1 Frame 29 – The Exponential Function

1(a) The value is
$$e^{2\pm3\pi i} = e^2 e^{\pm3\pi i} = e^2(-1) = e^2$$

1(b) The value is
$$e^{1/2+i\pi/4} = e^{1/2}e^{i\pi4} = \sqrt{e}\frac{1+i}{\sqrt{2}} = \sqrt{\frac{e}{2}}(1+i)$$

1(c) This expression can be split up as
$$e^{z+\pi i} = e^z e^{i\pi} = e^z \cdot -1 = -e^z$$

3 The function can be written as
$$e^{\bar{z}} = e^x e^{-iy} = e^x \cos y - ie^x \sin y$$

so the partial derivatives are
$$u_x = e^x \cos y$$
$$u_y = -e^x \sin y$$
$$v_x = -e^x \sin y$$
$$v_y = -e^x \cos y$$

These don't satisfy Cauchy-Riemann, so the function is differentiable nowhere and thus analytic nowhere.

8(a) The equation $e^z = -2$ can be written as
$$e^z = 2e^{i\pi}$$

so
$$e^x = 2 \rightarrow x = \ln 2$$
$$e^{iy} = e^{i\pi} \rightarrow y = (2n+1)\pi$$

and
$$z = \ln 2 + (2n+1)i\pi$$

8(b) The equation $e^z = 1 + i\sqrt{3}$ can be written as
$$e^z = 2e^{i\pi/3}$$

so
$$x = \ln 2$$
$$y = \left(\frac{1}{3} + 2n\right)\pi$$

and

$$z = \ln 2 + \left(\frac{1}{3} + 2n \right) i\pi$$

8(c) If

$$e^{2z-1} = 1$$

then

$$2z - 1 = i2n\pi$$

or, solving for z,

$$z = \frac{1}{2} + in\pi$$

2   Frame 31 – Logarithms

1(a) Evaluating the logarithm,

$$\text{Log}(-ei) = \ln(e) + \text{Arg}(-ei) = 1 - i\frac{\pi}{2}$$