Т4

$$p(x) = \frac{\theta}{2}\{(-1,1)/\{0\}\} + \frac{1-\theta}{2}\{0\} + \frac{1-\theta}{2}\{2\}$$

$\theta \in (0,1)$

ОММ:

$$\alpha_1 = M\xi = \int_{-1}^{1} x\frac{\theta}{2}dx + 2\frac{1-\theta}{2} = 1-\theta$$

$$\alpha_2 = \int_{-1}^{1} x^2\frac{\theta}{2}dx + 4\frac{1-\theta}{2} = \frac{2}{3}\cdot\frac{\theta}{2} + 2 -$$

$$-2\theta = 2 - \frac{5}{3}\theta$$

$$D\xi = M\xi^2 - (M\xi)^2 = 2 - \frac{5}{3}\theta - 1 + 2\theta - \theta^2 =$$

$$= 1 + \frac{\theta}{3} - \theta^2$$

$$1 - \theta = \alpha_1 = \hat{\alpha}_1 = \overline{x}$$

$$\hat{\theta}_1 = 1 - \overline{x}$$

1) несм.:

$$M[\theta_1^2] = M[1-\bar{x}] = 1 - M[\xi] = \theta \Rightarrow$$

$\Rightarrow$ несм.

2) сост.:

$$D[\theta_1^2] = D[\bar{x}] = \frac{1}{n} D\xi = \frac{1}{n}\left(1 + \frac{\theta}{3} - \theta^2\right) \xrightarrow[n\to\infty]{} 0$$

$\Rightarrow$ по дост. усл. сост.

3) регулярн.:

рег. модели:

а) $p_1(x,\theta)$ и $p_k(\theta)$ — непр. диф.

б) $$\int_{-1}^{1} \frac{\partial}{\partial\theta}\left(\frac{\theta}{2}\right) dx + \frac{\partial}{\partial\theta}(1-\theta) =$$

$$= 1 - 1 = 0$$

в) $$I(\theta) = \int^{1}\left(\frac{\partial \ln\left(\frac{\theta}{2}\right)}{\partial\theta}\right)^2 \frac{\theta}{2} dx +$$

$$+ 2\left(\frac{\partial \ln\frac{1-\theta}{2}}{\partial\theta}\right)^2 \left(\frac{1-\theta}{2}\right) = \int_{-1}^{1}\left(\frac{1}{\theta}\right)^2 \cdot \frac{\theta}{2} dx +$$

$$+ 2\left(\frac{-2}{1-\theta}\right)^2\left(\frac{1-\theta}{2}\right) = \int_{-1}^{1} \frac{1}{2\theta} dx +$$

$$+ \frac{4}{1-\theta} = \frac{1}{\theta} + \frac{4}{1-\theta} > 0 \quad \forall \theta \in (0,1),$$

непр.

рег. оценки:

$\tilde{\theta}_1$ - несм.

$D[\tilde{\theta}_1]$ огр. на $\forall$ комп. из Θ

$\Rightarrow$ оценка рег. по дост. усл.

нер-во К-Р:

$$D[\tilde{\theta}_1] \geq \frac{1}{nI(\theta)} = \frac{1}{n} \cdot \frac{\theta - \theta^2}{1+3\theta}$$

$$\frac{3+\theta-3\theta^2}{3} \geq \frac{\theta-\theta^2}{1+3\theta} \Rightarrow ?$$

ОМП:

m - суммарное кол-во „0" и „2"

$$L(\theta) = \left(\frac{\theta}{2}\right)^{n-m} \left(\frac{1-\theta}{2}\right)^{m}$$

$$\ln L = (n-m)\ln\frac{\theta}{2} + m\ln\frac{1-\theta}{2}$$

$$(\ln L)'_{\theta} = (n-m)\frac{2}{\tilde{\theta}_2} + m\frac{2}{1-\tilde{\theta}_2}(-1) = 0$$

$$\frac{n-m-n\tilde{\theta}_2 \cancel{+m\tilde{\theta}_2 - m\tilde{\theta}_2}}{\tilde{\theta}_2(1-\tilde{\theta}_2)} = 0$$

$$n\tilde{\theta}_2 = n-m \qquad \tilde{\theta}_2 = 1 - \frac{m}{n} = 1-\nu$$

$$(\ln L)''_{\theta} = (n-m)\frac{-2}{\tilde{\theta}_2^2} + m\frac{-2}{(1-\tilde{\theta}_2)^2} =$$

$$= \frac{-2\cancel{(n-m)}\cdot n}{(n-m)^{\cancel{2}}} - 2m\left(\frac{n}{m}\right)^2 = -2\left(\frac{n}{(n-m)} + \frac{n^2}{m}\right) < 0 \Rightarrow max$$

несм. !

$M[\tilde{\theta}_2] = M[1-\bar{\nu}] = 1-1+\theta = \theta \Rightarrow$ несм.

сост. !

$D[\tilde{\theta}_2] = D[1-\bar{\nu}] = D\bar{\nu} = \frac{(1-\theta)\cdot\theta}{n} \xrightarrow[n\to\infty]{} 0$

$\Rightarrow$ сост.

нер-во К-Р:

оценка рег. по доб. усл.

$\frac{(1-\theta)\cdot\theta}{n} \geq \frac{\theta-\theta^2}{1+3\theta} \frac{1}{n} \Rightarrow ?$