Wells' partial memory model

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28/10/2019

Ranking Probabilities: Wells partial memory model

For an n-item lineup consisting of one target and $n-1$ foils, let t be the probability that the eyewitness will detect the target (as the target) and let f be the probability that they will detect a foil (as a foil). Let $[n] = \{1, \ldots, n\}$.

Suppose for a given eyewitness that $k \in [n-1]$ foils are not detected. We assume that the target is never detected as a foil. Then the *effective lineup size* is $x = k+1$. Let $s(x)$ be the probability that the effective lineup size is x. Then

$$s(x) = \binom{n-1}{x-1}(1-f)^{x-1} f^{n-x}.$$

Suppose that the target is not detected as a target and the effective lineup size is x items. Then the target has an equal probability of being assigned a rank from 1 to x. Let $p(r)$ be the probability that the target is assigned rank $r \in [n]$, given it is not detected. Then

$$p(r) = \sum_{x=r}^{n} \frac{s(x)}{x}. \tag{1}$$

The function $p(r)$:

```
b <- function(r, n, f){
  for (i in r:n)
    x[i] <- choose(n-1, i-1)*(1-f)^(i-1)*f^(n-i)/i
  return(sum(x, na.rm = TRUE))
}

p <- function(t,n, f){
  for (i in 1:n)
    x[i] <- b(i, n, f)
  return(x)
}
```

We assume that if the target is detected it is assigned rank 1. Let $q(r)$ be the probability that a target is assigned rank $r \in [n]$. Then

$$q(r) : \begin{cases} t + (1-t)p(r), & r = 1 \\ (1-t)p(r), & r > 1 \end{cases} \tag{2}$$

The function $q(r)$:

```
qr <- function(t,n,f){
  x[1] <- t + (1-t)*(b(1, n, f))
  for (i in 2:n){
    x[i] <- b(i, n, f)*(1-t)
  }
```

```
    return(x)
}
```

Conditional Ranking Probabilities

Let $Q(r)$ be the cumulative sum,

$$Q(r) = \sum_{i=1}^{r} q(i). \tag{3}$$

Let $c(r)$ be the conditional probability that the target is assigned rank r given that it has not been assigned any rank less than r. Then

$$c(r) : \begin{cases} q(r), \; r = 1 \\ \dfrac{q(r)}{(1 - Q(r-1))}, \; r > 1 \end{cases} \tag{4}$$

Theorem

We want to show that $c(2) < c(3)$. Suppose $c(2) < c(3)$. I thought I would try a brute force approach (although unsuccessfully). From Equations (3) and (4),

$$\frac{q(2)}{1 - q(1)} < \frac{q(3)}{1 - q(1) - q(2)}$$

From Equation (4),

$$\frac{(1-t)p(2)}{1 - t - (1-t)p(1)} < \frac{(1-t)q(3)}{1 - t - (1-t)p(1) - (1-t)p(2)}$$

For $t < 1$,

$$\frac{p(2)}{1 - p(1)} < \frac{p(3)}{1 - p(1) - p(2)}$$

If the theorem is true, then $c(2)/c(3)$ is < 1 for all values of n and f.

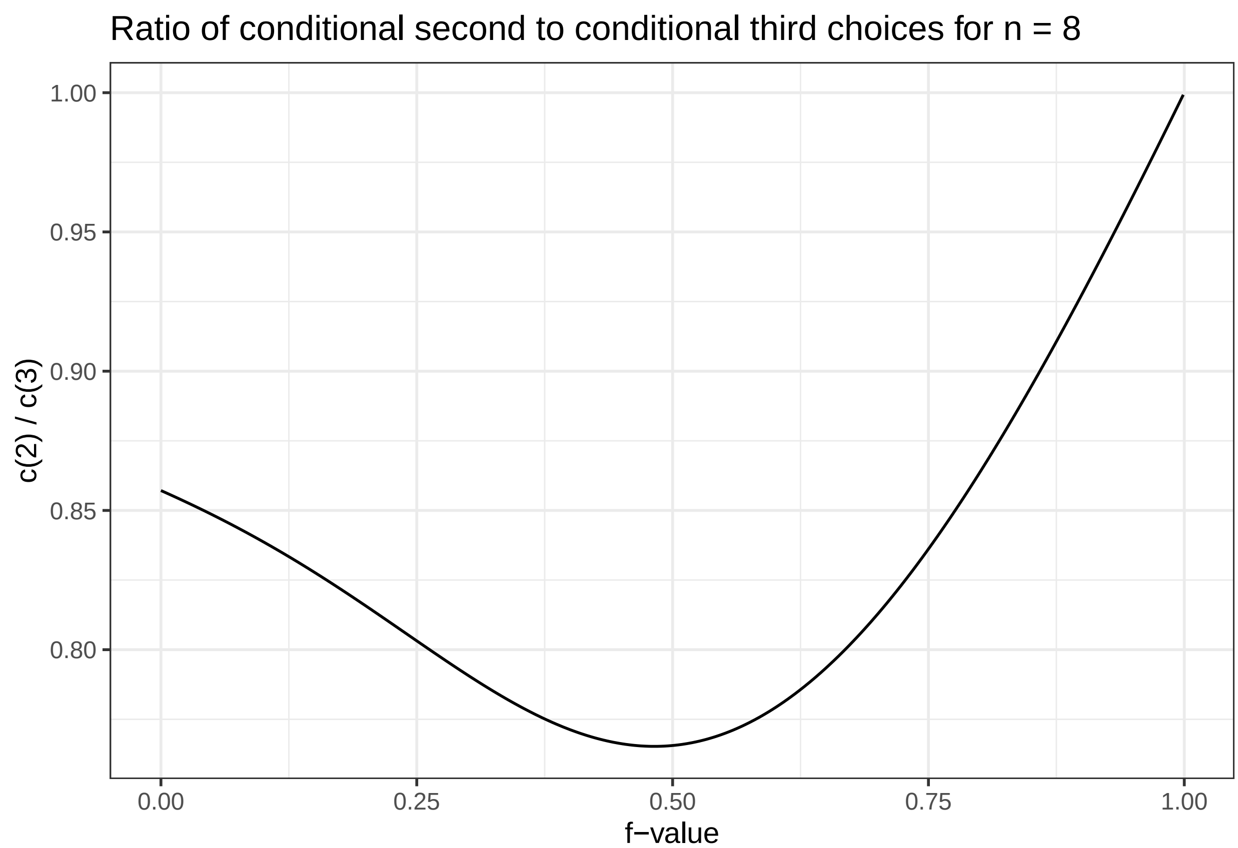
Demonstration of theorem

The function $c(2)/c(3)$:

```
c2c3ratio <- function(t,n, f){
  x <- qr(t,n,f)
  x[n+1] <- (x[2]/(1-x[1]))/(x[3]/(1-x[1]-x[2]))
  return(x[n+1])
}
```

Ranking probability: Thurstone's rank orker model

Draw the function $c(2)/c(3)$ for $n = 8$:



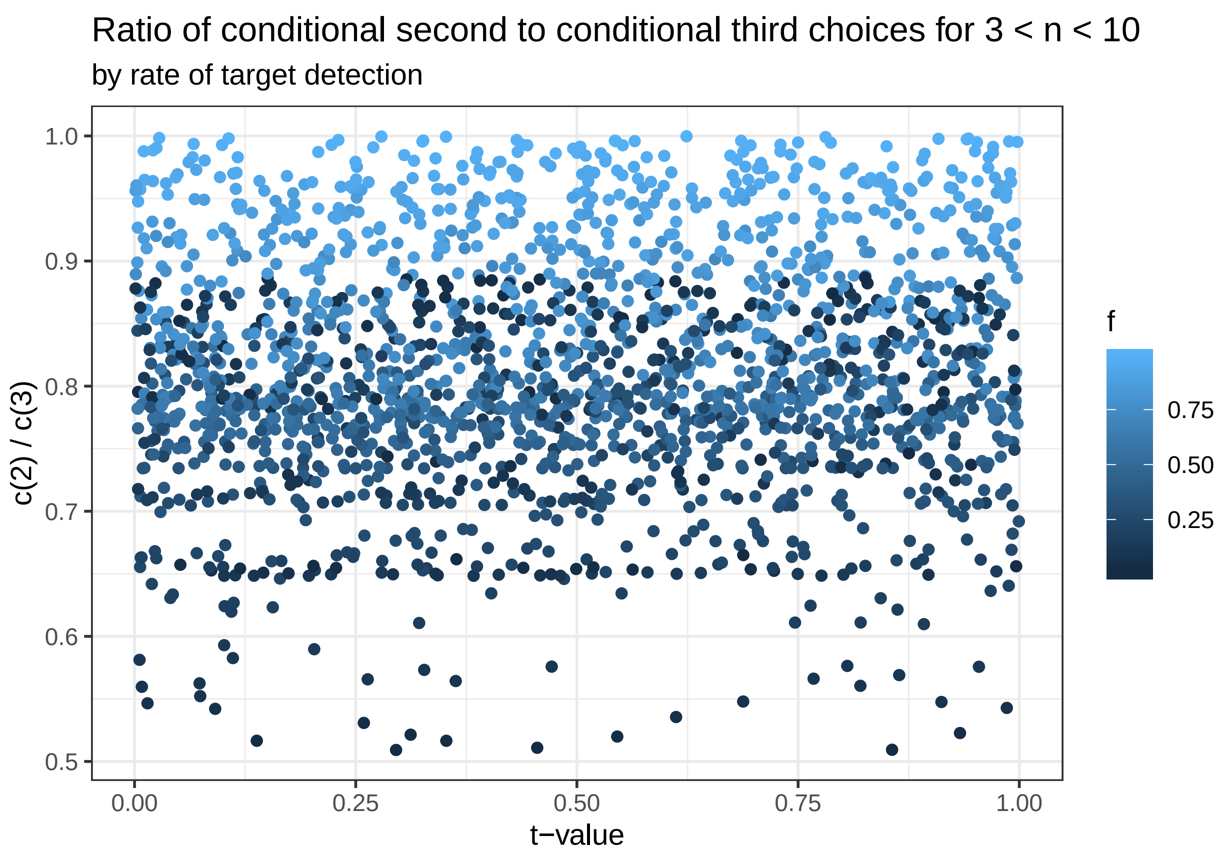

```
minimum <- optimise(f=c2c3ratio, c(0,1), maximum = FALSE, n=n, t=0)
```

The minimum value of $c(2)/c(3)$ for $n = 8$ is 0.7652849, which occurs at $f = 0.4821476$

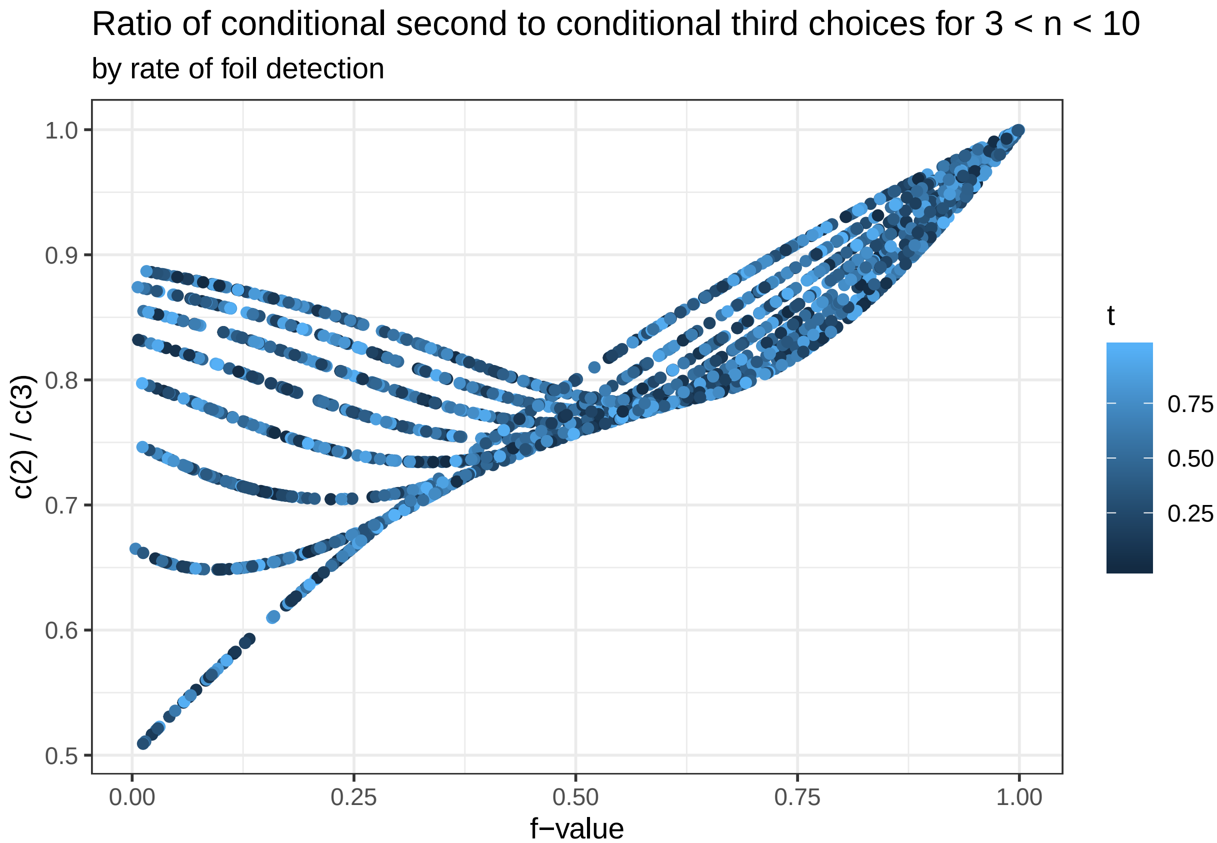
Monte Carlo analysis of $c(2)/c(3)$ for $n \in \{3, ..., 10\}$, all values of t and f.

```
# Set number of interations required
h <- 2000

# Create vectors of uniformly random values for each variable
foil <- runif(h)
lineup_size <- floor(runif(h, min = 3, max = 11))
target <- runif(h, min =0, max = 1)

# Create an empty vector for the Monte Carlo generated numbers
MC <- vector()

# Run the Monte Carlo simulation

 for (i in 1:length(lineup_size)) {
    MC[i] <- c2c3ratio(target[i],lineup_size[i],foil[i])
}

# Build data frame of simulaiton values for ggplot
MC <- as.data.frame(MC)
MC$n <- lineup_size
```

```
MC$f <- foil
MC$t <- target

# Plot simulation values by values of t
ggplot(MC,aes(x=t, y=MC))+
  geom_point(aes(color=f))+
  labs(title="Ratio of conditional second to conditional third choices for 3 < n < 10",
       subtitle = "by rate of target detection",
       x="t-value",
       y="c(2) / c(3)"
       ) +
  theme_bw()
```




```
# Plot simulation values by values of f
ggplot(MC,aes(x=f, y=MC))+
  geom_point(aes(color=t))+
  labs(title="Ratio of conditional second to conditional third choices for 3 < n < 10",
        subtitle = "by rate of foil detection",
       x="f-value",
       y="c(2) / c(3)"
       ) +
  theme_bw()
```


Ratio of conditional second to conditional third choices for 3 < n < 10
by rate of foil detection
1.0
0.9
0.8
0.7
0.6
0.5
c(2) / c(3)
0.00
0.25
0.50
0.75
1.00
f−value
t
0.75
0.50
0.25