University of Toronto at Scarborough
CSCC73 - Algorithm Design and Analysis

Assignment #5: Dynamic Programming and Network Flow

Due: November 6, 2018 at 11.59pm This exercise is worth 5% of your final grade.

Warning: Your electronic submission on MarkUs affirms that this exercise is your own work and no one else's, and is in accordance with the University of Toronto Code of Behaviour on Academic Matters, the Code of Student Conduct, and the guidelines for avoiding plagiarism in CSCC73. Late assignments will not be accepted. If you are working with a partner your partners' name must be listed on your assignment and you must sign up as a "group" on MarkUs. Recall you must not consult **any outside sources except your partner, textbook, TAs and instructor**.

1. (10 marks) Recall the optimal binary search tree algorithm we saw in class that minimizes the expected weight of the tree. Lets define a weight balanced tree as a binary tree such that at every non-leaf node, v, the number of nodes in each of its subtrees is at least one quarter of the total number of nodes in the subtree NOT INCLUDING THE root v. Modify the optimal binary search tree algorithm such that the resulting tree is a weight balanced tree. That is, the optimization has to be limited to weight balanced trees.

2. The following questions deal with how changing capacity affects flow.

   (a) (5 marks) Determine whether the following statement is true or false. If it is true, give a short proof. If it is false, give a counterexample.

   > *Let (A, B) be a minimum s,t-cut with respect to these capacities $\{c_e : e \in E\}$. Now suppose we add 1 to every capacity, then (A, B) is still a minimum s,t-cut with respect to these new capacities $\{1 + c_e : e \in E.\}$*

   (b) (5 marks) Now assume that we are given a max flow f on G. Suppose that the capacity of a single edge $(u, v) \in E$ is increased by k. Give an $\mathcal{O}(V + E)$-time algorithm to update the maximum flow. Justify the correctness of your algorithm.

3. (10 marks) Consider the following scenario. Due to population growth, a shift in demographics and a push to have children walk to school to increase fitness, the city of Toronto needs to rezone it's schools (some schools currently have too many students and some not enough, some are just too far away). That is, for each school the catchment area needs to be adjusted so that children in the neighbourhoods are assigned to schools within 1km of their home while ensuring that schools are not overloaded with too many students. Suppose that the census has identified that there are n children in Toronto and k schools. Each of the n children needs to be assigned to a school such that no school has more than $\lceil \frac{n}{k} \rceil$ students and the child's home is within 1km of their school.

   Give a polynomial-time algorithm that given the distances for each child from all schools, determines whether it is possible to assign each child to a school. Hint: Set up a network flow...