how one can deduce the behavior of the many interesting fields under the Hamiltonian's discrete symmetries.

3.1 Primaries

The $\mathbb{Z}_3$ CFT has central charge $c = 4/5$ [10] and is a rational conformal field theory. The fundamental characteristic of a rational conformal field theory is that all the operators/states of the theory can be expressed in terms of a finite set of operators dubbed primary fields. That is, every state in the Hilbert state may be constructed by acting with a primary field and the generators of the (possibly extended) conformal algebra.

With appropriate boundary conditions, the left- and right-moving conformal symmetries are independent. When space-time is written in terms of complex coordinates, the corresponding generators are the holomorphic and antiholomorphic parts of the energy-momentum tensor, respectively. Thus one can decompose any field into representations of these independent symmetries. A given field therefore can be characterized by left and right scaling dimensions $(h, \bar{h})$, so that its total scaling dimension is $h + \bar{h}$ while its conformal spin is $h - \bar{h}$. Local fields possess integer conformal spin and exhibit correlators that remain invariant under 2π rotations; parafermions (and fermions for that matter) do not represent local fields in this sense.

The $\mathbb{Z}_3$ CFT supports additional spin ± 3 currents denoted W and $\overline{W}$. It is then useful to extend the usual conformal (Virasoro) algebra by these generators to obtain what is known as the "$\mathcal{W}_3$ algebra" [30]. This is the simplest non-trivial CFT with this symmetry algebra. Fortunately, for our purposes here the intricacies of the extended algebra are largely unimportant. All we need to know is the list of primary fields and that the field content can be generated by operator product expansions (OPEs) of the primaries with the left- and right-moving stress-energy tensors T, $\overline{T}$, and with W, $\overline{W}$. The "descendant fields" obtained in this fashion yield all the operators/states in the theory.

The chiral building blocks of the fields are known as primary chiral vertex operators [31]; we call these chiral primaries for short. All primary fields, both local and non-local, can be built from linear combinations of products of chiral and anti-chiral primaries. It is important to note that in any conformal field theory other than that of a free boson, this decomposition is non-trivial. Some of the fields are not simply the product of holomorphic and antiholomorphic fields; they are the *sum* of such products.

The six *local* primary fields of the $\mathbb{Z}_3$ CFT have long been known [32]. A set of local fields has the property that all their correlators remain unchanged under 2π rotations of the system; i.e., their conformal spin $h - \bar{h}$ is an integer. For a given CFT, there is not a unique such set. As with the Ising model [1], in parafermion theories one can form a set of local fields containing either the spin *or* the disorder field, but not both: the OPE of the two contains fractional powers of z [cf. Eq. (25)]. By convention we view the spin field as local. This choice uniquely determines the set of local primaries, which we denote by 1, s, $s^\dagger$, E, $\psi\bar{\psi}$, and $\psi^\dagger\bar{\psi}^\dagger$.

There is of course the identity field, labeled 1. The spin fields $s(z, \overline{z})$ and $s^\dagger(z, \overline{z})$ each have dimensions $(1/15, 1/15)$, and correspond to the scaling limit of the spin operators $\hat{\sigma}_a$, $\hat{\sigma}_a^\dagger$ described above. Charge conjugation $\mathcal{C}$ interchanges them, so they form a doublet under the S_3 symmetry [33]. The energy field $E(z, \overline{z})$ possesses dimensions $(2/5, 2/5)$. Perturbing the critical theory by this field describes the scaling limit of the three-state Potts model away from criticality with $f/J \neq 1$ [10]. We denote the chiral primaries comprising $s(z, \overline{z})$ as σ; a full labeling includes the fusion channels [31], but we will not need this information. We likewise label the chiral primaries that are part of $s^\dagger$ and E by $\sigma^\dagger$ and ϵ, respectively, with the antichiral primaries labeled as $\bar{\sigma}$, $\bar{\sigma}^\dagger$, and $\bar{\epsilon}$.

As opposed to the spin and energy fields, the remaining two primaries of conformal spin zero split into a simple product of holomorphic and antiholomorphic fields. It is thus convenient to