

Problem Sequence - Solutions

This document will be filled up with the solutions from the problem sequence.

Solution 1 (Greg)

From Axiom 1, there are three cases to consider:

1. $p = 0$

p is equal to the only element of M . From the definition of limit point, every segment containing p must contain a point of M different from p . However, there are no points in M different from p , so p must not be a limit point of M .

2. $p > 0$

From Axiom 3, there exists a point a such that $0 < a < p$. There also exists a point b such that $b > p$. Since $p > 0$ and $b > p$, Axiom 2 tells us that $b > 0$. We can then form the segment $S = (a, b)$. Since $a > 0$ and $b > 0$, 0 is not between a and b , so S does not contain 0 . However, $a < p < b$, so S contains p . S is a segment containing p that does not contain any element of M , so p is not a limit point of M .

3. $p < 0$

(symmetric to the $p > 0$ case)

Therefore, regardless of our choice of p , we can construct a segment that contradicts the requirements in the limit point definition, so p is not a limit point of M .

Solution 2 (Jeff)

According to Definition 4, we can prove that p is not a limit point of M if we can construct a segment containing p but not a different point of M . Construct this segment as follows:

1. *If all points of M are on the opposite side of p , choose any value as an endpoint.*
2. *If any point of M is on the same side of p , choose a point between the nearest point of M and p . (Axiom 3 confirms that there will be such a point.)*

This segment contains p but will not contain any points of M (with the exception of p , if p is 0 or 1). Therefore, we have found a segment that does not fulfill the requirements of Definition 4, so p is not a limit point of M .

Solution 3

Solution 4 (Erin)

Solution 5 (Greg)

Choose any point p from S . Construct a segment (x, y) that contains p . (ie: $x < p < y$) Put no other condition on y . According to Axiom 1, one of these three cases is true:

1. $x > a$

Choose a point q between x and p ($x < q < p$; this exists by Axiom 3).
Apply Axiom 2 three times:

- $a < x$ and $x < q$, so $a < q$
- $q < p$ and $p < b$, so $q < b$
- $q < p$ and $p < y$, so $q < y$

so $a < q < b$ and $x < q < y$. Therefore, q is an element of S inside (a, b) that is different from p .

2. $x = a$

Repeat the proof for $x > a$ with one change:

- $a = x$ and $x < q$, so $a < q$

3. $x < a$

Choose q between a and p . Change:

- $x < a$ and $a < q$, so $x < q$

(same conclusion as $x > a$)

In each of these three cases, every (x, y) contains a point q from (a, b) . Therefore, we have satisfied Definition 4, so p is a limit point of (a, b) .

Solution 6 (Zack)**Solution 7 (Amber)****Solution 8 (Greg)****Solution 9****Solution 10****Solution 11****Solution 12****Solution 13****Solution 14****Solution 15****Solution 16****Solution 17****Solution 18**