the amplitude of the magnetic field. Thus, if the latter is multiplied by 100, so will be the maximum energy too.

Such a large amplification of the ambient magnetic field around SNRs is indeed what has been suggested in the last decade, both on theoretical grounds, with resonant streaming instability as well as non-resonant, CR current driven instabilities (see e.g. the works of [32, 33, 34, 35, 36, 37, 38, 39]), and on observational grounds, with the interpretation of the extreme thinness of the X-ray rims observed at the forward shock of SNRs as due to the synchrotron losses of the highest-energy electrons in the local, intense magnetic fields [40, 41, 42, 43, 44, 45, 46].

However, even with such a large field amplification, by a factor of order 100, and even if one assumes that this can be maintained at the appropriate length scale to be fully profitable to particle acceleration, it remains very difficult for DSA to accelerate protons up to 1 PeV in SNRs. Now, even if it were possible, this would still be very far from what is needed to validate, at least in principle, the SNR-GCR connection. Indeed, the above study of the GCR/EGCR transition, in the light of the experimental data on the highest energy cosmic-rays, shows that the GCR sources must be able to accelerate protons at least up to $\sim 10^{17}$ eV (and Fe nuclei up to the ankle). This is (at least) two orders of magnitude higher than what can be optimistically achieved by standard SNRs (note that, in most cases where a maximum proton energy could be estimated, this energy is actually much lower than 1 PeV, e.g. [44], even though this may simply be because the acceleration phase during which PeV energies are reached is very short in time).

4.2. The "other source" solution

One solution to save the SNR-GCR connection is to invoke an additional source of GCRs. Basically, the SNRs would account for the GCR spectrum up to the knee, while another type of sources would account for the spectrum above that. This other component could be either Galactic, or extragalactic (as investigated recently in [21]). This does not seem very likely, however, because the matching of the two independent components in a seamless knee-like structure is particularly difficult to achieve in practice (see above).

In the case of an additional Galactic component, one may also refer to Ockham's razor and note that, if another type of sources, still to be identified, is needed to explain the cosmic-ray flux up to the ankle, then there might be no need for SNRs at all (at least not as the main contributors to the GCRs), as these new sources might just as well dominate at lower energy too, instead of suddenly starting above the PeV energy range. This would solve the problem of matching two components through a knee.

4.3. The "subset" solution

Another interesting solution to save the SNR-GCR connection consists in invoking a subset of the SNRs, rather than a new type of sources, to fill the gap between the knee and the ankle. This is an attractive solution because there could then be no need to match two independent components in a seamless way. If most SNRs accelerate particles up to the knee, and a smaller and smaller subset manages to accelerate particles up to higher and higher energies (eventually reaching $\sim Z \times 10^{17}$ eV or above, the schematic picture presented above would be globally satisfied. The knee would then be the consequence of the reduction of the number of contributing sources rather than an effect of particle propagation in (and escape out of) the Galaxy.

For instance, the flux of GCRs above the knee could be due some exceptional SN explosions or failed gamma-ray bursts, or to SNRs which interact with the wind of the progenitor (massive) star during the first few years after the explosion ([47, 48, 49]).

Of course, it remains to be confirmed that this "subset solution" is viable in practice, i.e. that some SNRs can indeed accelerate particles up to $Z \times 10^{17}$ eV. Such a demonstration would be a remarkable achievement, and it certainly is an important astrophysical question. According to [49], in the case of SN 1993J, the proton maximum energy could have been as high as 2–3 10^{16} eV, one or two days after the outburst. Although this is still short of what is needed, and although, according to [49], it does not take into account some nonlinear effects pointed out by [50] which could reduce further the maximum energy, it suggests that the maximum GCR energies could be reached in a few exceptional sources, in some particular circumstances, so that the usual limitations pertaining to the most studied SNRs (in a later phase of their evolution) may not be directly relevant.

However, while this remains an open question, we wish to point out that the subset of sources needed to account for the whole GCR spectrum should not be a small subset, as discussed below.

4.4. The size of the subset in the "subset solution"

Let us adopt the point of view that all SNRs accelerate protons with a power-law in E^{-x} up to the knee, at $E_{\rm knee} = 3 \times 10^{15}$ eV, or above, while only a smaller fraction of SNRs, $\alpha(E)$, accelerate protons with the same power-law up to an energy $E > E_{\rm knee}$ or above. What is then the fraction of the SNRs, $\alpha(10^{17}\,\mathrm{eV})$, which must accelerate protons up to 10^{17} eV or above?