

(It is interesting to note that the same 1936–1937 issue of *Comptes Rendus des Séances de la Société des Sciences et des Lettres de Varsovie* in which Ślupecki’s paper was published, contains the paper [We2] of Donald L. Webb, a fresh PhD from Caltech and the author of the already mentioned [We1], in which he extends the results of the latter paper. This, among other things, suggests that developments in this branch of logical calculi on both sides of Atlantic did not proceed in isolation at that time<sup>†</sup>). Earlier, another Łukasiewicz student, Mordchaj Wajsberg has reported at the Łukasiewicz–Tarski seminar<sup>‡</sup> some related results – see a brief description at the very end (“Anmerkung”) of [Wa] (cf. also Surma [Su, §ix]). In the same paper, more binary maps generating all maps of arbitrary arity on a finite set are presented without proof. Some of the Wajsberg’s results apparently go back as early as 1927, i.e. almost a decade before Webb.

More logical calculi, both complete and incomplete, were developed around this time, first of all, by Emil Leon Post (born in Poland, otherwise not related to that country<sup>§</sup>), as well as by Łukasiewicz, Ślupecki, Bolesław Sobociński (another PhD student of Łukasiewicz), Eustachy Żyliński (professor of the Lvov University in 1919–1941), Zygmunt Zawirski, and others. Post’s purely formal treatment of many-valued logics (“a combinatorial scheme” in the words of Mostowski [Mo, p. 3]) is, perhaps, closer in spirit to mathematical questions considered here than the philosophical attitude of Łukasiewicz.

Another feature of the work of Łukasiewicz’s school and Polish logicians in general, was the constant quest for minimal, as far as possible, systems of axioms for that or another logical calculus. Minimality was understood both in terms of the number of axioms and their length (cf., e.g., [Wo1, pp. 390–391] and [Su, §v])<sup>¶</sup>. Banach’s claim and question resonate well with this line of thought, as they can be phrased as follows: if a certain 3-term operation in an  $n$ -valued logic is not generated by a 2-term one (alas, as we have seen, this is wrong), the next best thing to ask is to extend the 3-term operation so that it will be generated that way.

Of course, many further results about Hilbert’s 13th problem, from one side, and functional completeness and incompleteness of various logical calculi, from the other, were obtained afterwards (cf., e.g., the survey by Vitushkin [V] for the former, and the book by Lau [L] for the latter), but their discussion will bring us far away from our main topic.

## 2. CLONES OF ANALYTIC AND ORDERED MAPS, COUNTING SUPERPOSITIONS, EXAMPLES TO BANACH’S CLAIM

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<sup>†</sup> The personal contacts started, probably, with the visit of Willard Van Orman Quine (then at Harvard) to Warsaw in 1932. Ernest Nagel (then at Columbia) made public (cf. [Nag]) his interesting impressions of visiting Poland in 1935, but, written from the philosopher’s rather than logician’s standpoint (“the logical researches both at Warsaw and Lwów are extraordinary specialized and technical”), these impressions, probably, contributed little to interchange of logical and mathematical ideas between the countries. Ulam was moving back and forth between US (Princeton and then Harvard) and Lvov in 1935–1939, but his interests, at least at that time, were outside logic. Webb’s thesis advisor, Eric Temple Bell, an enthusiastic, albeit not always precise, writer of popular mathematical books, praised Łukasiewicz’s 3-valued logic as one of the four major contributions “on the nature of truth” during the last 6000 years (cf. [Be, pp. 258–262]), so it is, perhaps, not accidental that Webb, a young Californian, published in a relatively obscure Warsaw journal.

<sup>‡</sup> “Professor Łukasiewicz’s seminar at Warsaw was crowded with competent young men, incomparably better equipped in logic than students of like age in America, who were expected to write as seminar exercises papers which elsewhere would be thought important enough for publication”, reports Nagel.

<sup>§</sup> A quote from [Wo1]: “When Tarski met Emil Post for the first time (in 1939 or 1940) he told him: ‘You are the only logician who achieved something important in propositional calculus without having anything to do with Poland’. Post answered: ‘Oh, no, I was born in Białystok’ ”.

<sup>¶</sup> This fascination with minimal systems of logical axioms was not shared by everyone in Poland. An anti-utopian novel “Nienasycenie” by Stanisław Ignacy Witkiewicz (a close friend of Leon Chwistek, professor of logic at the Lvov University, as well as of Alfred Tarski), written in 1927 and depicting conquest of Poland by enemy forces and establishment there of a totalitarian regime by the end of XX century, features a grotesque figure of logician Afanasol Benz (a Jew, stresses Witkiewicz) who invented a single axiom that nobody but him could understand, and from which all mathematics follows by a mere formal combination of symbols (cf. [Wi, p. 93]).