1 Frame 29 – The Exponential Function

1(a) The value is

$$e^{2\pm 3\pi i} = e^2 e^{\pm 3\pi i} = e^2(-1) = e^2$$

1(b) The value is

$$e^{1/2+i\pi/4} = e^{1/2}e^{i\pi 4} = \sqrt{e}\frac{1+i}{\sqrt{2}} = \sqrt{\frac{e}{2}}(1+i)$$

1(c) This expression can be split up as

$$e^{z+\pi i} = e^z e^{i\pi} = e^z \cdot -1 = -e^z$$

3 The function can be written as

$$e^{\overline{z}} = e^x e^{-iy} = e^x \cos y - ie^x \sin y$$

so the partial derivatives are

$$\begin{aligned} u_x &= e^x \cos y \\ u_y &= -e^x \sin y \\ v_x &= -e^x \sin y \\ v_y &= -e^x \cos y \end{aligned}$$

These don't satisfy Cauchy-Riemann, so the function is differentiable nowhere and thus analytic nowhere.

8(a) The equation $e^z = -2$ can be written as

$$e^z = 2e^{i\pi}$$

so

$$\begin{aligned} e^x = 2 &\to x = \ln 2 \\ e^{iy} = e^{i\pi} &\to y = (2n+1)\pi \end{aligned}$$

and

$$z = \ln 2 + (2n+1)i\pi$$

8(b) The equation $e^z = 1 + i\sqrt{3}$ can be written as

$$e^z = 2e^{i\pi/3}$$

so

$$\begin{aligned} x &= \ln 2 \\ y &= \left(\frac{1}{3} + 2n\right)\pi \end{aligned}$$

and

$$z = \ln 2 + \left(\frac{1}{3} + 2n\right) i\pi$$

8(c) If

$$e^{2z-1} = 1$$

then

$$2z - 1 = i2n\pi$$

or, solving for z,

$$z = \frac{1}{2} + in\pi$$