As in Ref. [6], let us consider the five-point correlation function with one degenerate field

$$\langle V_{-\frac{1}{2b}}(z) V_{\alpha_1}(0) V_{\alpha_2}(1) V_{\alpha_3}(\infty) V_{\alpha_4}(x)\rangle. \tag{2.1}$$

Due to the presence of the degenerate field, this correlation function satisfies a partial differential equation, second order in z, first order in x. For the purposes of this section it is convenient to define the function $\Psi(u|q)$ as

$$\langle V_{-\frac{1}{2b}}(z) V_{\alpha_1}(0) V_{\alpha_2}(1) V_{\alpha_3}(\infty) V_{\alpha_4}(x)\rangle = z^{\frac{1}{2b^2}}(z-1)^{\frac{1}{2b^2}} \frac{\big(z(z-1)(z-x)\big)^{\frac{1}{4}}}{\big(x(x-1)\big)^{\frac{2\Delta(\alpha_4)}{3}+\frac{1}{12}}} \frac{\Theta_1(u)^{b^{-2}}}{\Theta_1'(0)^{\frac{b^{-2}+1}{3}}} \Psi(u|q), \tag{2.2}$$

where the variable u is related with the variables z and x as

$$u = \frac{\pi}{4K(x)} \int_0^{\frac{z-x}{x(z-1)}} \frac{dt}{\sqrt{t(1-t)(1-xt)}}, \tag{2.3}$$

$K(x)$ is the elliptic integral of the first kind

$$K(x) = \frac{1}{2} \int_0^1 \frac{dt}{\sqrt{t(1-t)(1-xt)}}, \tag{2.4}$$

$$\tau = \mathrm{i}\, \frac{K(1-x)}{K(x)} = \frac{\log q}{\mathrm{i}\pi}, \tag{2.5}$$

and $\Theta_1(u)$ is the Jacobi theta function (see definitions in appendix).

The function $\Psi(u|q)$ defined by Eq.(2.2) satisfies the non-stationary Schrödinger equation with doubly periodic potential

$$\left[\partial_u^2 - \mathbb{V}(u) + \frac{4\mathrm{i}}{\pi b^2}\partial_\tau\right]\Psi(u|q) = 0, \tag{2.6}$$

where the potential $\mathbb{V}(u)$ is given by

$$\mathbb{V}(u) = \sum_{j=1}^{4} s_j(s_j+1)\wp(u-\omega_j) \tag{2.7}$$

and the parameters s_k are related to the parameters α_k as

$$\alpha_k = \frac{Q}{2} - \frac{b}{2}\left(s_k + \frac{1}{2}\right). \tag{2.8}$$

In Eq.(2.7) $\wp(u)$ is the Weierstraß elliptic function with periods π and $\pi\tau$ defined by the infinite sum