$v \in \mathsf{ext}(L_{k,\alpha})$ and every n, there is a word x such that $|x| > |u_n|$ and $xv \in \mathsf{ext}(L_{k,\alpha})$; since $\mathbf{u}$ contains the factor xv, there is an occurrence of v in $\mathbf{u}$ outside the prefix u_n. Hence v occurs in $\mathbf{u}$ infinitely many times. Thus $\mathbf{u}$ is recurrent and we proved this implication.

Now turn to the backward implication. For arbitrary words $u, v \in \mathsf{ext}(L_{k,\alpha})$ each of them occurs in $\mathbf{u}$ infinitely often, so we can find a factor of the form uwv. This factor also occurs in $\mathbf{u}$ infinitely often, allowing us to find arbitrarily long words x, y such that $xuwvy$ is a factor of $\mathbf{u}$. Then $uwv \in \mathsf{ext}(L_{k,\alpha})$. Hence we proved the Restivo-Salemi property for $L_{k,\alpha}$. □

Remark 39. It is worth mentioning that for small binary languages Theorem 37 works in an extremal form. Since $2^+ \le \alpha \le 7/3$ implies $\mathsf{ext}(L_{2,\alpha}) = \mathrm{Fac}(\mathbf{t})$, the language $L_{2,\alpha}$ trivially has the Restivo-Salemi property; as we know from Corollary 3, *all* α-power-free infinite binary words contain all words of $\mathsf{ext}(L_{2,\alpha})$ as factors.

The following conjecture is based on extensive numerical studies.

Conjecture 40. [30, Conjecture 1] All power-free languages satisfy the Restivo-Salemi property.

As an approach to Conjecture 40, we suggest the following.

Open Question 41. *Prove the converse of Corollary 38.*

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