Problem Sequence - Solutions

This document will be filled up with the solutions from the problem sequence.

Solution 1 *(Greg)*

From Axiom 1, there are three cases to consider:

1. $p = 0$

   p is equal to the only element of M. From the definition of limit point, every segment containing p must contain a point of M different from p. However, there are no points in M different from p, so p must not be a limit point of M.

2. $p > 0$

   From Axiom 3, there exists a point a such that $0 < a < p$. There also exists a point b such that $b > p$. Since $p > 0$ and $b > p$, Axiom 2 tells us that $b > 0$. We can then form the segment $S = (a, b)$. Since $a > 0$ and $b > 0$, 0 is not between a and b, so S does not contain 0. However, $a < p < b$, so S contains p. S is a segment containing p that does not contain any element of M, so p is not a limit point of M.

3. $p < 0$

   (symmetric to the $p > 0$ case)

Therefore, regardless of our choice of p, we can construct a segment that contradicts the requirements in the limit point definition, so p is not a limit point of M.

Solution 2 *(Jeff)*

According to Definition 4, we can prove that p is not a limit point of M if we can construct a segment containing p but not a different point of M. Construct this segment as follows:

1. *If all points of M are on the opposite side of p, choose any value as an endpoint.*

2. *If any point of M is on the same side of p, choose a point between the nearest point of M and p. (Axiom 3 confirms that there will be such a point.)*

This segment contains p but will not contain any points of M (with the exception of p, if p is 0 or 1). Therefore, we have found a segment that does not fulfill the requirements of Definition 4, so p is not a limit point of M.

Solution 3 *(Erin, solved after Problem 4)*

Let $a = 0$ and $b = 1$. Then, according to Problem 4, b is a limit point of (a, b). Therefore, 1 is a limit point of $(0, 1)$.

Solution 4 *(Erin)*

Solution 5 *(Greg)*

Choose any point p from S. Construct a segment (x, y) that contains p (ie: $x < p < y$). Put no other condition on y. According to Axiom 1, one of these three cases is true:

1. $x > a$

   Choose a point q between x and p ($x < q < p$; this exists by Axiom 3). Apply Axiom 2 three times:

   - $a < x$ *and* $x < q$, *so* $a < q$
   - $q < p$ *and* $p < b$, *so* $q < b$
   - $q < p$ *and* $p < y$, *so* $q < y$

   so $a < q < b$ and $x < q < y$. Therefore, q is an element of S inside (a, b) that is different from p.

2. $x = a$

   Repeat the proof for $x > a$ with one change:

   - $a = x$ *and* $x < q$, *so* $a < q$

3. $x < a$

   Choose q betweem a and p. Change:

   - $x < a$ *and* $a < q$, *so* $x < q$

   (same conclusion as $x > a$)

In each of these three cases, every (x, y) contains a point q from (a, b). Therefore, we have satisfied Definition 4, so p is a limit point of (a, b).

Solution 6 *(Zack)*

Solution 7 *(Amber)*

Solution 8 *(Greg)*

Solution 9

Solution 10

Solution 11

Solution 12

Solution 13

Solution 14

Solution 15

Solution 16

Solution 17

Solution 18