A remark on the Gaussian lower bound for the Neumann heat kernel of the Laplace-Beltrami operator

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Abstract. We adapt in the present note the perturbation method introduced in [3] to get a lower Gaussian bound for the Neumann heat kernel of the Laplace-Beltrami operator on an open subset of a compact Riemannian manifold.

Keywords. Neumann heat kernel, Laplace-Beltrami operator, Riemannian manifold.

Mathematics Subject Classification. 35K08

1. Introduction

The study of heat kernels is an important problem in the theory of parabolic PDE's. The properties of heat kernels give an efficient tool to answer to some central questions both in analysis and probability theory. One of the main questions is to know whether a heat kernel admits Gaussian bounds. An upper Gaussian bound is for instance an useful tool for getting L^p-L^q estimates, the analyticity of the corresponding semigroups in L^p for any finite $p \geq 1$ or bounded functional calculus, whereas one can get a strong maximum principle or a Harnack inequality from a lower Gaussian bound. We refer to the textbooks [5], [11] and [12] and references therein for more details on the subject.

In the preceding work [3], starting from the classical parametrix method, we constructed the Neumann heat kernel of a general parabolic operator as a perturbation of the fundamental solution of the same operator by a single-layer potential. From this construction, the two-sided Gaussian bounds for the fundamental solution and taking into account the smoothing effect in time of the single-layer potential, we succeeded in proving a lower Gaussian bound for the Neumann Green function. We adapt in the present note this