Lecture Summary 19

Now consider the model

$$k(x|\theta, M) = \sum_{j=1}^{M} w_{j,M}\, N(x|\mu_j, \sigma^2)$$

and write $w^{(M)} = (w_{1,M}, \ldots, w_{M,M})$. In addition here, as now we also need to move weights from one model to another, we need to include

$$f(w^{(M)}) \times \prod_{l=1}^{M-1} f(w^{(l)}|w^{(l+1)}) \times \prod_{j=M+1}^{\infty} f(w^{(l)}|w^{(l-1)}).$$

Assume we are at $(M, \mu_1, \ldots, \mu_M, w^{(M)})$ and in anticipation of moving either to $M+1$ or $M-1$ we sample $w^{(M+1)}$ from $f(w^{(M+1)}|w^{(M)})$ and $w^{(M-1)}$ from $f(w^{(M-1)}|w^{(M)})$. We also sample μ_{M+1} from $f(\mu_{M+1}|\mu_M)$. Note that these $f(\cdot|\cdot)$ can be anything, but trying to be good proposals as to where the correct values might be.

Recall that the posterior $f(M, \mu_1, \ldots, \mu_M, w^{(M)}, \lambda|\text{data})$ is given by

$$f(\lambda)\, f(M)\, f(\mu_1, \ldots, \mu_M)\, f(w^{(M)}) \prod_{i=1}^{n} \sum_{j=1}^{M} w_{j,M}\, N(x_i|\mu_j, \sigma^2).$$

If we propose the move to $M+1$ then the acceptance probability of the move involves

$$\frac{f(M+1, \mu_1, \ldots, \mu_{M+1}, w^{(M+1)}, \lambda)\, f(w^{(M)}|w^{(M+1)})}{f(M, \mu_1, \ldots, \mu_M, w^{(M)}, \lambda)\, f(\mu_{M+1}|\mu_M)\, f(w^{(M+1)}|w^{(M)})}.$$

The $f(\cdot|\cdot)$ can be as follows: split w_j, where j is chosen at random, into two parts; ww_j and $(1-w)w_j$ with w coming from some density $f(w)$ such as a beta. For the extra component we sample μ_{M+1} from $f(\mu_{M+1}|\mu_M)$. Then

$$f(w^{(M+1)}|w^{(M)}) = \frac{1}{M} \times f(w).$$

If we propse the move to $M-1$ then a possible choice is given by: combine w_j and w_k, for $j \neq k$, together. The mean for this component can be $\frac{1}{2}\mu_j + \frac{1}{2}\mu_k$, for example. Hence

$$f(w^{(M-1)}|w^{(M)}) = \frac{1}{M} \times \frac{1}{M-1}.$$

For this move the acceptance probability involves

$$\frac{f(M-1, \mu_1, \ldots, \mu_{M-1}, w^{(M-1)}, \lambda)\, f(w^{(M)}|w^{(M-1)})\, f(\mu_M|\mu_{M-1})}{f(M, \mu_1, \ldots, \mu_M, w^{(M)}, \lambda)\, f(w^{(M-1)}|w^{(M)})}.$$