University of Toronto at Scarborough
CSCC73 - Algorithm Design and Analysis

Assignment #8: Simplex Method, Dual LP and Review

Due: Dec 3, 2018 at 11.59pm This exercise is worth 5% of your final grade.

Warning: Your electronic submission on MarkUs affirms that this exercise is your own work and no one else's, and is in accordance with the University of Toronto Code of Behaviour on Academic Matters, the Code of Student Conduct, and the guidelines for avoiding plagiarism in CSCC73. Late assignments will not be accepted. If you are working with a partner your partners' name must be listed on your assignment and you must sign up as a "group" on MarkUs. Recall you must not consult **any outside sources except your partner, textbook, TAs and instructor**.

1. (10 marks) Consider the following LP problem:

   $$\max 18x_1 + 12.5x_2$$

   subject to

   $$\begin{aligned} x_1 + x2 &\leq 20 \\ x_1 &\leq 12 \\ x_2 &\leq 16 \\ x_1, x_2 &\geq 0 \end{aligned}$$

   (a) Solve the program using the SIMPLEX method.

   (b) Write the dual LP and show that your solution in part (a) is optimal.

2. (10 marks) Given an unlimited supply of coins of denominations $x_1, x_2, \ldots, x_n$, we wish to make change for a value v; that is, we wish to find a set of coins whose total value is v. This might not be possible: for instance, if the denominations are 5 and 10 then we can make change for 15 but not for 12.

   Give an $\mathcal{O}(nv)$ dynamic-programming algorithm for the following problem.

   INPUT: $x_1, \ldots, x_n; v$.

   Is it possible to make change for v using coins of denominations $x_1, \ldots, x_n$?

3. (10 marks) Give a divide and conquer algorithm to find a *"1/3-majority"* element in an array A, if such an element exists (i.e., find an element that appears in more than $n/3$ positions of A). Your algorithm is only allowed to compare array elements for equality (i.e., it cannot tell if one element is less than or greater than another, only if it is equal or not).

   Analyze the running time of your algorithm, which should be as efficient as possible.