$$y_{S,m} \in \{2,4\}, \forall \nu_P(k) \leq m < n. \tag{152}$$

In fact, if $y_{S,m} \in \{1,3\}$ for some $\nu_P(k) \leq m < n$, then the PU packet is successfully decoded in slot m; similarly, if $y_{S,m} \in \{5,6,7\}$ for some $\nu_P(k) \leq m < n$, then the PU packet becomes connected to the root of the CD graph. Specializing (151) to this case and using (152), we need to prove the recursion

$$M_{S,n+1} + v_{S,n+1} = M_{S,n} + v_{S,n} + \chi(y_{S,n} \in \{1,2\}). \tag{153}$$

In fact, if $y_{S,n} \in \{1,2\}$, the root is successfully decoded and the full CD potential is released, resulting in

$$M_{S,n+1} = M_{S,n} + v_{S,n}. \tag{154}$$

The new root of the CD graph becomes $\rho_S(\mathcal{G}_{n+1}) = n+1$ (new SU packet), with CD potential $v_{S,n+1} = 1$, so that

$$M_{S,n+1} + v_{S,n+1} = M_{S,n} + v_{S,n} + 1. \tag{155}$$

Otherwise, the root of the CD graph remains unchanged, $\rho_S(\mathcal{G}_{n+1}) = \rho_S(\mathcal{G}_n)$, with CD potential $v_{S,n+1} = v_{S,n}$, and no SU packets are decoded, so that $M_{S,n+1} = M_{S,n}$. It follows that

$$M_{S,n+1} + v_{S,n+1} = M_{S,n} + v_{S,n}. \tag{156}$$