Lab05-Linear Programming

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2019.

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1. A company intends to invest 0.3 million dollars in 2018, with a proper combination of the following 3 projects:

   - **Project 1:** Invest at the beginning of a year, and can receive a 20% profit of the investment in this project at the end of this year. Both the capital and profit can be invested at the beginning of next year;
   - **Project 2:** Invest at the beginning of 2018, and can receive a 50% profit of the investment in this project at the end of 2019. The investment in this project cannot exceed 0.15 million dollars;
   - **Project 3:** Invest at the beginning of 2019, and can receive a 40% profit of the investment in this project at the end of 2019. The investment in this project cannot exceed 0.1 million dollars.

   Assume that the company will invest *all* its money at the beginning of a year. Please design a scheme of investment in 2018 and 2019 which maximizes the overall sum of capital and profit at the end of 2019.

   (a) Formulate a linear programming with necessary explanations.

   (b) Transform your LP into its standard form and slack form.

   (c) Transform your LP into its dual form.

   (d) Use the simplex method to solve your LP by step.

Solution.

(a) Assume that at the beginning of 2018, We invest x_1 (million dollars) in **Project 1**, y in **Project 2**. While at the beginning of 2019, investing x_2 (million dollars) in **Project 1** and z in **Project 3**.
In this case, we can give required constraint equations as follows:

$$\max\ f(x_1, y, z_1, x_2, z_2) = 1.5y + 1.2x_2 + 1.4z$$

s.t.

$$x_1 + y = 0.3, \tag{1}$$

$$y \leq 0.15, \tag{2}$$

$$x_2 + z = 1.2x_1, \tag{3}$$

$$z \leq 0.1, \tag{4}$$

$$x_1, y, x_2, z \geq 0. \tag{5}$$

Explaination: Equation (1) and equation (3) means we will use all the money to invest at the beginning of every year. Equation (2),(4) and (5) is the original constrain.

(b) **Standard Form:**

$$\max\ f(x_1, y, z_1, x_2, z_2) = 1.5y + 1.2x_2 + 1.4z$$

s.t.

$$x_1 + y \leq 0.3,$$
$$-x_1 - y \leq -0.3,$$
$$y \leq 0.15,$$
$$x_2 + z - 1.2x_1 \leq 0,$$
$$-x_2 - z + 1.2x_1 \leq 0,$$
$$z \leq 0.1,$$
$$x_1, y, x_2, z \geq 0.$$

Slack Form:

$$\max\ f(x_1, y, z_1, x_2, z_2, m, n, p, q, w, s) = 1.5y + 1.2x_2 + 1.4z$$

s.t.

$$0.3 - x_1 - y = m,$$
$$-0.3 + x_1 + y = n,$$
$$0.15 - y = p,$$
$$-x_2 - z + 1.2x_1 = q,$$
$$x_2 + z - 1.2x_1 = w,$$
$$0.1 - z = s,$$
$$x_1, y, x_2, z, m, n, p, q, w, s \geq 0.$$

(c) **Dual Form:**
Multiplier can refers to Table.1, So we can get dual form as follows:

$$\max\ f(k_1, k_2, k_3, k_4, k_5, k_6) = 0.3k_1 - 0.3k_2 + 0.15k_3 + 0.1k_6$$

s.t.

$$k_1 - k_2 + k_3 \leq 1.5,$$
$$k_4 - k_5 \leq 1.2,$$
$$k_4 - k_5 + k_6 \leq 1.4,$$
$$k_1, k_2, k_3, k_4, k_5, k_6 \geq 0.$$

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Multiplier	Constraint Equation (Standard Form)
k_1	$x_1 + y \leq 0.3$
k_2	$-x_1 - y \leq -0.3$
k_3	$y \leq 0.15$
k_4	$x_2 + z - 1.2x_1 \leq 0$
k_5	$-x_2 - z + 1.2x_1 \leq 0$
k_6	$z \leq 0.1$

Figure 1: Dual form multiplier

(d) **Step 1:**
Base on the slack form:

$$\max\ f(x_1, y, z_1, x_2, z_2, m, n, p, q, w, s) = 1.5y + 1.2x_2 + 1.4z$$

s.t.

$$0.3 - x_1 - y = m,$$
$$-0.3 + x_1 + y = n,$$
$$0.15 - y = p,$$
$$-x_2 - z + 1.2x_1 = q,$$
$$x_2 + z - 1.2x_1 = w,$$
$$0.1 - z = s,$$
$$x_1, y, x_2, z, m, n, p, q, w, s \geq 0.$$

Then we can get the basic solution:

$$(x_1, y, x_2, z, m, n, p, q, w, s) = (0, 0, 0, 0, 0.3, -0.3, 0.15, 0, 0, 0.1)$$

Step 2:
We can use the tightest constrain $-z + 1.2x_1 - q = x_2$ to replace x_2 to get a new basic solution as follows:

$$\max\ f(x_1, y, z_1, x_2, z_2, m, n, p, q, w, s) = 1.44x_1 + 1.5y + 1.2(-q) + 0.4z$$

s.t.

$$0.3 - x_1 - y = m,$$
$$-0.3 + x_1 + y = n,$$
$$0.15 - y = p,$$
$$-z + 1.2x_1 - q = x_2,$$
$$x_2 + z - 1.2x_1 = w,$$
$$0.1 - z = s,$$
$$x_1, y, x_2, z, m, n, p, q, w, s \geq 0.$$

Then we can get the basic solution:

$$(x_1, y, x_2, z, m, n, p, q, w, s) = (0, 0, 0, 0, 0.3, -0.3, 0.15, 0, 0, 0.1)$$

Step 3:
We can use the tightest constrain $0.3 - m - y = x_1$ to replace x_1 to get a new basic solution as follows:

$$\max\ f(x_1, y, z_1, x_2, z_2, m, n, p, q, w, s) = 1.44(0.3 - m) + 0.06y + 1.2(-q) + 0.4z$$

s.t.

$$0.3 - m - y = x_1,$$
$$-0.3 + x_1 + y = n,$$
$$0.15 - y = p,$$
$$-z + 1.2x_1 - q = x_2,$$
$$x_2 + z - 1.2x_1 = w,$$
$$0.1 - z = s,$$
$$x_1, y, x_2, z, m, n, p, q, w, s \geq 0.$$

Then we can get the basic solution:

$$(x_1, y, x_2, z, m, n, p, q, w, s) = (0.3, 0, 0.36, 0, 0, 0, 0.15, 0, -0.36, 0.1)$$

Step 4:
We can use the tightest constrain $0.15 - p = y$ to replace y to get a new basic solution as follows:

$$\max\ f(x_1, y, z_1, x_2, z_2, m, n, p, q, w, s) = 1.44(0.3 - m) + 0.06(0.15 - p) + 1.2(-q) + 0.4z$$

s.t.

$$0.3 - m - y = x_1,$$
$$-0.3 + x_1 + y = n,$$
$$0.15 - p = y,$$
$$-z + 1.2x_1 - q = x_2,$$
$$x_2 + z - 1.2x_1 = w,$$
$$0.1 - z = s,$$
$$x_1, y, x_2, z, m, n, p, q, w, s \geq 0.$$

Then we can get the basic solution:

$$(x_1, y, x_2, z, m, n, p, q, w, s) = (0.15, 0.15, 0.18, 0, 0, 0, 0, 0, 0, 0.1)$$

Step 4:
We can use the tightest constrain $0.1 - s = z$ to replace z to get a new basic solution as follows:

$$\max\ f(x_1, y, z_1, x_2, z_2, m, n, p, q, w, s) = 1.44(0.3-m)+0.06(0.15-p)+1.2(-q)+0.4(0.1-s)$$

s.t.

$$0.3 - m - y = x_1,$$
$$-0.3 + x_1 + y = n,$$

$$
\begin{aligned}
0.15 - p &= y, \\
-z + 1.2x_1 - q &= x_2, \\
x_2 + z - 1.2x_1 &= w, \\
0.1 - s &= z, \\
x_1, y, x_2, z, m, n, p, q, w, s &\geq 0.
\end{aligned}
$$

Then we can get the basic solution:

$$(x_1, y, x_2, z, m, n, p, q, w, s) = (0.15, 0.15, 0.08, 0.1, 0, 0, 0, 0, 0, 0)$$

Conclusion: Now we can find that the optimal solution for original problem is:

$$(x_1, y, x_2, z) = (0.15, 0.15, 0.08, 0.1)$$

it means that, at the beginning of 2018, we invest **Project 1** 0.15 million dollars while **Project 2** 0.15 million dollars. At the end of 2018, we can get total 0.18 million dollars. Then at the beginning of 2019, we invest **Project 1** 0.08 million dollars while **Project 3** 0.1 million dollars.
We can use the optimal solution to get the maximal profit

$$f(x_1, y, z_1, x_2, z_2) = 1.5y + 1.2x_2 + 1.4z = 1.5 * 0.15 + 1.2 * 0.08 + 1.4 * 0.1 = 0.461$$

(scale: *million dollars*).

2. An engineering factory makes seven products (PROD 1 to PROD 7) on the following machines: four grinders, two vertical drills, three horizontal drills, one borer and one planer. Each product yields a certain contribution to profit (in £/unit). These quantities (in £/unit) together with the unit production times (hours) required on each process are given below. A dash indicates that a product does not require a process.

	PROD 1	**PROD 2**	**PROD 3**	**PROD 4**	**PROD 5**	**PROD 6**	**PROD 7**
Contribution to profit	10	6	8	4	11	9	3
Grinding	0.5	0.7	-	-	0.3	0.2	0.5
Vertical drilling	0.1	0.2	-	0.3	-	0.6	-
Horizontal drilling	0.2	-	0.8	-	-	-	0.6
Boring	0.05	0.03	-	0.07	0.1	-	0.08
Planing	-	-	0.01	-	0.05	-	0.05

There are marketing limitations on each product in each month, given in the following table:

	PROD 1	**PROD 2**	**PROD 3**	**PROD 4**	**PROD 5**	**PROD 6**	**PROD 7**
January	500	1000	300	300	800	200	100
February	600	500	200	0	400	300	150
March	300	600	0	0	500	400	100
April	200	300	400	500	200	0	100
May	0	100	500	100	1000	300	0
June	500	500	100	300	1100	500	60

It is possible to store up to 100 of each product at a time at a cost of £0.5 per unit per month (charged at the end of each month according to the amount held at that time). There are no stocks at present, but it is desired to have a stock of exactly 50 of each type of product at the end of June. The factory works six days a week with two shifts of 8h each day. It may be assumed that each month consists of only 24 working days. Each machine must be down for maintenance in one month of the six. No sequencing problems need to be considered.

When and what should the factory make in order to maximize the total net profit?

(a) Use *CPLEX Optimization Studio* to solve this problem. Describe your model in *Optimization Programming Language* (OPL). Remember to use a separate data file (.dat) rather than embedding the data into the model file (.mod).

(b) Solve your model and give the following results.

  i. For each machine:

    A. the month for maintenance.

  ii. For each product:

    A. The amount to make in each month.

    B. The amount to sell in each month.

    C. The amount to hold at the end of each month.

  iii. The total selling profit.

  iv. The total holding cost.

  v. The total net profit (selling profit minus holding cost).

Solution.

(a) **production.mod** and **production.dat** is attached in **.zip** file, which is tested in Mac OS X available.

(b) We can use Cplex to get optimal solution, which can be give by follows:

  – The following chart represent repairing month for all types of machines:

	grinder	vertDrill	horiDrill	borer	planer
January	0	0	1	0	0
February	1	1	0	0	0
March	0	0	2	0	0
April	3	1	0	1	1
May	0	0	0	0	0
June	0	0	0	0	0

Figure 2: maintenance table

  – All arangement can be described by a chart attached in the last page:

  – In Cplex solver, we can easily get this Three variables:
  The total selling profit: 109330 £
  The total holding cost: 475 £
  The total net profit: 108855 £

	Manufacture	Sell	Hold
January	500 PROD1	500 PROD1	0 PROD1
	1000 PROD2	1000 PROD2	0 PROD2
	300 PROD3	300 PROD3	0 PROD3
	300 PROD4	300 PROD4	0 PROD4
	800 PROD5	8000 PROD5	0 PROD5
	200 PROD6	200 PROD6	0 PROD6
	100 PROD7	100 PROD7	0 PROD7
February	600 PROD1	600 PROD1	0 PROD1
	500 PROD2	500 PROD2	0 PROD2
	200 PROD3	200 PROD3	0 PROD3
	0 PROD4	0 PROD4	0 PROD4
	400 PROD5	400 PROD5	0 PROD5
	300 PROD6	300 PROD6	0 PROD6
	150 PROD7	150 PROD7	0 PROD7
March	400 PROD1	300 PROD1	100 PROD1
	700 PROD2	600 PROD2	100 PROD2
	100 PROD3	0 PROD3	100 PROD3
	100 PROD4	500 PROD4	100 PROD4
	600 PROD5	400 PROD5	100 PROD5
	400 PROD6	100 PROD6	0 PROD6
	200 PROD7	100 PROD7	100 PROD7
April	0 PROD1	100 PROD1	0 PROD1
	0 PROD2	100 PROD2	0 PROD2
	0 PROD3	100 PROD3	0 PROD3
	0 PROD4	100 PROD4	0 PROD4
	0 PROD5	100 PROD5	0 PROD5
	0 PROD6	0 PROD6	0 PROD6
	0 PROD7	100 PROD7	0 PROD7
May	0 PROD1	0 PROD1	0 PROD1
	100 PROD2	100 PROD2	0 PROD2
	500 PROD3	500 PROD3	0 PROD3
	100 PROD4	100 PROD4	0 PROD4
	1000 PROD5	1000 PROD5	0 PROD5
	300 PROD6	300 PROD6	0 PROD6
	0 PROD7	0 PROD7	0 PROD7
June	550 PROD1	500 PROD1	50 PROD1
	550 PROD2	500 PROD2	50 PROD2
	150 PROD3	100 PROD3	50 PROD3
	350 PROD4	300 PROD4	50 PROD4
	1150 PROD5	1100 PROD5	50 PROD5
	550 PROD6	500 PROD6	50 PROD6
	110 PROD7	60 PROD7	50 PROD7

Figure 3: maintenance table