this paper. The literature on KAM theory is enormous and so there are many potential applications; we will only describe here some of those that may have some interest.

First, and more importantly, the technical estimates we derive in Appendix for Gevrey functions actually hold true for a larger class of ultra-differentiable functions that includes Gevrey (and thus analytic) functions as a particular case. This not only leads to a further extension of the KAM theorems we state and prove here, but also allows us to generalize other perturbative results such as the Nekhoroshev theorem (extending the result of [MS02] in the convex case and [Bou11] in the steep case). To keep this paper to a reasonable length, all these results will be derived in a subsequent article [BF17].

Then, our main result Theorem A deals with the persistence of Lagrangian tori; KAM theory also deals with lower-dimensional tori (see, for instance, [Rüs01] for a comprehensive treatment in the analytic case), and one may expect that our result extend to such a setting.

Finally, one may consider the problem of reducibility of quasi-periodic cocycles close to constant. In the analytic case, the Bruno-Rüssmann condition is sufficient, as was shown in [CM12]; in the α-Gevrey case, the α-Bruno-Rüssmann condition is sufficient. In fact, this setting is simpler from a technical point of view and our Gevrey estimates are not necessary to obtain such a result; one simply needs to go through the proof of [CM12]. A possible explanation for this is that for quasi-periodic cocycles, composition occur in a linear Lie group, thus only estimates for linear composition (product of matrices) are necessary and so everything boils down to good estimates for the product of two functions.

1.5 Plan of the paper

The plan of the paper is as follows.

In Section , we describe precisely the setting, namely we properly define the Gevrey norms we will use and the α-Bruno-Rüssmann condition. In Section we state our main results:

- Theorem A about the persistence of a torus in a non-degenerate Hamiltonian system under the α-Bruno-Rüssmann condition;
- Theorem B, the iso-energetic version of Theorem A;
- Theorem C, the non-autonomous time-periodic version;
- Theorem D about the destruction of a torus in the same context not assuming a condition weaker that the α-Bruno-Rüssmann condition;
- Theorem E about linearization of vector fields on the torus close to constant (we will also discuss necessary arithmetic conditions here, albeit in a restricted context);
- Theorem F, the discrete version of Theorem A, about the persistence of a torus in a non-degenerate exact-symplectic map;
- Theorem G, the discrete version of Theorem E, about the linearization of diffeomorphisms of the torus close to a translation.