$$\frac{\partial C}{\partial \tilde{T}} = \frac{\partial^2 C}{\partial X^2}, \qquad 1 < X < \infty, \quad \tilde{T} > 0, \tag{2.32}$$

$$\frac{[X]^2}{\delta}\frac{\partial C}{\partial \tilde{T}} = \frac{\partial^2 C}{\partial \tilde{X}^2}, \qquad \tilde{X} > 0, \quad \tilde{T} > 0, \tag{2.33}$$

subject to

$$C \to 0 \quad \text{as } X \to \infty, \tag{2.34}$$

$$C = 1, \quad -\frac{\partial C}{\partial \tilde{X}} = \frac{[X]^2}{\delta}\frac{\mathrm{d}\tilde{S}}{\mathrm{d}\tilde{T}}(1 - \frac{c_0}{c_s}), \quad \text{at } \tilde{X} = \tilde{S}\left(\tilde{T}\right), \tag{2.35}$$

$$\tilde{S} = 0, \quad C = C_a\left(X\right), \quad \text{at } \tilde{T} = 0,\ X > 1\left(\tilde{X} < 0\right). \tag{2.36}$$

In addition, we have

$$[C]_-^+ = 0 \quad \text{at } X = 1, \left(\tilde{X} = 0\right) \tag{2.37}$$

$$-\frac{\delta}{[X]}\left(\frac{\partial C}{\partial \tilde{X}}\right)_{\tilde{X}=0} = \left(\frac{\partial C}{\partial X}\right)_{X=1}. \tag{2.38}$$

We must now choose $[X]$ so that (2.32)-(2.38) constitute a self-consistent system. There are basically only two possibilities: $[X] \sim \delta$ and $[X] \sim \delta^{1/2}$. We try these in turn.

2.1.1 $[X] \sim \delta$

Equation (2.33) gives

$$\frac{\partial^2 C}{\partial \tilde{X}^2} = 0, \tag{2.39}$$

subject to, from (2.35),

$$C = 1, \quad \frac{\partial C}{\partial \tilde{X}} = 0, \quad \text{at } \tilde{X} = \tilde{S}\left(\tilde{T}\right) \tag{2.40}$$

and

$$[C]_-^+ = 0 \quad \text{at } X = 1, \tag{2.41}$$

$$-\left(\frac{\partial C}{\partial \tilde{X}}\right)_{\tilde{X}=0} = \left(\frac{\partial C}{\partial X}\right)_{X=1}. \tag{2.42}$$

Thus, (2.39) and (2.40) give just $C \equiv 1$ for $X < 1$, which means that (2.41) and (2.42) would become