Problem Sequence - Solutions

This document will be filled up with the solutions from the problem sequence.

Solution 1 *From Axiom 1, there are three cases to consider:*

1. $p = 0$

   p is equal to the only element of M. From the definition of limit point, every segment containing p must contain a point of M different from p. However, there are no points in M different from p, so p must not be a limit point of M.

2. $p > 0$

   From Axiom 3, there exists a point a such that $0 < a < p$. There also exists a point b such that $b > p$. Since $p > 0$ and $b > p$, Axiom 2 tells us that $b > 0$. We can then form the segment $S = (a, b)$. Since $a > 0$ and $b > 0$, 0 is not between a and b, so S does not contain 0. However, $a < p < b$, so S contains p. S is a segment containing p that does not contain any element of M, so p is not a limit point of M.

3. $p < 0$

   (symmetric to the $p > 0$ case)

Therefore, regardless of our choice of p, we can construct a segment that contradicts the requirements in the limit point definition, so p is not a limit point of M.

Solution 2

Solution 3

Solution 4

Solution 5

Solution 6

Solution 7

Solution 8

Solution 9

Solution 10 *According to Axiom 4, there is a largest integer a such that $a < p$ and a smallest integer b such that $p < b$. Then, we find a point x between a and p and a point y between p and b ($a < x < p < y < b$).*

1. *p is an integer*

2. p is not an integer

(need to think about these)

Solution 11

Solution 12

Solution 13

Solution 14

Solution 15

Solution 16

Solution 17

Solution 18