University of Toronto at Scarborough

CSCC73H3 Algorithm Design and Analysis, FALL 2018

Assignment No.5.Q1: Dynamic Programming and Network Flow

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1. **Description**

   The maximum s, t-flow in G has value k. Therefore there are k disjoint path from s to t. Then say each path is consisted of nodes $v_1, v_2, ...v_n$. Then make a *pulse* operation to find where it is disconnected. So first make a *pulse* on the $v_{\frac{n}{2}}$. If this signal sends that it is not connected, we look for first half $(v_1, v_2, ...v_{\frac{n}{2}})$. If this is not, then look for the second half. This is similar to binary searching. If we find node v_i such that $v_1, v_2, ...v_i$ is connected as $v_{i+1}, ..., v_n$ is not connected, we know that the nodes from $v_1, v_2, ...v_i$ is connected nodes, and $v_{i+1}, ..., v_n$ is not. Do the algorithm to the next disjoint path.

   Complexity

   For each of the path, finding the node v_i takes $O(logn)$ time complexity, given that the number of node in that path is at most n. And we have k disjoint path. Therefore the complexity is $O(klogn)$.

2. **Description**

First, use Fold-Fulkerson algorithm to make a max flow out of the flow network G. Then, set A consisted of nodes that are reachable from source by residual graph is *upstream*. And set B consisted of nodes that are reachable to sink by residual graph is *downstream*. Any node else is *central*.

Complexity

The complexity of my algorithm is same as the time required to compute a single maximum flow. We just need to find A by bfs starting from s. And we just need to find B by bfs starting from t with reversed residual graph.

Proof

Let's say that the set S contains s and $a_1, a_2, ...a_i$. Then at the residual graph, if we consider A and $V - A$

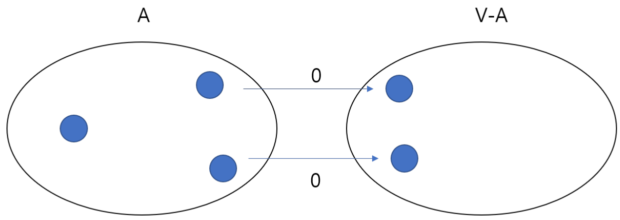

Figure 1: A and V-A

Then the edges from A to $V - A$ will be 0, by the definition of A. And there is no set that has less nodes than A and still be minimum cut. Also, every minimum cut has to include nodes in A. If not, say A' is the minimum cut without one node in A. then the min-cut capacity will increase by the residual graph edge from A' to that one node. Therefore every min cut has to include nodes in A. Similary, every *downstream* should include B. Therefore the *upstream* is A and *downstream* is B, and *central* is $V - A - B$.