$\boxed{\text{問 1}}$

$$\sin 75^\circ = \sin(45^\circ + 30^\circ)$$
$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$
$$= \boldsymbol{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

$$\tan 75^\circ = \tan(45^\circ + 30^\circ)$$
$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$$
$$= \frac{1+\frac{1}{\sqrt{3}}}{1 - 1\cdot\frac{1}{\sqrt{3}}}$$
$$= \frac{\left(1+\frac{1}{\sqrt{3}}\right)\times\sqrt{3}}{\left(1-\frac{1}{\sqrt{3}}\right)\times\sqrt{3}}$$
$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$
$$= \frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)}$$
$$= \frac{3+2\sqrt{3}+1}{3-1}$$
$$= \frac{4+2\sqrt{3}}{2} = \boldsymbol{2+\sqrt{3}}$$

【別解】

$$\tan 75^\circ = \frac{\sin 75^\circ}{\cos 75^\circ} = \frac{\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{\sqrt{6}-\sqrt{2}}{4}}$$
$$= \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \frac{(\sqrt{6}+\sqrt{2})^2}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})}$$
$$= \frac{6+2\cdot 2\sqrt{3}+2}{6-2}$$
$$= \frac{8+4\sqrt{3}}{4} = \boldsymbol{2+\sqrt{3}}$$

$$\sin 15^\circ = \sin(45^\circ - 30^\circ)$$
$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$
$$= \boldsymbol{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

$$\cos 15^\circ = \cos(45^\circ - 30^\circ)$$
$$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$
$$= \boldsymbol{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

$$\tan 15^\circ = \tan(45^\circ - 30^\circ)$$
$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$
$$= \frac{1-\frac{1}{\sqrt{3}}}{1 + 1\cdot\frac{1}{\sqrt{3}}}$$
$$= \frac{\left(1-\frac{1}{\sqrt{3}}\right)\times\sqrt{3}}{\left(1+\frac{1}{\sqrt{3}}\right)\times\sqrt{3}}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$
$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$$
$$= \frac{3-2\sqrt{3}+1}{3-1}$$
$$= \frac{4-2\sqrt{3}}{2} = \boldsymbol{2-\sqrt{3}}$$

【別解】

$$\tan 15^\circ = \frac{\sin 15^\circ}{\cos 15^\circ} = \frac{\frac{\sqrt{6}-\sqrt{2}}{4}}{\frac{\sqrt{6}+\sqrt{2}}{4}}$$
$$= \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \frac{(\sqrt{6}-\sqrt{2})^2}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}$$
$$= \frac{6-2\cdot 2\sqrt{3}+2}{6-2}$$

$$=\frac{8-4\sqrt{3}}{4}=\mathbf{2-\sqrt{3}}$$

問2

$$\text{与式}=\frac{\tan\theta+\tan\frac{\pi}{4}}{1-\tan\theta\tan\frac{\pi}{4}}$$

$$=\frac{\tan\theta+1}{1-\tan\theta\cdot 1}$$

$$=\boldsymbol{\frac{1+\tan\theta}{1-\tan\theta}}$$

問3

$\alpha$は第2象限の角だから，$\cos\alpha<0$

よって

$$\cos\alpha=-\sqrt{1-\sin^2\alpha}$$

$$=-\sqrt{1-\left(\frac{1}{3}\right)^1}$$

$$=-\sqrt{1-\frac{1}{9}}$$

$$=-\sqrt{\frac{8}{9}}=-\frac{2\sqrt{2}}{3}$$

$\beta$は第2象限の角だから，$\cos\beta>0$

よって

$$\cos\beta=\sqrt{1-\sin^2\beta}$$

$$=\sqrt{1-\left(-\frac{2}{5}\right)^2}$$

$$=\sqrt{1-\frac{4}{25}}$$

$$=\sqrt{\frac{21}{25}}=\frac{\sqrt{21}}{5}$$

したがって

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$=\frac{1}{3}\cdot\frac{\sqrt{21}}{5}+\left(-\frac{2\sqrt{2}}{3}\right)\cdot\left(-\frac{2}{5}\right)$$

$$=\frac{\sqrt{21}}{15}+\frac{4\sqrt{2}}{15}$$

$$=\boldsymbol{\frac{\sqrt{21}+4\sqrt{2}}{15}}$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$=\left(-\frac{2\sqrt{2}}{3}\right)\cdot\left(\frac{\sqrt{21}}{5}\right)-\frac{1}{3}\cdot\left(-\frac{2}{5}\right)$$

$$=-\frac{2\sqrt{42}}{15}+\frac{2}{15}$$

$$=\boldsymbol{\frac{2-2\sqrt{42}}{15}}$$

問4

$0<\alpha<\frac{\pi}{2}$，$0<\beta<\frac{\pi}{2}$の辺々を加えると

$$0<\alpha+\beta<\frac{\pi}{2}+\frac{\pi}{2}$$

すなわち，$0<\alpha+\beta<\pi$・・・①

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}$$

$$=\frac{\frac{5}{6}}{1-\frac{1}{6}}=\frac{\frac{5}{6}}{\frac{5}{6}}=\mathbf{1}$$

また，①より，$\alpha+\beta=\boldsymbol{\frac{\pi}{4}}$

問5

$\alpha$は第4象限の角だから，$\sin\alpha<0$

よって

$$\sin\alpha=-\sqrt{1-\cos^2\alpha}$$

$$=-\sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$=-\sqrt{1-\frac{9}{25}}$$

$$=-\sqrt{\frac{16}{25}}=-\frac{4}{5}$$

したがって

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$

$$=2\cdot\left(-\frac{4}{5}\right)\cdot\frac{3}{5}$$

$$=-\boldsymbol{\frac{24}{25}}$$

$$\cos 2\alpha=2\cos^2\alpha-1$$

$$=2\cdot\left(\frac{3}{5}\right)^2-1$$

$$=\frac{18}{25}-1=-\boldsymbol{\frac{7}{25}}$$

$$\tan 2\alpha=\frac{\sin 2\alpha}{\cos 2\alpha}$$

$$=\frac{-\frac{24}{25}}{-\frac{7}{25}}=\mathbf{\frac{24}{7}}$$

問 6

$$\cos^2\frac{\pi}{8}=\cos^2\frac{\frac{\pi}{4}}{2}$$

$$=\frac{1+\cos\frac{\pi}{4}}{2}$$

$$=\frac{1+\frac{1}{\sqrt{2}}}{2}$$

$$=\frac{\left(1+\frac{\sqrt{2}}{2}\right)\times 2}{2\times 2}$$

$$=\frac{2+\sqrt{2}}{4}$$

$\cos\frac{\pi}{8}>0$ であるから

$$\cos\frac{\pi}{8}=\sqrt{\frac{2+\sqrt{2}}{4}}=\boldsymbol{\frac{\sqrt{2+\sqrt{2}}}{2}}$$

問 7

$\frac{\pi}{2}<\alpha<\pi$より，$\frac{\pi}{4}<\frac{\alpha}{2}<\frac{\pi}{2}$ ・・・①

$$\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}$$

$$=\frac{1-\left(-\frac{7}{9}\right)}{2}$$

$$=\frac{\frac{16}{9}}{2}=\frac{8}{9}$$

①より，$\sin\frac{\alpha}{2}>0$ であるから

$$\sin\frac{\alpha}{2}=\sqrt{\frac{8}{9}}=\boldsymbol{\frac{2\sqrt{2}}{3}}$$

また

$$\cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}$$

$$=\frac{1+\left(-\frac{7}{9}\right)}{2}$$

$$=\frac{\frac{2}{9}}{2}=\frac{1}{9}$$

①より，$\cos\frac{\alpha}{2}>0$ であるから

$$\cos\frac{\alpha}{2}=\sqrt{\frac{1}{9}}=\boldsymbol{\frac{1}{3}}$$

$$\tan\frac{\alpha}{2}=\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$$

$$=\frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}}=\boldsymbol{2\sqrt{2}}$$

問 8

（１）与式 $=\frac{1}{2}\{\cos(2\theta+5\theta)+\cos(2\theta-5\theta)\}$

$$=\frac{1}{2}\{\cos 7\theta+\cos(-3\theta)\}$$

$$=\boldsymbol{\frac{1}{2}(\cos 7\theta+\cos 3\theta)}$$

（２）与式 $=-\frac{1}{2}\{\cos(4\theta+3\theta)-\cos(4\theta-3\theta)\}$

$$=-\frac{1}{2}(\cos 7\theta-\cos\theta)$$

$$=\boldsymbol{\frac{1}{2}(\cos\theta-\cos 7\theta)}$$

（３）与式 $=\frac{1}{2}\{\sin(5\theta+3\theta)-\sin(5\theta-3\theta)\}$

$$=\boldsymbol{\frac{1}{2}(\sin 8\theta-\sin 2\theta)}$$

問 9

（１）与式 $=2\sin\frac{5\theta+3\theta}{2}\cos\frac{5\theta-3\theta}{2}$

$$=2\sin\frac{8\theta}{2}\cos\frac{2\theta}{2}$$

$$=\boldsymbol{2\sin 4\theta\cos\theta}$$

（２）与式 $=2\cos\frac{2\theta+4\theta}{2}\cos\frac{2\theta-4\theta}{2}$

$$=2\cos\frac{6\theta}{2}\cos\frac{-2\theta}{2}$$

$$=2\cos 3\theta\cos(-\theta)$$

$$=\boldsymbol{2\cos 3\theta\cos\theta}$$

（３）与式 $=2\cos\frac{6\theta+2\theta}{2}\sin\frac{6\theta-2\theta}{2}$

$$=2\cos\frac{8\theta}{2}\sin\frac{4\theta}{2}$$

$$=\boldsymbol{2\cos 4\theta\sin 2\theta}$$

問 10

（１）$y=\sqrt{1^2+1^2}\sin(x+\alpha)$

$=\sqrt{2}\sin(x+\alpha)$

ここで,

$\cos\alpha=\dfrac{1}{\sqrt{2}}$, $\sin\alpha=\dfrac{1}{\sqrt{2}}$ より, $\alpha=\dfrac{\pi}{4}$

よって, $\boldsymbol{y=\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)}$

（２）$y=\sqrt{1^2+\left(\sqrt{3}\right)^2}\sin(x+\alpha)$

$=2\sin(x+\alpha)$

ここで,

$\cos\alpha=\dfrac{1}{2}$, $\sin\alpha=-\dfrac{\sqrt{3}}{2}$ より, $\alpha=-\dfrac{\pi}{3}$

よって, $\boldsymbol{y=2\sin\left(x-\dfrac{\pi}{3}\right)}$

問 11

$y=\sqrt{1^2+(-1)^2}\sin(x+\alpha)$

$=\sqrt{2}\sin(x+\alpha)$

また, $\cos\alpha=\dfrac{1}{\sqrt{2}}$, $\sin\alpha=-\dfrac{1}{\sqrt{2}}$ より, $\alpha=-\dfrac{\pi}{4}$

$y=\sqrt{2}\sin\left(x-\dfrac{\pi}{4}\right)$

ここで, $0\leqq x\leqq 2\pi$であるから

$-\dfrac{\pi}{4}\leqq x-\dfrac{\pi}{4}\leqq 2\pi-\dfrac{\pi}{4}$

$x-\dfrac{\pi}{4}=\dfrac{\pi}{2}$ すなわち, $x=\dfrac{3}{4}\pi$のとき

最大値　$\sqrt{2}\sin\dfrac{\pi}{2}=\sqrt{2}$

$x-\dfrac{\pi}{4}=\dfrac{3}{2}\pi$ すなわち, $x=\dfrac{7}{4}\pi$のとき

最小値　$\sqrt{2}\sin\dfrac{3}{2}\pi=-\sqrt{2}$

したがって,

**最大値**　$\boldsymbol{\sqrt{2}}$　$\boldsymbol{\left(x=\dfrac{3}{4}\pi\right)}$

**最大値**　$\boldsymbol{-\sqrt{2}}$　$\boldsymbol{\left(x=\dfrac{7}{4}\pi\right)}$