Homework 9

Problem 1. *Explain how the volume of a ball in high dimensions can simultaneously be in a narrow slice at the equator and also be concentrated in a narrow annulus at the surface of the ball.*

Solution. We know that no matter what distribution we use, the s scattering points will near the surface because, if we build the n-dimension ball with radius $r = 1$, for its volume

$$V \propto r^n\ ,\ \forall x < r, \lim_{n\to\infty} V' = 0$$

Therefore, we just need to let the scattering points locate at the equator. We know that once $x \sim N(0,1)$ random d-dimensional x_1, x_2 are approximately orthogonal. So we can let x_1 be pole, then x_2 locates at narrow slice at the equator.

Problem 2. *Find the singular value decomposition of the following matrix*

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}.$$

Solution. According to SVD, we can get

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \times \begin{pmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{\sqrt{12}} - \frac{1}{\sqrt{12}} & \frac{\sqrt{3}}{2} + \frac{1}{2} & \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{6}} \\ \frac{\sqrt{-3}}{\sqrt{12}} - \frac{1}{\sqrt{12}} & \frac{\sqrt{3}}{-2} + \frac{1}{2} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}} \end{pmatrix}$$