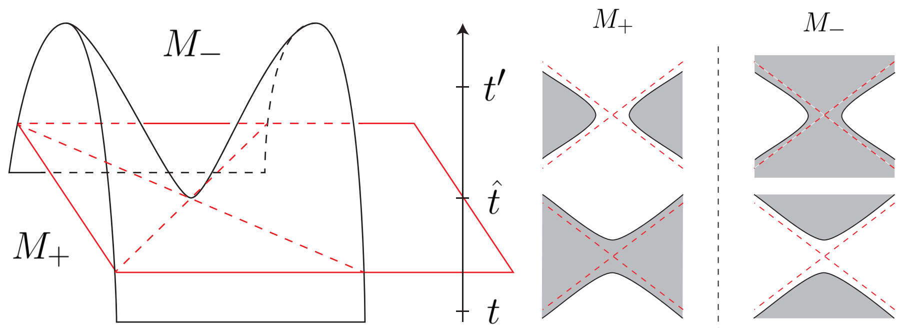

FIGURE 1.

Claim 2. *There are no compressing disks for $S_{t'} \cap M_+$ in $H^1_{t'}$.*

Proof. If such a compressing disk exists in $H^1_{t'}$, then there exists a compressing disk in $S_t \cap M_+$ in H^1_t for $t < \hat{t}$ by Claim 1, contradicting the assumption that the first compressing disk for $S_{t'} \cap M_+$ in $H^1_{t'}$ appears for $t' > \hat{t}$. □

Thus the side M_- must be where the first compressing disk in $H^1_{t'}$ appears at the level t' ($t < \hat{t} < t'$). That is, $S_{t'} \cap M_-$ has a compressing disk in $H^1_{t'}$. Then, there are no compressing disks for $S_{t'} \cap M_+$ in $H^0_{t'}$, for S is strongly irreducible. Together with Claim 2, we conclude that, for $t' > \hat{t}$ and t' very close to $\hat{t}$, there are no compressing disks for $S_{t'} \cap M_+$ in M_+, i.e., $S_{t'} \cap M_+$ is incompressible in M_+.

Also, at the level t ($t < \hat{t} < t'$), there cannot be any compressing disks in $S_t \cap M_-$ in H^0_t. Because if such a compressible disk exists, then it gives a compressing disk in $S_{t'} \cap M_-$ in $H^0_{t'}$ by Claim 1, contradicting that $S_{t'} \cap M_-$ does not have compressing disks on both sides. Also, at the level t, there cannot be any compressing disks in $S_t \cap M_-$ in H^1_t, for the first compressing disk for $S_{t'} \cap M_-$ in $H^1_{t'}$ appears for $t' > \hat{t}$. We conclude that there cannot be any compressing disks for $S_t \cap M_-$ in M_- at the level t, i.e., $S_t \cap M_-$ is incompressible in M_-. This gives the second case of the theorem.

Finally the third case occurs when $S_t \cap M_+$ has no compressing disk in H^0_t for t very close to $\hat{t}$ and $t < \hat{t}$. In this case, there are no compressing disk for $S_t \cap M_+$ in H^1_t, for the first compressing disk for $S_{t'} \cap M_+$ in $H^1_{t'}$ appears for $t' > \hat{t}$. This implies that $S_t \cap M_+$ is incompressible in M_+. In the same way, $S_t \cap M_-$ has no compressing disk in H^1_t.

Claim 3. *There are no compressing disks for $S_t \cap M_-$ in H^0_t.*

Proof. If such a compressing disk exists for $S_t \cap M_-$ in H^0_t, then it extends to a compressing disk in $S_{t'} \cap M_-$ in $H^0_{t'}$ for $t' > \hat{t}$ by Claim 1. This gives a contradiction to the strong irreducibility of the splitting (resp. the assumption that $S_{t'} \cap M_-$ does not have compressing disks on both sides) in the case that the first compressing disk at the level $t' > \hat{t}$ appears in the M_+ side (resp. the M_- side). □

Therefore we conclude that, for $t < \hat{t}$ and t arbitrarily close to $\hat{t}$, there are no compressing disks for either $S_t \cap M_+$ or $S_t \cap M_-$. □