(T5)

$\xi \sim R(\theta, 2\theta)$ $\qquad \xi \sim p(x) = \frac{1}{\theta}\{[\theta, 2\theta]\}$

а и в)

ОММ:

$$\alpha_1 = M[\xi] = \int_{\theta}^{2\theta} \frac{1}{\theta} x \, dx = 1{,}5\theta$$

$$\alpha_2 = M[\xi^2] = \int_{\theta}^{2\theta} \frac{1}{\theta} x^2 dx = \frac{8\theta^2}{3} - \frac{\theta^2}{3} = \frac{7\theta^2}{3}$$

$$D[\xi] = \alpha_2 - \alpha_1^2 = \frac{7\theta^2}{3} - \frac{9\theta^2}{4} = \frac{\theta^2}{12}$$

$$\alpha_1 = \hat{\alpha}_1 = \frac{1}{n} \sum x_i = \bar{x}$$

$$\frac{3}{2}\theta = \bar{x} \longrightarrow \tilde{\theta}_1 = \frac{2}{3}\bar{x}$$

1) несм.

$$M[\hat{\theta}_1] = M\left[\frac{2}{3}\cdot\frac{1}{n}\sum x_i\right] = \frac{2}{3}\cdot M\xi = \frac{2}{3}\cdot\frac{3}{2}\theta = \theta \Rightarrow \text{несм.}$$

2) сост.

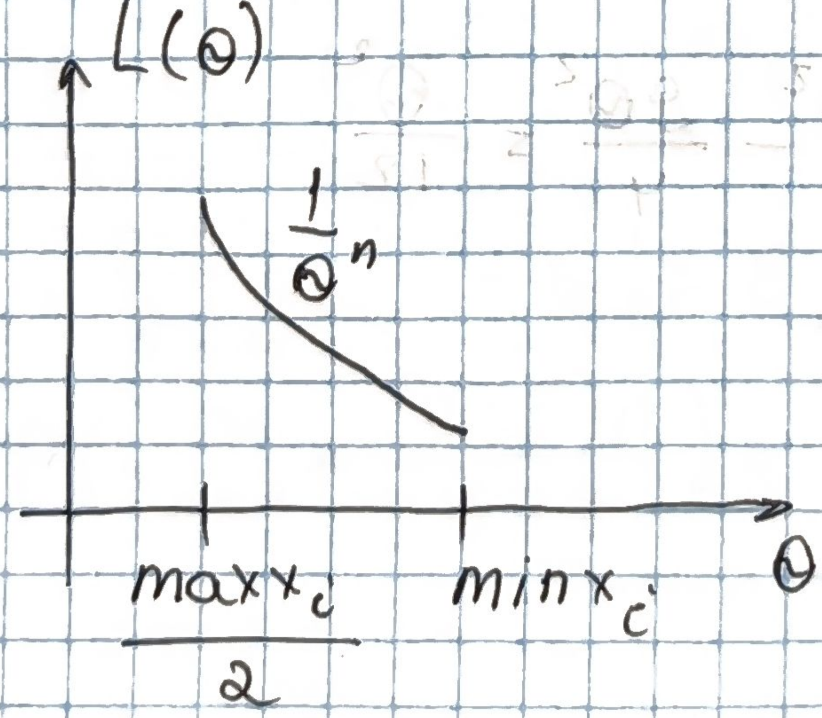
$$D[\hat{\theta}_1] = D\left[\frac{2}{3}\cdot\frac{1}{n}\sum x_i\right] = \frac{4}{9}\cdot\frac{1}{n}D\xi = \frac{4}{9n}\cdot\frac{\theta^2}{12}\xrightarrow[n\to\infty]{} 0 \Rightarrow \text{по дост. усл. сост.}$$

ОМП:

$$L(\theta) = \prod p(x_i, \theta) = \left(\frac{1}{\theta}\right)^n \{\forall i, \theta < x_i < 2\theta\} =$$

$$\cancel{\ln L = n\ln\frac{1}{\theta} \to \max_{\theta\in R}}$$

$$= \frac{1}{\theta^n}\{\max x_i < 2\theta, \ \min x_i > \theta\}$$



$$\frac{\max x_i}{2} \le \min x_i$$

$$x_i \in (\theta, 2\theta)$$

$\tilde{\Theta}_2 = \frac{x_{max}}{2}$

1) несм.

$M[\tilde{\Theta}_2] = M[\frac{1}{2} x_{max}] = \frac{1}{2}\int_{\Theta}^{2\Theta} x \cdot n\left(\frac{x}{\Theta}-1\right)^{n-1}\frac{1}{\Theta}dx =$

$= \frac{1}{2}\int_0^1 (t+1)\Theta n (t)^{n-1} dt = \frac{\Theta}{2}\left(\int_0^1 t^{n} \cdot n \cdot dt +\right.$

$\left.+\int_0^1 t^{n-1} n dt\right) = \frac{\Theta}{2}\left(\frac{n}{n+1}+1\right) = \frac{\Theta}{2}\left(\frac{2n+1}{n+1}\right) \Rightarrow$ смещ.

$\tilde{\Theta}_2' = \frac{2n+2}{2n+1} \cdot \frac{x_{max}}{2} = \frac{n+1}{2n+1} \cdot x_{max}$

2) сост.

$D[\tilde{\Theta}_2'] = \left(\frac{\cancel{2n}\ n+1}{2n+1}\right)^2 \cdot \frac{n\Theta^2}{(n+2)\cancel{(n+1)^2}} \xrightarrow[n\to\infty]{} 0 \Rightarrow$ сост.

c) $D[\tilde{\Theta}_1] = \frac{\Theta^2}{27n}$

$D[\tilde{\Theta}_2'] = \frac{n\Theta^2}{(2n+1)^2(n+2)}$

при $n=1$ и $n>3$ $\tilde{\Theta}_2'$ эффект. $\tilde{\Theta}_1$

d) $f = \frac{x_{max}}{\Theta}$

$f \sim \Phi(y) = P\left(\frac{x_{max}}{\Theta} < y\right) = P(x_{max} < \Theta y) =$

$= (F(\Theta y))^n = \left(\frac{\Theta y - \Theta}{\Theta}\right)^n = (y-1)^n \quad y \in [1, 2]$

$g(y) = \varphi'(y) = n(y-1)^{n-1}$

$t_1 = g_{\frac{1-\beta}{2}}$, $\beta = 0{,}95$ $\quad t_2 = g_{\frac{1+\beta}{2}}$

$(t_1 - 1)^n = \frac{1-\beta}{2}$ $\quad t_2 = 1 + \sqrt[n]{\frac{1+\beta}{2}}$

$t_1 = 1 + \sqrt[n]{\frac{1-\beta}{2}}$

$P\left(t_1 < \frac{x_{max}}{\theta} < t_2\right) = \beta$

$\frac{x_{max}}{t_2} < \theta < \frac{x_{max}}{t_1}$ $\quad \frac{x_{max}}{1+\sqrt[n]{0{,}975}} < \theta < \frac{x_{max}}{1+\sqrt[n]{0{,}025}}$

e) $R(\theta, 2\theta)$ – не рег. => не можем использовать ОМП

ОММ:

$g(\alpha_1) = \tilde{\theta} = \frac{2}{3}\alpha_1$ $\quad \beta = 0{,}95$ $\quad g(\alpha_1) = \theta$

$$\frac{\sqrt{n}(\tilde{\theta} - \theta)}{\sqrt{\frac{4}{9}(\alpha_2 - \alpha_1^2)}} \rightsquigarrow N(0,1)$$

$\nabla g(\alpha_1) = \frac{2}{3}$ $\quad K_{11} = \alpha_2 - \alpha_1^2$

$t_1 = u_{0{,}025} = -1{,}96$

$t_2 = u_{0{,}975} = 1{,}96$

$$t_1 < \frac{\sqrt{n}\,(\Theta^2 - \Theta)}{\sqrt{\frac{4}{9}(\bar{x}_2 - \bar{x}_1^2)}} < t_2$$

$$\frac{-t_2\sqrt{\frac{4}{9}(\bar{x}_2 - \bar{x}_1^2)}}{\sqrt{n}} + \Theta^2 < \Theta < \frac{-t_1\sqrt{\frac{4}{9}(\bar{x}_2 - \bar{x}_1^2)}}{\sqrt{n}} + \Theta^2$$

$$\frac{-1{,}96\sqrt{\frac{4}{9}(\bar{x}_2 - \bar{x}_1^2)}}{\sqrt{n}} + \frac{2}{3}\bar{x}_1 < \Theta < \frac{1{,}96\sqrt{\frac{4}{9}(\bar{x}_2 - \bar{x}_1^2)}}{\sqrt{n}} + \frac{2}{3}\bar{x}_1$$