1 Frame 38 – Derivatives and Integrals

1(b) Breaking the derivative into its complex components,

$$\begin{aligned}\frac{d}{dt}[w(t)]^2 &= \frac{d}{dt}[u(t)+iv(t)]^2\\ &= 2[u(t)+iv(t)][u(t)+iv(t)]'\\ &= 2w(t)w'(t)\end{aligned}$$

2(a) Evaluating the integral,

$$\begin{aligned}\int_1^2 \left(\frac{1}{t}-i\right)^2 dt &= \int_1^2 \frac{1}{t^2}-1-i\frac{2}{t}dt\\ &= -\frac{1}{t}-t-2i\ln t\Big|_1^2\\ &= -(\frac{1}{2}-1)-(2-1)-2i(\ln 2-0)\\ &= -\frac{1}{2}-i\ln 4\end{aligned}$$

2(b)

$$\begin{aligned}\int_0^{\pi/6} e^{i2t} &= \frac{1}{2i}e^{i2t}\Big|_0^{\pi/6}\\ &= \frac{1}{2i}(e^{i\pi/3}-1)\\ &= \frac{1}{2i}\left(i\frac{\sqrt{3}}{2}-\frac{1}{2}\right)\\ &= \frac{\sqrt{3}}{4}+i\frac{1}{4}\end{aligned}$$

2(c) Converting this improper integral into a limit,

$$\begin{aligned}\int_0^{\infty} e^{-zt} &= \lim_{L\to\infty}\int_0^L e^{-zt}\\ &= \lim_{L\to\infty} -\frac{1}{z}e^{-zt}\Big|_0^L\\ &= -\frac{1}{z}\lim_{L\to\infty} e^{-zL}-1\\ &= \frac{1}{z}\end{aligned}$$

4 Evaluating the left-side integral,

$$\begin{aligned}
\int_0^\pi e^{1+i} x dx &= \frac{1}{1+i} e^{1+i} x \Big|_0^\pi \\
&= \frac{1}{1+i}(e^{\pi+i\pi} - 1) \\
&= \frac{1-i}{2}(-e^\pi - 1) \\
&= -\frac{1}{2}(e^\pi + 1) + \frac{i}{2}(e^\pi + 1)
\end{aligned}$$