Correlation functions which include the ghost fields are generated by adding to the action analogous source terms for the ghosts.

1.1.2 Standard perturbative expansion (MLPE)

Due to the presence of the non-linear interaction terms in $\mathcal{S}$, the partition function (1.14) cannot be evaluated exactly and one has to resort to perturbative methods for its computation. As long as the coupling constant g is assumed to be small, one can aim to obtain a perturbative expansion of Z in powers of g.

The g-dependence of the partition function comes from the exponential of the interaction terms $\mathcal{S}_{int} := \mathcal{S}_{YM,int} + \mathcal{S}_{FP,int}$,

$$\mathcal{S}_{int} = \int d^dx \ \Big\{ - g\, f_{abc}\ \partial_\mu A^a_\nu\, A^{b\,\mu}\, A^{c\,\nu} - \frac{g^2}{4}\ f_{abc}\, f_{ade}\ A^b_\mu\, A^c_\nu\, A^{d\,\mu}\, A^{e\,\nu} + g\, f_{abc}\, \partial^\mu \overline{c}^a\ A^b_\mu c^c \Big\} \tag{1.20}$$

By Taylor-expanding the interaction exponential $e^{i\mathcal{S}_{int}}$, the partition function can be rewritten as

$$Z = \left(\int \mathcal{D}A^a_\mu\, \mathcal{D}\overline{c}^a\, \mathcal{D}c^a \ \ e^{i\mathcal{S}_0} \right) \ \left(\sum_{n=0}^{+\infty} \frac{1}{n!}\ \frac{\int \mathcal{D}A^a_\mu \mathcal{D}\overline{c}^a \mathcal{D}c^a \ \ e^{i\mathcal{S}_0}\ (i\mathcal{S}_{int})^n}{\int \mathcal{D}A^a_\mu\, \mathcal{D}\overline{c}^a\, \mathcal{D}c^a \ \ e^{i\mathcal{S}_0}} \right) \tag{1.21}$$

Here $\mathcal{S}_0 = \mathcal{S} - \mathcal{S}_{int}$,

$$\mathcal{S}_0 = \int d^dx \ \left\{ -\frac{1}{2}\ \partial_\mu A^a_\nu\, (\partial^\mu A^{a\,\nu} - \partial^\nu A^{a\,\mu}) - \frac{1}{2\xi}\ (\partial^\mu A^a_\mu)^2 + \partial^\mu \overline{c}^a \partial_\mu c^a \right\} \tag{1.22}$$

is the action for a free gauge vector field and a free ghost field. Eq. (1.22) can be expressed in momentum space as

$$\mathcal{S}_0 = i \int \frac{d^dk}{(2\pi)^d}\ \left\{ \frac{1}{2}\ A^a_\mu(k)\ \left[\Delta^{\mu\nu}_{0\perp\,ab}(k)^{-1} + \Delta^{\mu\nu}_{0\parallel\,ab}(k)^{-1} \right]\ A^b_\nu(k)^* + \overline{c}^a(k)\, \mathcal{G}_{0\,ab}(k)^{-1}\, c^b(k) \right\} \tag{1.23}$$