Main Course Notes

There are 5 axioms that we will use in this class:

Axiom 1 *If each of a and b is a number, then exactly one of the following is true:*

1. $a = b$
2. $a < b$
3. $a > b$

Axiom 2 *If each of a, b, c is a number, $a < b$, and $b < c$, then $a < c$.*

Axiom 3 *If a and b are two points on the number line, then there is a point between them.*

Axiom 4 *If a is a point, then there is a smallest integer b such that $b > a$ and a largest integer c such that $c < a$.*

Axiom 5 *If n is an integer, then $n + 1$ and $n - 1$ are integers, and n is the only integer between $n - 1$ and $n + 1$.*

Then, as the course continues, we will make various definitions and notation for these definitions.

Definition 1 *If a and b are two points and $a < b$, the statement that the point p is* ***between*** *the points a and b means that $a < p$ and $p < b$.*

Definition 2 *A* ***point set*** *is a set of one or more points.*

Definition 3 *The statement that the point set S is a* ***segment*** *means that there are two points a and b, called the endpoints of S, such that S is the set of all points between a and b.*

Notation 1 *If a and b are two points and $a < b$ then (a, b) denotes the segment consisting of all points between a and b.*

Definition 4 *If M is a point set and p is a point, the statement that p is a* ***limit point*** *of the point set M means that every segment that contains p contains a point of M different from p.*

Definition 5 *The statement that the point set I is an* ***interval*** *means that there are two points a and b, called the endpoints of I, such that I is the set containing a, b, and (a, b). I is denoted by $[a, b]$.*

Notation 2 *If a and b are two points and $a < b$ then $[a, b]$ denotes the interval with endpoints a and b.*

Definition 6 *The statement that the point set H is a* ***subset*** *of the point set K means that if p is a point of H, then p is a point of K.*

Notation 3 *If H is a point set and K is a point set then $H \subseteq K$ means that H is a subset of K.*

Definition 7 *If each of H and K is a point set and there is a point that is in both of them, then the* ***intersection*** *of H and K is the set to which a point p belongs if and only if p is in both H and K.*

Notation 4 *If each of H and K is a point set and there is a point that is in both of them, then $H \cap K$ denotes the intersection of H and K.*

Definition 8 *If each of H and K is a point set, the* ***union*** *of H and K is the set to which the point p belongs if and only if p is in H or p is in K.*

Notation 5 *If each of H and K is a point set, then $H \cup K$ denotes the union of H and K. Thus $H \cup K$ is the set of all points in H together with the points in K.*