which is naturally identified with a corresponding element $\psi_A \in V^* \otimes V^* \otimes V$. If $\mathbf{e} = \{e_1, \dots, e_n\}$ is an arbitrary basis in A, where $n = \dim_K A$, then ψ_A induces a K-quadratic polynomial map $\Psi_A : K^n \to K^n$ defined by

$$\psi_A \circ \epsilon = \epsilon \circ \Psi_A, \tag{1}$$

where ϵ is the coordinatization map

$$\epsilon(x) := \sum_{i=1}^{n} x_i e_i : K^n \to A, \qquad x = (x_1, \dots, x_n) \in K^n.$$

In this setting, Ψ_A is a bilinear map on K^n. Then an element $c = \epsilon(x) \in A$ is idempotent if and only if the corresponding $x \in K^n$ is a fixed point of $\Psi_A(x, x)$, i.e.

$$\Psi_A(x, x) - x = 0. \tag{2}$$

It is convenient to consider the projectivization of the latter system. Namely, let

$$\Psi_A^{\mathbf{P}}(X) = \Psi_A(x, x) - x_0 x,$$

where $X = (x_0, x_1, \dots, x_n) \in K^{n+1}$. The modified equation

$$\Psi_A^{\mathbf{P}}(X) = 0 \tag{3}$$

is homogeneous of degree 2. By the made assumption on K, we can consider both (2) and (3) as equations over the complex numbers. Furthermore, (3) defines a variety in $\mathbb{CP}^n$. Clearly, if x solves (2) then $X = (1, x)$ is a solution of (3), and, conversely, $X = (x_0, x)$ solves (3) with $x_0 \neq 0$ then $\frac{1}{x_0}x$ is a solution of (2). In the exceptional case $x_0 = 0$, one has $\Phi(x) = 0$, i.e. $\epsilon(x)$ is a 2-nilpotent in A.

In summary, there exists a natural bijection (depending on a choice of a basis in A) between the set $\mathbf{P}(A_{\mathbb{C}})$ and all solutions of (3) in $\mathbb{CP}^n$. In this picture, 2-nilpotents correspond to the 'infinite' part of solutions of (2) (i.e. solutions of (3) with $x_0 = 0$).

Then the classical Bezóut's theorem implies the following dichotomy: either there are infinitely many solutions of (3) or the number of distinct solutions is less or equal to 2^n, where $n = \dim_K A$. Therefore if the set $\mathbf{P}(A_{\mathbb{C}})$ is finite then necessarily

$$\operatorname{card} \mathbf{P}(A_{\mathbb{C}}) \leq 2^n \tag{4}$$

We point out that one should interpret a solution to (3) in the projective sense.