Given:

- T_1 is the transformation matrix of the object with respect to the camera.
- T_2 is the transformation matrix of the robot base with respect to the camera.

a/ The rotation matrix for a $90°$ counterclockwise rotation about the z-axis is:

$$R_z(90°) = \begin{bmatrix} \cos 90° & -\sin 90° & 0 & 0 \\ \sin 90° & \cos 90° & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The new position and orientation of the camera with respect to the robot's base coordinate system:

$$T_c' = T_2 . R_z(90°) = \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & -1 & 0 & 20 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -10 \\ -1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b/ After rotated happened in a, the object is also rotated by $90°$ about the x-axis:

$$R_x(90°) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90° & -\sin 90° & 0 \\ 0 & \sin 90° & \cos 90° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The object also translated by 4 units along the rotated y-axis:

$$T_{trans} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The position and orientation of the object with respect to the robot's base coordinate system:

$$T_{obj}' = T_1 . R_x(90°) . T_{trans} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 10 \\ 0 & 0 & -1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 10 \\ 0 & -1 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 10 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since T_c' is the transformation from the camera to the robot base, so we invert it so that we could have the transformation from the robot's base frame to the rotated camera's frame.

$$T_c'^{-1} = \begin{bmatrix} 0 & -1 & 0 & 20 \\ -1 & 0 & 0 & -10 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The position and orientation of the object with respect to the rotated camera coordinate system:

$$T_{obj}'' = T_c'^{-1} . T_{obj}' = \begin{bmatrix} 0 & -1 & 0 & 20 \\ -1 & 0 & 0 & -10 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 10 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 20 \\ 0 & 0 & 1 & -11 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$