University of Toronto at Scarborough
CSCC73 - Algorithm Design and Analysis

Assignment #2: Greedy Algorithms

Due: September 29, 2018 at 11.59pm This exercise is worth 5% of your final grade.

Warning: Your electronic submission on MarkUs affirms that this exercise is your own work and no one else's, and is in accordance with the University of Toronto Code of Behaviour on Academic Matters, the Code of Student Conduct, and the guidelines for avoiding plagiarism in CSCC73. Late assignments will not be accepted. If you are working with a partner your partners' name must be listed on your assignment and you must sign up as a "group" on MarkUs. Recall you must not consult **any outside sources except your partner, textbook, TAs and instructor**.

1. Consider a communications graph. Each edge of the connected graph $G = (V, E)$ represents a communication link between sites (represented as nodes). Each edge e has a bandwidth b_e.

   Each pair of nodes $u, v \in V$ needs to be able to communicate. For any $u, v - path\ P$ the *bottleneck transmission rate* $b(P)$ of P is the minimum bandwidth of any edge it contains. In other words, $b(P) = min_{e \in P} b_e$. The *best achievable bottleneck rate* for a pair $u, v \in V$ is the maximum, over all $u - v$ paths P in G, of the value $b(P)$. Our goal is to determine a set of $u - v$ paths for each pair $u, v \in V$ with best achievable bottleneck rate.

   Fortunately, we can construct a spanning tree T of G such that for every pair of nodes $u, v \in V$ the unique $u - v$ path in the tree T actually achieves the best achievable bottleneck rate for u, v in G.

   Give an efficient algorithm to construct such a spanning tree. Your algorithm should construct a spanning tree T in which, for each $u, v \in V$, the bottleneck rate of the $u - v$ path in T is equal to the best achievable bottleneck rate for the pair u, v in G. Prove the correctness of your algorithm.

2. A new store is opening and needs to stock their shelves with product. They have a budget of X dollars to initially spend to stock the store. The store manager needs to decide which products to buy (and then sell) in the hopes of turning the greatest profit. The catalogue of items lists for each item, the available quantity, wholesale cost for purchase as well as MSRP (manufacturers suggested retail price).

   The store manager decides to do his homework and creates a table listing for each item i, m_i the MSRP per unit, c_i the cost per unit and available quantity q_i in units. For example, the following might be the table for 4 items:

Item i	A	B	C	D
Cost/unit c_i	\$40	\$100	\$20	\$25
MSRP m_i	\$100	\$240	\$80	\$75
Available Quantity q_i	30	200	500	200

   The *Available Quantity* refers to the quantity in stock available to purchase (Note that any fraction of this quantity may be purchased). The *MSRP* is the suggested retail price and the *Cost* is the wholesale cost. The profit made by a store is defined to be the *MSRP - Cost*.

   Suppose for this example, the total budget is $X = \$16K$, one can verify that if the store keeper buys all of products D and C and 5/6 the available quantity of A, (s)he will have spent $\$16K$ and total profit will be maximized at \$41500.

   Help the store keeper by providing an algorithm to determine which items to select and in what quantity to maximize profit.