Lecture Summary 9

In this lecture we will look at some examples of Bayesian models which will need Markov chain methods in order to do inference. This will mean that we want to sample, using a Markov chain, the posterior distribution $f(\theta|\text{data})$.
1. If

$$k(x|\theta) = ax^{a-1}be^{-bx^a}, \quad x > 0$$

with $\theta = (a, b)$, then

$$f(\theta|x_1, \ldots, x_n) \propto \pi(\theta)\,(ab)^n \left(\prod_{i=1}^n x_i\right)^a \exp\left\{-b\sum_{i=1}^n x_i^a\right\}$$

which has a complicated form. Here $\pi(\theta)$ is the prior density for $\theta$.
2. If

$$k(x|\theta) = \sum_{j=1}^\infty w_j\, N(x|\mu_j, \sigma^2),$$

so $\theta = (w, \mu, \sigma^2)$, then $\theta$ is infnite dimensional.
3. If observations arrive according to

$$y_{ij} = \eta(\beta_i, x_{ij}) + \sigma\,\varepsilon_{ij}$$

for a nonlinear function $\eta$, and the $\beta_i$ are i.i.d. $N(\mu, \Sigma)$, then the density for the observations is intractable. Here $\theta = (\sigma, \mu, \Sigma)$.
4. If

$$k(x|\theta) = \sum_{j=1}^m w_{jm}\, N(\mu_j, \sigma^2)$$

where a prior distribution is assigned to $m$, then $\theta = (m, w_m, \mu, \sigma)$ and the issue here is that the dimension of $\theta$ is not fixed. Here $\sum_{j=1:m} w_{jm} = 1$.
5. If observations follow

$$y_i = \eta(x_i, \beta) + \sigma\varepsilon_i \quad \text{and} \quad x_i = \rho x_{i-1} + \tau\delta_i,$$

where the $(x_i)$ are unobserved, the density for the observations is highly intractable.
6. The density model is

$$k(x|\theta) = \frac{h(x, \theta)}{\int h(x, \theta)\, dx}$$

and the denominator is intractable; i.e. the integral is not computable directly.