where $\widetilde{\mathbf{\Pi}} = -\widetilde{p}\mathbb{1} + \eta\left(\widetilde{\nabla}\,\widetilde{\boldsymbol{v}} + \left[\widetilde{\nabla}\,\widetilde{\boldsymbol{v}}\right]^T\right)$ is the fluid stress tensor and $d\widetilde{\mathcal{S}} = ad\phi dz$, $d\widetilde{V} = rdrd\phi dz$ are the surface and volume elements in cylindrical coordinates respectively.

B. Non-dimensionalization

We express the displacements (r, z) in terms of half the length of the swimmer, l. We consider swimmers with concentration gradients generated by the colloids on a scale c^*/a. Hence we express the flux $\widetilde{J}$ of reactant (product) particles in units of c^*D/a, the concentration field $\widetilde{c}$ in units of c^*, (where D is the diffusion coefficient of the reactant/product particles), the interaction energy $\widetilde{\Psi}$ in the units of thermal energy k_BT, (k_B the Boltzmann constant and T is the temperature of the solution), the flow field $\widetilde{\boldsymbol{v}}$ with $U^* = \mu^* c^*/a$ (where $\mu^* = k_BTL^{*2}/\eta$ is the characteristic phoretic mobility coefficient, η is the viscosity of the fluid and L^* is the short-range interaction lengthscale of the reactant (product) molecules with the swimmer surface). We scale the pressure field $\widetilde{p}$ with $\eta U^*/l$. We note that the problem has three length-scales; the swimmer characteristic length $2l$, its cross-sectional radius a and the range L^* of the interaction between the molecules and swimmer surface. Consequently, we have three asymptotic near-field regions and in addition the scale on which the ends of the rod are rounded.

We therefore define the dimensionless parameters, $\epsilon = a/l$, the slenderness ratio and $\lambda = L^*/l$, the interaction layer thickness to the swimmer largest lengthscale, with $0 < \lambda \ll \epsilon \ll 1$. In addition, we define dimensionless fields $\boldsymbol{J} = \widetilde{\boldsymbol{J}}a/c^*D,\ c = \widetilde{c}/c^*,\ \boldsymbol{v} = \widetilde{\boldsymbol{v}}/U^*,\ p = \widetilde{p}l/\eta U^*,\ \psi = \widetilde{\psi}/k_BT$, and the dimensionless catalytic flux $\alpha = \widetilde{\alpha}a/c^*D$ on the swimmer surface.

$$\nabla\cdot\boldsymbol{J} = 0\ ; \qquad \boldsymbol{J} = -\epsilon\left(\nabla c + c\nabla\psi\right)\ , \tag{8}$$

$$0 = \nabla\cdot\boldsymbol{v}\ , \tag{9}$$
$$\mathbf{0} = \nabla^2\boldsymbol{v} - \nabla p - \epsilon\ \lambda^{-2}\ c\nabla\psi\ , \tag{10}$$

where $\lambda = L^*/l$ is the ratio of the interaction length-scale to half the length of the swimmer, $\epsilon = a/l$ is the swimmer slenderness ratio and $\psi(r - \epsilon S(z), z)$ is the short-range interaction potential between reactant (product) molecules and surface.

$$\hat{\boldsymbol{n}}\cdot\boldsymbol{J} = \alpha(z), \quad \text{at} \quad r = \epsilon\ S(z), \tag{11}$$

and the concentration decays to its value far from the swimmer, $c \to c_\infty,\ \sqrt{r^2+z^2} \to \infty$.

$$\boldsymbol{v} = \mathbf{0}, \quad \text{at} \quad r = \epsilon\ S(z), \tag{12}$$
$$\boldsymbol{v} \to -U\hat{\boldsymbol{e}}_z, \quad \sqrt{r^2+z^2} \to \infty. \tag{13}$$

The zero torque and force conditions are

$$\oiint_{r=\epsilon\ S(z)} \mathbf{\Pi}\cdot\hat{\boldsymbol{n}}\ d\mathcal{S} - \epsilon\ \lambda^{-2}\iiint_{-\infty}^{\infty} c\ \nabla\psi\ dV = \mathbf{0} \tag{14}$$

where $\mathbf{\Pi} = -p\mathbb{1} + \left(\nabla\boldsymbol{v} + [\nabla\boldsymbol{v}]^T\right)$ is the dimensionless stress tensor and $d\mathcal{S}$, dV are the surface and volume elements in cylindrical coordinates respectively.

C. The slender shape function