Problem Sequence - Problems

This document contains the problems that we are given throughout the course.

Problem 1. Show that if M is the set which contains only the number 0, and p is a point, then p is not a limit point of M.

Problem 2. Show that if M is the point set that contains only the two points 0 and 1, then no point is a limit point of M.

Problem 3. Show that 1 is a limit point of the segment $(0, 1)$.

Problem 4. Show that a and b are limit points of the segment (a, b).

Problem 5. If S is the segment (a, b), show that every point of S is a limit point of S.

Problem 6. Show that if M is an interval and p is a point not in M, then p is not a limit point of M.

Problem 7. Show that if H is a point set, and K is a point set, and $H \subseteq K$, and p is a limit point of H, then p is a limit point of K.

Problem 8. Show that if M is the set of all positive integers and p is a point, then p is not a limit point of M.

Problem 9. Show that if p is a limit point of the point set M and S is a segment containing p, then S contains 2 points of M.

Problem 10. Let H be a point set which has a limit point, and let K be the set of all limit points of H. Show that if p is a limit point of K, then p is also a limit point of H.

Problem 11. Show that if M is the set of all reciprocals of positive integers, then 0 is a limit point of M.

Problem 12. Show that if $p \neq 0$, then p is not a limit point of the set of all reciprocals of positive integers.

Problem 13. if H and K are two point sets having a common point, and p is a limit point of $H \cap K$, then p is a limit point of H and p is a limit point of K.

Problem 14. Suppose that M is a point set and p is a limit point of M. Must it be true that every interval containing p contains a point of M different from p?

Problem 15. Suppose that M is a point set and every interval containing p contains a point of M different from p. Must it be true that p is a limit point of M?

Problem 16. Show that if the point p is in each of the two segments S_1 and S_2, then $S_1 \cap S_2$ is a segment containing p.

Problem 17. Show that if p is not a limit point of the point set H and p is not a limit point of the point set K, then p is not a limit point of $H \cup K$.

Problem 18. Show that if H and K are two point sets and p is a limit point of $H \cup K$, then p is a limit point of H or p is a limit point of K.

Problem 19. Show that if M is a point set and there is no smallest number in M (or there is no largest number in M), then if n is a positive integer, M has n points so that M is infinite.

Problem 20. Show that if M is a point set and M is finite then there is a smallest (and a largest) number in M.

Problem 21. Show that if M is a point set, and p is a limit point of M, and S is a segment containing p, and n is a positive integer, then S contains n points so that $S \cap M$ is infinite.

Problem 22. What does it mean to say that a sequence $x_1, x_2, x_3, \ldots$ does not converge to the number c?

Problem 23. Show that the sequence $0, 1, 0, 1, \ldots$ does not converge to 0.

Problem 24. Show that the sequence $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ converges to 0.
(not assigned as course work)

Problem 25. Show that if the sequence $x_1, x_2, x_3, \ldots$ converges to the number c and (a, b) is a segment containing c then there are not infinitely many terms of the sequence $x_1, x_2, x_3, \ldots$ that are not in (a, b).

Problem 26. Does the sequence $\frac{1}{2}, \frac{1}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{6}, \frac{1}{5}, \ldots$ converge to 0? Note that this is the sequence x where $x_n = \frac{1}{n-1}$ if n is an even positive integer and $x_n = \frac{1}{n+1}$ if n is an odd positive integer.

Problem 27. Assume that the sequence $x_1, x_2, x_3, \ldots$ converges to the point c and d is a point different from c. Show that $x_1, x_2, x_3, \ldots$ does not converge to d.

Problem 28. Show that if the sequence $x_1, x_2, x_3, \ldots$ converges to the point c, and, for each positive integer n, $x_n \neq x_{n+1}$, then c is a limit point of the range of the sequence.

Problem 29. Find a sequence $x_1, x_2, x_3, \ldots$ such that the point c is a limit point of the range $\{x_1, x_2, \ldots\}$ of the sequence but $x_1, x_2, x_3, \ldots$ does not converge to c.

Problem 30. Show that if d is a number and $x_1, x_2, x_3, \ldots$ is a sequence which converges to the point c, then the sequence $d \cdot x_1, d \cdot x_2, d \cdot x_3, \ldots$ converges to $d \cdot c$.

Problem 31. Find examples of infinite sets, one having a first point and one not having a first point.

Problem 32. Assume that $x_1, x_2, x_3, \ldots$ is a sequence that converges to the point c and d is a point different from$c$. Show that d is not a limit point of the range of the sequence.

Problem 33. Show that if M is a point set, there cannot be both a rightmost point of M and a smallest number which is larger than each number in M.

Problem 34. If M is a point set and each point of M is to the left of p and p is the smallest number such that each point of M is to the left of p, then p is a limit point of M.

Problem 35. If the range of the increasing sequence $x_1, x_2, x_3, \ldots$ is bounded above, then $x_1, x_2, x_3, \ldots$ converges to some point.

Problem 36. Assume that $x_1, x_2, x_3, \ldots$ is a sequence and c is a point. Suppose that, given any positive number ϵ, we can find an integer n such that $x_n, x_{n+1}, x_{n+2}, \ldots$ are in $(c - \epsilon, c + \epsilon)$. Show that $x_1, x_2, x_3, \ldots$ converges to c.

Problem 37. Show that if the sequence $p_1, p_2, p_3, \ldots$ converges to c and the sequence $q_1, q_2, q_3, \ldots$ converges to d, then the sequence $p_1+q_1, p_2+q_2, p_3+q_3, \ldots$ converges to $c + d$.

Problem 38. If the range of the non decreasing sequence $x_1, x_2, x_3, \ldots$ is bounded above, then $x_1, x_2, x_3, \ldots$ converges to some point.

Problem 39. If M is a bounded point set then the lub(M) (and the glb(M)) is either a point of M or a limit point of M.

Problem 40. Give an example of a bounded point set such that lub(M) is both a point of M and a limit point of M.

Problem 41. If the sequence $x_1, x_2, x_3, \ldots$ converges to the point c, then $M = x_1, x_2, x_3, \ldots$ is bounded.

Problem 42. If M is an infinite and bounded point set, then M has a limit point.

Problem 43. Show that if f is the graph which contains only the two points $(0, 0)$ and $(1, 1)$, then f is continuous at the point $(0, 0)$.

Problem 44. Let f be the graph such that $f(x) = x^2$ for each number x. Show that f is continuous at the point $(2, 4)$.

Problem 45. Let f be the simple graph such that $f(x) = 1$ for all numbers $x > 0$, and $f(0) = 0$. Show that f is not continuous at the point $(0, 0)$.

Problem 46. If f is a simple graph and $x_1, x_2, x_3, \ldots$ is a sequence of points in the domain of f converging to the number c in the domain of f, and f is continuous at $(c, f(c))$, then $f(x_1), f(x_2), f(x_3), \ldots$ converges to $f(c)$.