$$\frac{F_p}{A} = \sum_i |n_i| \left(m_i^2(\phi_0) - m_i^2(\phi_n)\right) \int \frac{d^3p}{(2\pi)^3 2E_{i,p}(\phi_0)} f_{i,p}^{eq}(\phi_0) \tag{32}$$

where n_i is the number of degrees of freedom for species i, $m_i(\phi)$ are the field-dependent masses (including the thermal masses for the bosons), $E_{i,p}(\phi) = \sqrt{p^2 + m_i^2(\phi)}$, and where $f_{i,p}^{eq}$ are the *equilibrium* distribution functions. Since the wall's motion is assumed to be ultra-relativistic, the passage of the wall changes the masses sharply but leaves the distribution functions as they were in the symmetric phase to leading order in $1/\gamma$ (where $\gamma = 1/\sqrt{1-v^2}$) [95].

As pointed out in Ref. [92], the above expression is equivalent to the free-energy density difference between the minima in the so-called *mean field* $T \neq 0$ thermal effective potential, $\widetilde{V}_T$ [92, 95]. $\widetilde{V}_T$ is simply given by a Taylor expansion of $V_{1,T}$ around the symmetric minimum in field space, truncated at quadratic order in the field-dependent masses, i.e. [92, 95]

$$\widetilde{V}_T(\phi, T \neq 0) \equiv V_T(\phi_0, T \neq 0) + \sum_i \left(m_i^2(\phi) - m_i^2(\phi_0)\right) \frac{dV_T(\phi_0, T \neq 0)}{dm_i^2}. \tag{33}$$

With this definition, the condition that must be satisfied for the wall to run away can be re-phrased as $\widetilde{V}_T(\phi_n, T_n) - \widetilde{V}_T(\phi_0, T_n) < V_1(\phi_0, T = 0) - V_1(\phi_n, T = 0)$. Re-arranging, and defining the full 1-loop mean field effective potential $\widetilde{V}(\phi, T) \equiv V_0(\phi) + V_1(\phi, T = 0) + \widetilde{V}_T(\phi, T)$, the runaway condition becomes

$$\widetilde{V}(\phi_0, T_n) - \widetilde{V}(\phi_n, T_n) > 0, \quad \leftrightarrow \quad \text{Runaway solution exists.} \tag{34}$$

This suggests the following criterion for determining whether the wall is safe from runaway [92]: if tunneling to the broken minimum ϕ_n is not energetically favored in the mean field potential, the wall cannot run away. Thus, for all of our benchmarks, it suffices to compute both the full effective potential and the mean field potential along the tunneling direction; if the symmetry-breaking minimum disappears or is raised above the symmetric minimum in the mean field limit, the wall will remain sub-luminal. We will check against this criterion for all of the points we consider.