to estimate the covariance matrix over a longer period of time, say five years for example, and use it as an input for some risk estimation method. In this spirit, we want to know how well the homogeneous correlation matrix estimates the risk, taking fluctuating correlations into account. For different time horizons, we estimate the empirical covariance matrix for the monthly returns of the S&P 500 stocks. We compare the results for the empirical covariance matrix with the results for a covariance matrix with homogeneous correlation struture. For each time horizon we determine the parameter N as described in section . In addition, we estimate the volatilities and drift for each stock and the average correlation c. The parameters are shown in Table 2. For volatility and drift we only show the average values $\bar{\sigma}$ and $\bar{\mu}$ over all stocks. Notice that N must be an integer in our simulation. During the financial crisis a smaller value of $N_{\mathrm{emp}} = 7$ is necessary to model the higher than usual fluctuations of the volatilities.

Time horizon for estimation	K	N_{hom}	N_{emp}	$\bar{\sigma}$ in $\mathrm{month}^{-1/2}$	$\bar{\mu}$ in month^{-1}	c
2006-2010	465	5	12	0.11	0.009	0.40
2002-2004	436	5	14	0.10	0.015	0.30
2008-2010	478	5	7	0.12	0.01	0.46

Table 2: Parameters used for the different time horizons.

We calculate the relative deviation of the VaR and ETL for different quantiles $\alpha = 0.99, 0.995, 0.999$ from the empirical covariance matrix. We study two cases for the covariance matrix with homogeneous correlation structure. First, we use the average values of volatility and drift for each stock. This resembles the homogeneous case discussed in section and is shown in Table 3. Second, we use the empirically obtained volatilities and drifts for each stock, see Table 4. Positive values of the relative deviation indicate that the covariance matrix with homogeneous correlation structure overestimates VaR and ETL, while negative values show an underestimation. We round all values to an accuracy of 0.5. For homogeneous volatilities and drifts we find that the covariance matrix with homogeneous correlations underestimates the risk in most cases. If we use heterogeneous volatilities and drifts, we find that the covariance matrix with homogeneous correlations is an appropriate fit and in most cases slightly overestimates the VaR and ETL. In all cases we observe decreasing deviations from the empirical covariance matrix for larger leverages F/V_0. This shows that the structure of the correlation matrix plays a minor role and underlines the importance of getting the volatilities right.

In Figure 9 we demonstrate how the VaR is underestimated by using stationary correlations. We calculate the relative deviation of the VaR for $N \to \infty$ and for different