Homework 2

517030910155

Problem 1. *Prove the formula*

$$\left\{ {r \atop r} \right\} + \left\{ {r+1 \atop r} \right\} + \left\{ {r+2 \atop r} \right\} + \cdots + \left\{ {n \atop r} \right\} = \left\{ {n+1 \atop r+1} \right\}$$

Solution.

1.

Problem 2. *Let* $(X, \preccurlyeq_1), (Y, \preccurlyeq_2)$ *be (partially) ordered sets. We say that they are isomorphic if there exists a bijection* $f : X \to Y$ *such that for every* x, $y \in X$, *we have* $x \preccurlyeq_1 y$ *if and only if* $f(x) \preccurlyeq_2 f(y)$.

Solution.

Problem 3. *Prove or disprove: If a partially ordered set* $(X, \preccurlyeq)$ *has a single minimal element, then it is a smallest element as well.*

Solution.

Problem 4. *Let* $(X, \preccurlyeq)$ *and* $(X\prime, \preccurlyeq\prime)$ *be partially ordered sets. A mapping* $f : X \to X\prime$ *is called an embedding of* $(X, \preccurlyeq)$ *into* $(X\prime, \preccurlyeq\prime)$ *if the following conditions hold:*

- f *is an injective mapping;*
- $f(x) \preccurlyeq\prime f(y)$ *if and only if* $x \preccurlyeq y$.

Now consider the following problem:

a) *Describe an embedding of the set* $\{1, 2\} \times \mathbb{N}$ *with the lexicographic ordering into the ordered set* $(\mathbb{Q}, \leq)$.

b) *Solve the analog of a) with the set* $N \times N$ *(ordered lexicographically) instead of* $\{1, 2\} \times N$.

Solution.

Problem 5. *Prove the following strengthening of the* ***Erdos-Szekeres Lemma****: Let $\mathscr{K}$, $\mathscr{L}$ be natural numbers. Then every sequence of real numbers of length $\mathscr{K}\mathscr{L} + 1$ contains an nondecreasing subsequence of length $\mathscr{K} + 1$ or a decreasing subsequence of length $\mathscr{L} + 1$.*

Proof.