Moreover, let $\mathcal{G}_{\mathrm{cp}}(C_\varphi, a, \rho, C_g, c_g, C)$ be the class of functions g such that X is a compound Poisson process, $C_4^{\pm} < C$,

$$\forall u \in \mathbb{R}: \ |\varphi_{X_1}(u)| \geq C_\varphi$$

and

$$\forall u \in \mathbb{R}: |\mathcal{F}g(u)| \leq C_g |u|^{-a} \exp(-c_g |u|^\rho).$$

The following result which describes the rates of convergence with respect to the prescribed smoothness classes introduced above is a direct consequence of Theorem 3.1.

Proposition 3.2. *Let* K *be the sinc-kernel. This is equivalent to stating that* $\mathcal{F}\mathrm{K}(u) = \mathbb{1}_{[-1,1]}(u)$.

(i) Let h^* *be implicitly defined, as the solution of the minimization equation*

$$\sum_{j=1}^{n} \Delta_j h^{2\Delta_j \beta + 2} = 1. \tag{3.2}$$

Then

$$\sup_{g \in \mathcal{G}_{pol}(\beta, C_\varphi, c_\varphi, C_g, C)} \mathbb{E}_g \big[\|g - \widehat{g}_{h^*}\|^2_{\mathrm{L}^2(\mathbb{R})} \big] = O(h^*).$$

(ii) Let h^* *be the solution of*

$$\sum_{j=1}^{n} \Delta_j e^{-2\Delta_j c_\varphi (1/h)^\alpha} h^{2(\alpha-1)} = 1.$$

Then

$$\sup_{g \in \mathcal{G}_{exp}(\alpha, C_\varphi, C_g, C)} \mathbb{E}_g \big[\|g - \widehat{g}_{h^*}\|^2_{\mathrm{L}^2(\mathbb{R})} \big] = O\big((h^*)^{1-2\alpha}\big).$$

(iii) Let h^* *be the solution of*

$$e^{2c_g (1/h)^\rho} h^{-2a} = \sum_{j=1}^{n} \Delta_j C_\varphi^{\Delta_j}.$$

a) Assume that $\rho > 0$ *holds and* $a = (1-\rho)/2$. *Then*

$$\sup_{g \in \mathcal{G}_{cp}(C_\varphi, a, \rho, C_g, c_g, C)} \mathbb{E}_g \big[\|g - \widehat{g}_{h^*}\|^2_{\mathrm{L}^2(\mathbb{R})} \big] = O\big(\exp(-2c_g (1/h^*)^\rho) \big).$$