University of Toronto at Scarborough

**CSCC73H3 Algorithm Design and Analysis, FALL 2018**

**Assignment No.5: Dynamic Programming and Network Flow**

**DUE:** November 8, 2018, at 11:59 pm

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1. On Assignment No.6
2. **Description**
   (a) The statement is false. The counter example is below.

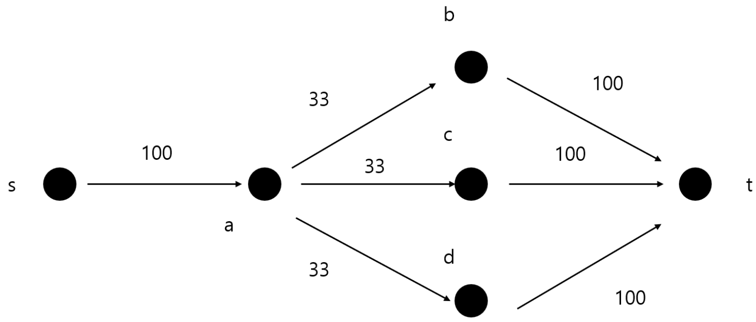


Figure 1: Original Network $G$

At Figure 1 the *minimum* $s-t$ *cut* is s and a. After adding 1 to each edges, the graph follows.

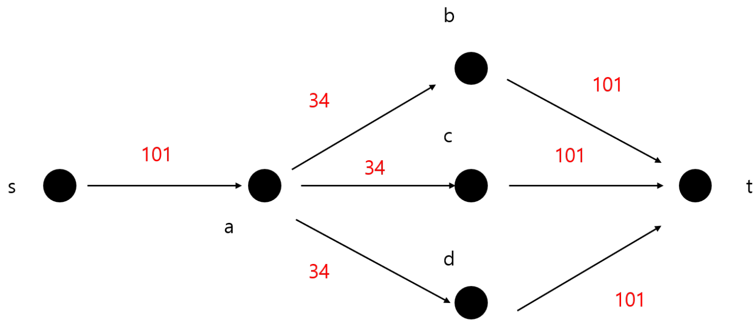


Figure 2: One Added Network $G'$

We can see that now the *minimum* $s-t$ *cut* is only s.
By showing counterexample, the statement is false.

(b) $(u, v)$ is increased by k. Then use the max flow algorithm covered in class. This is done by finding path from $s$ to $u$ and a path from $v$ to $t$. Finding path could be done by DFS, which has complexity of $O(V + E)$. Therefore the algorithm takes $O(V + E)$ time complexity.

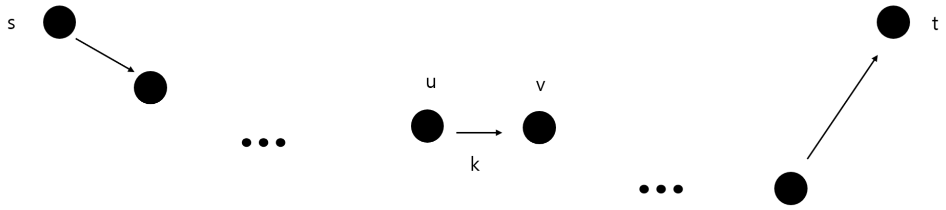


Figure 3: k added Network

This is enough because if the addition of $k$ only affects this one path $p$.

**Proof**

My algorithm will add residual graph by any value that $p$ allowed. Then any other possible path from $s$ to $p$ on the newly edited graph should use the reverse direction of $p$. If not, it contradicts that the original network is maximum network flow. If

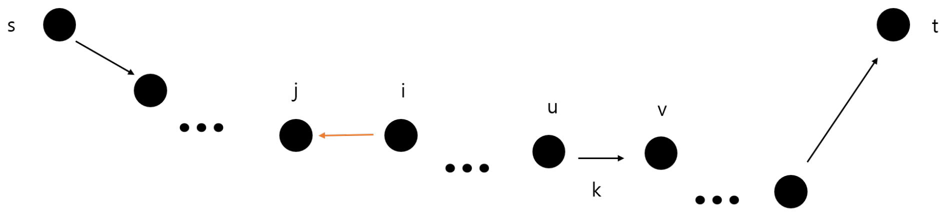


Figure 4: Network with i j

we use any residual direction path of reverse path $p$, let's call that edge $(i, j)$. Both $i$ and $j$ is in path $p$. Because there is a path from $s$ to $i$ to $j$ to $t$(this is what we have assumed). Then this means at the original network, there is a path from $s$ to $i$ to $t$ or $s$ to $j$ to $t$. This depends on the position of edge $(u, v)$. If it was before $(j, i)$ on $p$, it means that the original network has a path from $s$ to $i$ to $t$ and contradiction rises. Otherwise if the $(u, v)$ was after $(j, i)$, then there was path from $s$ to $j$ to $t$. This also rise contradiction.

3. **Description**

First set up a network by Figure5.

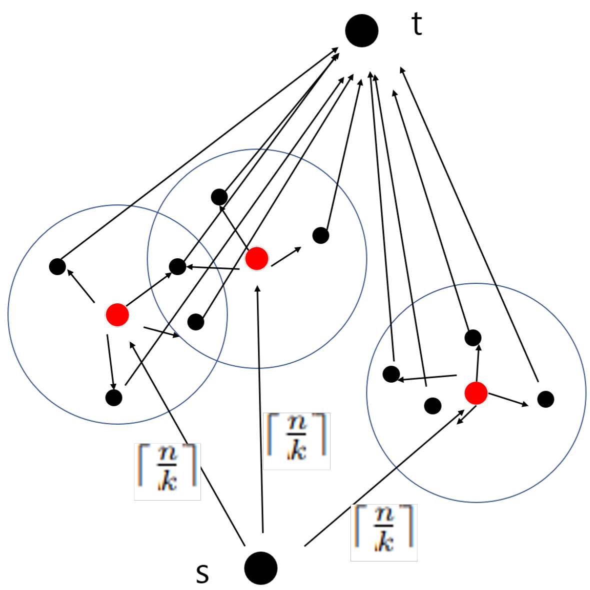


Figure 5: Network Flow of School and Houses

The boundary represents the school catchment area which is 1 km radius circle. The middle of the circle is school. The small circles represent houses of students.

We set up a network by adding $\lceil \frac{n}{k} \rceil$ to every edge from $s$ to each school. The school has edge 1 to every houses. The houses have edge 1 to $t$.

Then we do the maximum network flow algorithm covered in class.