University of Toronto at Scarborough

CSCC73H3 Algorithm Design and Analysis, FALL 2018

Assignment No.7: Linear Programming

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1. (a) Let's say the number of package A that the company decides to make is x, and the number of package B y. Then the objective function is the profit. The total dried fruits is $x + 2y$ lb and the total nuts is $1.5x + y$ lb. So buying the bulk is total $4x + 4y$ dollars. Also, packaging is $1.4x + 0.6y$ dollars. The money by selling is $7x + 6y$ dollars. So the profit is $(7x + 6y) - (4x + 4y) - (1.4x + 0.6y)$. Therefore the objective function is $1.6x + 1.4y$.

   Now the constraints.

   Because factory can only produce 110000 of A, $x \leq 110000$.

   Because there are total 240000 lb of dried fruts and 180000lb of nuts, $x + 2y \leq 240000$ and $1.5x + y \leq 180000$.

   And because there should be at least one package for A and B, $x \geq 0$ and $y \geq 0$.

   (b) Following is the plotted graph.

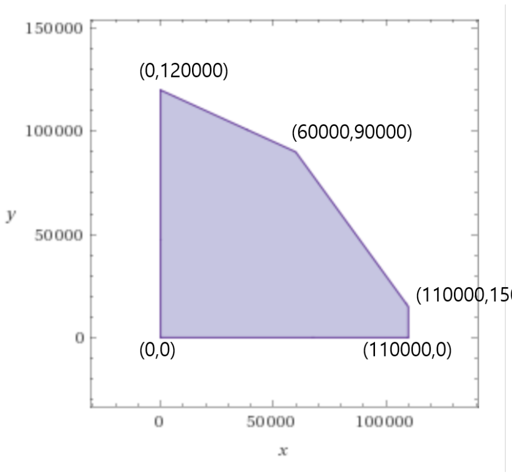

Figure 1: Feasible Reigion

   The vertex maximizing the profit is $(60000, 90000)$.

   (c) Let's use the simplex method.

The objective function is $1.6x+1.4y$. and the constraints are $x \leq 110000$, $x+2y \leq 240000$, $1.5x+y \leq 180000$, $x \geq 0$, $y \geq 0$. Let's label each constraints c_i. (i = 1, 2, 3, 4, 5)

Then increase y to make c_3 tight. so $y = 120000$. And lets shift to make a new origin. Let $x_1 = x$ and $y_1 = 240000 - x - 2y$. Then we have our constraints with x_1 and y_1. c_2 is tight, c_5 is released.

The new constraints are $x_1 \leq 110000, y_1 \geq 0, x_1 - 0.5y_1 \leq 60000, x_1 \geq 0, x_1 + y_1 \leq 240000$. And the new objective function is $0.9x_1 - 0.7y_1 + 168000$. Now x_1 has positive coefficient, so increasing x_1 increases the objective function.

Then c_4 is released, c_3 is tight. So let $x_2 = 60000 - x_1 + 0.5y_1$, $y_2 = y_1$. So the new constraints are $-x_2+0.5y_2 \leq 50000, y_2 \geq 0, x_2 \geq 0, x_2 - 0.5y_2 \leq 60000, x_2 - 1.5y_2 \geq -180000$. And the objective function is $222000 - 0.25y_2 - 0.9x_2$. The coefficients are all negative, and this is the optimal point. The optimal value is 222000.

2. All the value $|ax_i + by_i - c|$ must be bounded by the maximum value,$\max_{1 \leq i \leq 7} |ax_i + by_i - c|$. Finding the minimum value of this maximum value is to make a new variable k, which will be the maximum value.

   Then the objective function is k. We will try to minimize this.

   Because every $|ax_i + by_i - c|$ is bounded by k. The constraints are the following.

   $|10a + 19b - c| \leq k$

   $|8a + 15b - c| \leq k$

   $|7a + 14b - c| \leq k$

   $|5a + 11b - c| \leq k$

   $|3a + 7b - c| \leq k$

   $|2a + 5b - c| \leq k$

   $|1a + 3b - c| \leq k$

   But this is not linear, so making the constraints to follow linear programming, the real constraints are following.

   $10a + 19b - c \leq k$

   $10a + 19b - c \geq -k$

   $8a + 15b - c \leq k$

   $8a + 15b - c \geq -k$

   $7a + 14b - c \leq k$

   $7a + 14b - c \geq -k$

   $5a + 11b - c \leq k$

   $5a + 11b - c \geq -k$

   $3a + 7b - c \leq k$

   $3a + 7b - c \geq -k$

   $2a + 5b - c \leq k$

   $2a + 5b - c \geq -k$

$$a + 3b - c \leq k$$
$$a + 3b - c \geq -k$$