crystal without time-reversal symmetry. Once again, we emphasize that this effect would be absent in the linear approximation of the energy spectrum around the Weyl nodes.

E. Reversal of the direction of light propagation

Thus far, we have assumed that the direction of light propagation is along the positive z direction. If the direction of propagation is reversed, the roles of LCP and RCP are exchanged[32] and consequently the valley polarization is reversed. In other words, LCP and RCP are exchanged in Figs. 7 and 9, while the small and large black circles are exchanged in Figs. 10 and 11.

IV. ANALYTICAL RESULTS

The objective of this section is to support and supplement the numerical results of the preceding section with a simplified analytical solution of Eq. (40). Our approach is partly related to that of Ref. [33], which studied two-electron bound states. The main simplification consists of replacing the screened Coulomb potential in real space by a delta function potential. This approximation is valid at length scales that far exceed the screening length, i.e., for momenta that are small compared to the Thomas-Fermi screening wave vector k_s. If k_s is large compared to the momentum cutoff of the model (which is mathematically possible in the large g limit, or in the high-doping limit, or else in the neighborhood of a van-Hove singularity for the density of states), but still small compared to the separation between the Weyl nodes, then we can approximate Eq. (35) as

$$
\begin{aligned}
V_{m\tau}(k_\parallel, k_z; k'_\parallel, k'_z) &\simeq \frac{gv}{2k_s^2}\int_0^{2\pi}\frac{d\phi}{2\pi}e^{-im\phi}\left[\sin\theta_\tau\sin\theta'_\tau + (1+\cos\theta_\tau\cos\theta'_\tau)\cos\phi + i(\cos\theta_\tau+\cos\theta'_\tau)\sin\phi\right]\\
&= \frac{gv}{2k_s^2}\left[\sin\theta_\tau\sin\theta'_\tau\delta_{m,0} + \frac{1}{2}(1-\cos\theta_\tau)(1-\cos\theta'_\tau)\delta_{m,-1} + \frac{1}{2}(1+\cos\theta_\tau)(1+\cos\theta'_\tau)\delta_{m,1}\right]. \quad (46)
\end{aligned}
$$

In this approximation, only $m = 0, \pm 1$ channels contribute to the effective electron-hole attraction. Out of these, only the $m = \pm 1$ are active under irradiation by LCP and RCP lights. In addition, Eq. (46) becomes independent of the interaction strength because g/k_s^2 is independent of g in the Thomas-Fermi approximation. Finally, the interaction kernel is separable into "primed" and "non-primed" variables, which will enable an analytical solution of the corresponding Wannier equation. In fact, the problem at hand becomes a variation of the Cooper problem in the BCS theory of superconductivity.[34]

Let us consider the $m = 0$ channel first. Dividing both parts of Eq. (40) by $(2|\mathbf{B}_\tau(k_\parallel, k_z)| - \epsilon_{n,m=0,\tau})$ (which we assume to be nonzero), multiplying by $\sin\theta_\tau$ and integrating over $\mathbf{k}$, we arrive at the condition

$$
\frac{gv}{2k_s^2}\int'_{k_\parallel,k_z}\sin^2\theta_\tau\frac{\Theta(|\mathbf{B}_\tau(k_\parallel,k_z)| - |\epsilon_F|)}{2|\mathbf{B}_\tau(k_\parallel,k_z)| - \epsilon_{n,m=0,\tau}} = 1, \quad (47)
$$

where we have taken the zero temperature limit, and $\epsilon_F < 0$ (hole-doped WSM). Besides, for simplicity, we have neglected the self-energy correction to the energy bands, so that $|\tilde{\mathbf{B}}_\tau(k_\parallel, k_z)| \to |\mathbf{B}_\tau(k_\parallel, k_z)|$. We remind the reader that the integrals over momenta are constrained by the condition $|\mathbf{B}_\tau(k_\parallel, k_z)| < \Lambda$ (cf. Eq. (38)).

Equation(47) gives the electron-hole excitation energies corresponding to $m = 0$, at the valley τ. Proceeding in the same way, we find that the excitation energies for the $m = \pm 1$ channels must obey

$$
\frac{vg}{2k_s^2}\int'_{k_\parallel,k_z}\frac{(1\pm\cos\theta_\tau)^2}{2}\frac{\Theta(|\mathbf{B}_\tau(k_\parallel,k_z)| - |\epsilon_F|)}{2|\mathbf{B}_\tau(k_\parallel,k_z)| - \epsilon_{n,m=\pm1,\tau}} = 1. \quad (48)
$$

In order to obtain approximate analytical solutions of Eqs. (47) and (48), we begin by recognizing that

$$
\int'_{k_\parallel,k_z} F(k_\parallel,k_z) = \int^\Lambda dE\int'_{\mathbf{k}}F(\mathbf{k})\delta(E - |\mathbf{B}_\tau(\mathbf{k})|), \quad (49)
$$

where $\int'_{\mathbf{k}} \equiv \int d^3k/(2\pi)^3\Theta(\Lambda - |\mathbf{B}_\tau(\mathbf{k})|)$. Applying Eq. (49) to Eqs. (47) and (48), the latter become

$$
\begin{aligned}
&\frac{gv}{2k_s^2}\int_{|\epsilon_F|}^\Lambda dE\frac{\rho_\tau(E)}{2E - \epsilon_{n,m=0,\tau}}\langle\sin^2\theta_\tau\rangle_E = 1\\
&\frac{gv}{2k_s^2}\int_{|\epsilon_F|}^\Lambda dE\frac{\rho_\tau(E)}{2E - \epsilon_{n,m=\pm1,\tau}}\left\langle\frac{(1\pm\cos\theta_\tau)^2}{2}\right\rangle_E = 1,
\end{aligned} \quad (50)
$$

where $\rho_\tau(E) = \int'_{\mathbf{k}}\delta(E - |\mathbf{B}_\tau(\mathbf{k}|)$ is the valley-resolved density of states at energy E and

$$
\langle f_\tau(\mathbf{k})\rangle_E \equiv \frac{\int'_{\mathbf{k}} f(\mathbf{k})\delta(E - |\mathbf{B}_\tau(k_\parallel,k_z)|)}{\rho_\tau(E)} \quad (51)
$$