While α is fixed at 2 Å^{-1}, β is taken at the three different values 0.1, 0.3, and 1.02 Å corresponding to increasing couplings between the R and ϕ coordinates in the interaction region. E is kept at 0.5 eV, j_1 at 0, μ at 2/3 amu, m at 1/2 amu and r at 1 Å. R_1 and R_2 are both taken at 4 Å, beyond which $V(R,\phi)$ is negligibly small and the integrand in Eq. (11) is a constant of motion. The collisional systems resulting from the previous parameters involve strong interferences, as shown by the exact quantum state distributions $P_{j_2j_1}(E) = |S_{j_2j_1}(E)|^2$ displayed in Fig. 1 (blue circles connected by dotted segments; see Ref.[54] for some details on their calculations). Since $J_1 = 0$, we note from Eq. (4) that $\tilde{\phi}_1 = \phi_1$.

For $\beta = 0$, there is no coupling between R and ϕ and J keeps constantly equal to $\hbar j_1$ during the collision. Calling $t(R_1, R_2)$ the time to go from R_1 to the interaction region and back to R_2, we have

$$\phi_2 = \phi_1 + \frac{\hbar j_1}{I}\, t(R_1, R_2). \tag{13}$$

Using Eq. (4), we thus arrive at

$$\tilde{\phi}_2 = \phi_1 + \frac{\hbar j_1}{I}\, t(R_1, R_2) - \frac{\mu R_2 \hbar j_1}{P_1 I} \tag{14}$$

with P_1 given by Eq. (5). From Eqs. (7) and (14), and the fact that $t(R_1, R_2)$ does not depend on ϕ_1, we obtain

$$\left.\frac{\partial \tilde{\phi}_2}{\partial \tilde{\phi}_1}\right|_{J_1} = 1. \tag{15}$$

$\left.\frac{\partial \tilde{\phi}_2}{\partial \tilde{\phi}_1}\right|_{J_1}$ is represented in Fig. 2 for $\beta = 0$ and the three values previously considered. For β equal 0.1, the coupling is small and $\left.\frac{\partial \tilde{\phi}_2}{\partial \tilde{\phi}_1}\right|_{J_1}$ slightly oscillates around 1. For β equal 0.3, the coupling is stronger, thus leading to oscillations around 1 of larger amplitude. In both cases, however, $\left.\frac{\partial \tilde{\phi}_2}{\partial \tilde{\phi}_1}\right|_{J_1}$ is found to be positive. Hence, the SC-I$^-$ and SC-I$^+$ approaches lead