Lab00-Proof

CS214-Algorithm and Complexity, Xiaofeng Gao, Spring 2019.

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1. Prove that for any integer $n > 2$, there is a prime p satisfying $n < p < n!$. (Hint: consider a prime factor p of $n! - 1$ and prove by contradiction)

   Proof. As for $n! - 1$, it follows that $n < n! - 1 < n!$.

   If $n! - 1$ is a prime number, the original statement is true.

   If $n! - 1$ is not a prime number, we can suppose p is a prime number of $n! - 1$. Then, we assume $p \leq n$, so p can be a factor of $n!$, but $n!$ and $n! - 1$ don't have any matual factor more than 1. The contradiction negate the assumption that $p \leq n$, so $n < p < n!$.

   Above all, the original statement is true. □

2. Use the minimal counterexample principle to prove that for any integer $n > 17$, there exist integers $i_n \geq 0$ and $j_n \geq 0$, such that $n = i_n \times 4 + j_n \times 7$.

   Proof. If the statement is not true for every $n > 17$, there must be a smallest such false value, say $n = k$.

   Since n=18, $n = 1 \times 4 + 2 \times 7$,which $i_0 = 1$ and $j_0 = 2$.

   Since n=k is the smallest false one, so $k - 1 = i_k \times 4 + j_k \times 7$, in which $i_k \geq i_0 = 1$ and $j_k \geq j_0 = 2$.

   However, $k = (k-1) + 1 = (i_k + 1) \times 4 + (j_k - 1) \times 7$, so we can derived a contradiction,which allows us to conclude that our original assumption is false. □

3. Suppose $a_0 = 1$, $a_1 = 2$, $a_2 = 3$, and $a_k = a_{k-1} + a_{k-2} + a_{k-3}$ for $k \geq 3$. Use the strong principle of mathematical induction to prove that $a_n \leq 2^n$ for any integer $n \geq 0$.

   Proof. As for k=3, $a_3 = a_0 + a_1 + a_2 = 6 < 2^3 = 8$,the original statement is true.

   We can assume that for $3 \leq k \leq n - 1$, the statement is true.

   As for k=n, $a_n = a_{n-1} + a_{n-2} + a_{n-3} \leq 2^{n-1} + 2^{n-2} + 2^{n-3} = 7 \times 2^{n-3} < 2^n$ Above all, the original statement is true. □

4. Prove, by mathematical induction, that

$$(n+1)^2 + (n+2)^2 + (n+3)^2 + \cdots + (2n)^2 = \frac{n(2n+1)(7n+1)}{6}$$

   is true for any integer $n \geq 1$.

   Proof. For $n = 1$, the equation clearly holds.

   Suppose the equation holds when $n = k$, it means $(k+1)^2 + (k+2)^2 + (k+3)^2 + \cdots + (2k)^2 = \dfrac{k(2k+1)(7k+1)}{6}$.

As for $n = k+1$, $(k+2)^2 + (k+3)^2 + (k+4)^2 + \cdots + (2(k+1))^2$

$$= \frac{k(2k+1)(7k+1)}{6} + (2k+1)^2 + (2k+2)^2 - (k+1)^2$$

$$= \frac{(k+1)(2k+3)(7k+8)}{6}.$$

Above, for any integer $n \geq 1$, the original statement is true. $\square$

Remark: You need to include your .pdf and .tex files in your uploaded .rar or .zip file.