$$\nabla e^{\frac{-|x-x_0|^2}{4r}} = \frac{-2(x-x_0)}{4r} e^{\frac{-|x-x_0|^2}{4r}} \tag{3.12}$$

Since by lemma 2.5 the entropy for a compact hypersurface will be attained by an F functional centered in its convex hull, without loss of generality x_0 is in the convex hull of $M^{\#}$. Since for such x_0 we have $|x-x_0| \leq D < \infty$[2], we see for a lower bound c on r we have $\nabla e^{\frac{-|x-x_0|^2}{4r}}| \leq \frac{D}{2c} < \infty$ for any choice of $x \in M_T$. Denote this upper bound by ρ.

We also note similarly for $r > c$ that the Gaussian weight of a $F_{x_0,r}$ functional (with x_0 in the convex hull of M) is bounded below by $e^{\frac{-D^2}{4c}} > 0$; denote this lower bound by σ. Also denote by $m_{x_0,r}$ and $M_{x_0,r}$ the minimum and maximum respectively of the Gaussian weight of $F_{x_0,r}$ in U_e. Then the following is true:

$$1 \geq \frac{m_{x_0,r}}{M_{x_0,r}} \geq \frac{m_{x_0,r}}{m_{x_0,r} + r_3\rho} \geq \frac{\sigma}{\sigma + r_e\rho} = 1 - \frac{r_e\rho}{\sigma + r_e\rho} \tag{3.13}$$

Since $\sigma > 0$ and $\rho < \infty$ we can make this quotient as close to one as we like by making r_e sufficiently small; in other words we can make the ratio of the minimum to the maximum of the weight in these F functionals as close to 1 as we want in U_e by increasing H_{neck}. Switching to the translated and rescaled picture (the ratio persists under rescaling), we have for $x_0 \in \widetilde{U}_f$ and for $r > c_1$ the following:

$$\begin{aligned}
&F_{x_0,r}(\widetilde{M}^+) \\
&= \int_{\widetilde{M}^+} \frac{1}{(4\pi r)^{\frac{n}{2}}} e^{\frac{-|x-x_0|^2}{4r}} \\
&\leq \int_{\widetilde{M}^+ \setminus \widetilde{U}_e} \frac{1}{(4\pi r)^{\frac{n}{2}}} e^{\frac{-|x-x_0|^2}{4r}} + \int_{\widetilde{M}^+ \cap \widetilde{U}_e} \frac{1}{(4\pi r)^{\frac{n}{2}}} e^{\frac{-|x-x_0|^2}{4r}} \\
&\quad \text{(because surgery only happens in } \widetilde{U}_e) \\
&= \int_{\widetilde{M}^{\#} \setminus \widetilde{U}_e} \frac{1}{(4\pi r)^{\frac{n}{2}}} e^{\frac{-|x-x_0|^2}{4r}} + \int_{\widetilde{M}^+ \cap \widetilde{U}_e} \frac{1}{(4\pi r)^{\frac{n}{2}}} e^{\frac{-|x-x_0|^2}{4r}} \\
&\leq \int_{\widetilde{M}^{\#} \setminus \widetilde{U}_e} \frac{1}{(4\pi r)^{\frac{n}{2}}} e^{\frac{-|x-x_0|^2}{4r}} + \int_{\widetilde{M}^+ \cap \widetilde{U}_e} \frac{1}{(4\pi r)^{\frac{n}{2}}} M_{x_0,r}
\end{aligned} \tag{3.14}$$

[2]of course, the diameter is decreasing under the flow so is uniformly bounded by the diameter of the initial time slice