdot g_i,\,\hat{h}) \ \text{then} \\ \text{return False} \\ \text{else} \\ \text{return True} \\ \text{end if} \end{array}
```

Algorithm 9 $issue_3(P_k, P_h, s_h, P_{PC}, P_{NR})$

```
(n,S,Z,\{R_i\}_{\forall i\in[1,l]}) \leftarrow P_k; \ (U,c,\hat{v}',\{m_i\}_{\forall i\in\mathcal{A}_h},n_1,U_r) \leftarrow P_h \\ (\{m_i\}_{\forall i\in\mathcal{A}_k},A,e,v'',s_e,c'\} \leftarrow P_{PC}; \ (I_A,\sigma,c,s'',wit_i,g_i,g_i',i) \leftarrow P_{NR} \\ \text{if $e$ is not prime OR $e\notin[2^{596},2^{596}+2^{119}]$ then} \\ \text{return null} \\ \text{end if} \\ (v',s') \leftarrow s_h; \ v\leftarrow v'+v''; \ s\leftarrow s'+s'' \\ Q\leftarrow Z(S^v\prod_{i\in(\mathcal{A}_k\cup\mathcal{A}_h)}R_i^{m_i})^{-1} \ (\text{mod }n) \\ \text{if $Q\neq A^e$ then} \\ \text{return null} \\ \text{end if} \\ \hat{A}\leftarrow A^{c'+s_e\cdot e} \\ \text{if $c'\neq H(Q||A||\hat{A}||n_1)$ then} \\ \text{return null} \\ \text{else} \\ C_P\leftarrow (\{m_i\}_{\forall i\in(\mathcal{A}_k\cup\mathcal{A}_h)},A,e,v); \ C_{NR}\leftarrow (I_A,\sigma,c,s,wit_i,g_i,g_i',i) \\ \text{return $(C_P,C_{NR})$} \\ \text{end if} \\ \end{cases}
```

Algorithm 10 revoke(V, acc, i)

```
 \begin{array}{c} \mathcal{V} \leftarrow \mathcal{V} \backslash \{i\} \\ acc \leftarrow acc \cdot (g'_{L+1-i})^{-1} \\ \mathbf{return} \ (\mathcal{V}, acc) \end{array}
```