in Fig. 10 (a). Even if the entire Fermi surface is taken into account the selection rules still work in the Fe-based materials. For instance, in either case full cancellation is found for B_{2g} symmetry[24].

Explicitly written out, the fermionic loop is given by[22–24,41]

$$\begin{aligned}\theta_{i,\mu}(\mathbf{q}_c,\Omega,\omega_m) &= \theta^{(1)}_{i,\mu}(\mathbf{q}_c,\Omega,\omega_m)+\theta^{(2)}_{i,\mu}(\mathbf{q},\Omega,\omega_m),\\ \theta^{(1)}_{i,\mu}(\mathbf{q}_c,\Omega,\omega_m) &= T\sum_n\int_{\mathbf{k}}\gamma^{\mu}_{\mathbf{k}}G_\Gamma(\mathbf{k},\varepsilon_n-\Omega)G_\Gamma(\mathbf{k},\varepsilon_n)\\ &\quad\times G_i(\mathbf{k}-\mathbf{q}_c,\varepsilon_n-\omega_m),\\ \theta^{(2)}_{i,\mu}(\mathbf{q}_c,\Omega,\omega_m) &= T\sum_n\int_{\mathbf{k}}\gamma^{\mu}_{\mathbf{k}}G_i(\mathbf{k},\varepsilon_n-\Omega)G_i(\mathbf{k},\varepsilon_n)\\ &\quad\times G_\Gamma(\mathbf{k}-\mathbf{q}_c,\varepsilon_n-\Omega+\omega_m),\end{aligned}\tag{D1}$$

where $\gamma^{\mu}_{\mathbf{k}}$ is the form factor ($\mu=B_{1g},\ A_{1g}$ etc.), and G_i is the electron propagator on band $i=\Gamma,\ X,\ Y$. ε_n is the electronic energy and Ω is the energy difference between the incoming and scattered photons. Experimentally, pure symmetries can be obtained from linear combinations of the response measured at appropriate polarizations of the incoming and scattered photons $\hat{e}_i$ and $\hat{e}_s$.

For illustration purposes the fermionic loop θ is approximated in the hot-spot approximation. Hot-spots are regions in momentum space where both $\mathbf{k}$ and $\mathbf{k}\pm\mathbf{q_c}$ lie on the Fermi surface [Fig 10 (b)]. Since the loop θ contains the symmetry factor $\gamma(\mathbf{k})$ linearly inside the momentum integral the sign of $\gamma(\mathbf{k})$ is crucial. If $\gamma(\mathbf{k})$ changes sign for different hot spots connected by $\mathbf{q_c}$ (Fig. 10 (c), (d), and (e) for $A_{1g},\ B_{1g},\ B_{2g}$, respectively) there will be full or partial cancelation within θ. Full cancelation is observed for the first two (and also higher) orders of B_{2g} symmetry [Fig. 10 (e)]. In contrast, $\gamma(\mathbf{k})$ does not change sign across different hot-spots for the B_{1g} channel. Consequently, in B_{1g} and B_{2g} the fluctuations are Raman active and inactive, respectively.

The A_{1g} symmetry is more complicated in that the first order contribution, proportional to $\cos(k_x)+\cos(k_y)$ [upper row of Fig. 10 (c)], would be as strong as the B_{1g} contribution [Fig. 10 (d)] whereas the second order contribution $(\cos(k_x)\cos(k_y))$ [second row of Fig. 10 (c)] shows cancelation. For clarifying the relative magnitude of the two orders we analyze the effective mass vertices on the Fermi surfaces (second derivative or curvature of the band structure), that are the best approximations for the sensitivity away from resonances, in a way similar to what was proposed in Ref. 28. The last row of Fig. 10 (c) shows that the band curvatures corresponding to the A_{1g} vertex

$$\gamma_{i,A1g}(\mathbf{k})=\frac{\partial^2\varepsilon_{i,\mathbf{k}}}{\partial k_x\partial k_x}+\frac{\partial^2\varepsilon_{i,\mathbf{k}}}{\partial k_y\partial k_y}\tag{D2}$$

on the Fermi surface of the hole and the electron bands (i) are predomininantly negative and positive, respectively, as expected already for simple parabolic bands with masses $m_h\approx -m_e$ although there are various near nodes on both bands. This result shows that $\cos(k_x)\cos(k_y)$ is the leading order. We note that $\cos(k_x)\cos(k_y)$ predicts a stronger mixing of the particle-hole response from the electron and hole bands than $\cos(k_x)+\cos(k_y)$ as already outlined by Mazin *et al.* Ref. 28.

Appendix E: Subtraction of the continuum

The fluctuation response is superposed on the particle-hole continuum that essentially reflects symmetry-resolved transport properties[31]. Since the contribution of the fluctuations is relatively strong here they can be isolated with little uncertainty. The simplest way is to use the continuum at or slightly above the crossover temperature T_f and subtract it from all spectra measured below T_f. This was sufficient for $ErTe_3$[42] but created negative intensities in the case of $La_{2-x}Sr_xCuO_4$[43]. Here, we wish to compare the temperature dependence of the fluctuations to a theoretical prediction and have to improve on the subtraction of the continuum. To this end we make the analytical phenomenology for the B_{1g} continuum temperature dependent in a way that yields $\Gamma_{0,B1g}(T)\propto\rho(T)$. This seems sensible since the proportionality holds for the A_{1g} results in the entire temperature range above $T_{\rm SDW}$ and for the B_{1g} spectra above T_f. Fig. 11 shows the steps and checks necessary for the procedure. The analytical function used reads

$$\begin{aligned}\chi''_{\rm cont}(\Omega,T)=[\alpha_1+\alpha_2\cdot T]\tanh\left(\frac{\Omega}{\tilde{\Gamma}_0(T)}\right)+\\ [\beta_1+\beta_2\cdot T]\left(\frac{\Omega}{\tilde{\Gamma}_0(T)}\right)\end{aligned}\tag{E1}$$

which obeys $\chi''_{\rm cont}(-\Omega,T)=-\chi''_{\rm cont}(\Omega,T)$ as required by causality. α_1, α_2, β_1 and β_2 depend only on doping x. For $x=0.025$ we used $\alpha_1=0.82379$, $\alpha_2=-0.00138$, $\beta_1=-0.00923$, and $\beta_2=0.00028$. $\tilde{\Gamma}_0(T)$ is a fitting parameter that is selected in a way that the inverse slope $\Gamma_c(0,T)$ of $\chi''_{\rm cont}(\Omega,T)$ follows the resistivity (orange diamonds in Fig. 11 d). If a constant continuum is used the fluctuations can be isolated in a qualitatively similar fashion. However, the experimental data in Fig. 2 vary more slowly close to T_s.

Below T_s the uncertainties increase since surface layers accumulate rapidly in the presence of twin boundaries where the surface assumes a more polar character. This can be seen directly in Fig. 6 (c).

Appendix F: Initial slope

For being a causal function the Raman response is antisymmetric and, as long as there is no gap, linear around the origin. Then Eq. (C1) can be approximated as