Lecture Summary 8

When the state space is continuous; for example $\mathbb{R}$, then a Markov chain (X_n) is described by a transition density $p(y|x)$. That is,

$$P(X_{n+1} \in A | X_n = x) = \int_A p(y|x)\, dy.$$

The n step transition is written as $p^n(y|x)$ and is defined as

$$p^n(y|x) = \int p(y|z)\, p^{n-1}(z|x)\, dz.$$

So for example

$$p^2(y|x) = \int p(y|z)\, p(z|x)\, dz.$$

If $q_0(y)$ is the starting density for X_0, the distribution of X_1 is given by

$$q_1(y) = \int p(y|x)\, q_0(x)\, dx$$

and in general it is

$$q_n(y) = \int p^n(y|x)\, q_0(x)\, dx.$$

The stationary density for p satisfies

$$f(y) = \int p(y|x)\, f(x)\, dx$$

and the question is whether q_n converges to f and in what sense.

Under certain conditions, which we will go through in class, we have

$$\int |p^n(y|x) - f(y)|\, dy \to 0$$

for any initial choice of x.

There are also conditions under which

$$\hat{I}_N = N^{-1} \sum_{n=1}^{N} l(X_n) \to I = \int l(x)\, f(x)\, dx$$

and

$$\sqrt{N}(\hat{I}_N - I) \to N\Big(0, \sigma^2\Big)$$

where

$$\sigma^2 = \operatorname{Var} g(X_1) + 2 \sum_{i=1}^{\infty} \operatorname{Cov}(X_1, X_i)$$

with $X_1 \sim f$.