Homework 2

517030910155

Problem 1. *Prove the formula*

$$\left\{ {r \atop r} \right\} + \left\{ {r+1 \atop r} \right\} + \left\{ {r+2 \atop r} \right\} + \cdots + \left\{ {n \atop r} \right\} = \left\{ {n+1 \atop r+1} \right\}$$

Solution.

1.

Problem 2. *Let $(X, \preccurlyeq_1), (Y, \preccurlyeq_2)$ be (partially) ordered sets. We say that they are isomorphic if there exists a bijection $f : X \to Y$ such that for every x, $y \in X$, we have $x \preccurlyeq_1 y$ if and only if $f(x) \preccurlyeq_2 f(y)$.*

Solution.

Problem 3. *Prove or disprove: If a partially ordered set $(X, \preccurlyeq)$ has a single minimal element, then it is a smallest element as well.*

Solution.

Problem 4. *Let $(X, \preccurlyeq)$ and $(X\prime, \preccurlyeq\prime)$ be partially ordered sets. A mapping $f : X \to X\prime$ is called an embedding of $(X, \preccurlyeq)$ into $(X\prime, \preccurlyeq\prime)$ if the following conditions hold:*

- *f is an injective mapping;*
- *$f(x) \preccurlyeq\prime f(y)$ if and only if $x \preccurlyeq y$.*

Now consider the following problem:

a) Describe an embedding of the set $\{1, 2\} \times \mathbb{N}$ with the lexicographic ordering into the ordered set $(\mathbb{Q}, \leq)$.

b) Solve the analog of a) with the set $N \times N$ (ordered lexicographically) instead of $\{1, 2\} \times N$.

Solution.

Problem 5. *Prove the following strengthening of the* ***Erdos-Szekeres Lemma****: Let $\mathscr{K}$, $\mathscr{L}$ be natural numbers. Then every sequence of real numbers of length $\mathscr{K}\mathscr{L}+1$ contains an nondecreasing subsequence of length $\mathscr{K}+1$ or a decreasing subsequence of length $\mathscr{L}+1$.*

Proof.