

# Standard Code Library

Your TeamName

Your School

October 3, 2023

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## 一切的开始

### 快读

```
1  #define gc() (is==it?it=(is=in)+fread(in,1,Q,stdin),(is==it?EOF:*is++):*is++)
2  const int Q=(1<<24)+1;
3  char in[Q],*is=in,*it=in,c;
4  void read(long long &n){
5      for(n=0;(c=gc())<'0' || c>'9');
6      for(;c<='9'&&c>='0';c=gc())n=n*10+c-48;
7  }
```

### 代码模板

```
1  #include <bits/stdc++.h>
2
3  using namespace std;
4  #define dbg(x...) \
5      do { \
6          cout << #x << " -> "; \
7          err(x); \
8      } while (0)
9
10 void err() {
11     cout << endl;
12 }
13
14 template<class T, class... Ts>
15 void err(T arg, Ts &... args) {
16     cout << arg << ' ';
17     err(args...);
18 }
19
20 typedef long long ll;
21 typedef pair<int, int> pii;
22 const int N = 1e5 + 10, Log = 20, inf = 0x3f3f3f3f;
23
24 void solve() {
25
26 }
27
28 int main() {
29     int T = 1;
30     ios::sync_with_stdio(false);
31     cin >> T;
32     while (T--) solve();
33     return 0;
34 }
```

## 数据结构

### st 表

$st[i][j]$  表示区间  $[i, i + 2^j - 1]$  的 gcd

```
1  int st[N][Log + 5], logx[N];
2
3  void init(int n) {
4      logx[0] = -1;
5      for (int i = 1; i <= n; i++) logx[i] = logx[i >> 1] + 1;
6      for (int i = 1; i <= n; i++) st[i][0] = i;
7      for (int j = 1; (1 << j) <= n; j++) {
8          for (int i = 1; i + (1 << j) - 1 <= n; i++) {
9              st[i][j] = __gcd(st[i][j - 1], st[i + (1 << (j - 1))][j - 1]);
10         }
11     }
12 }
13
14 int query(int l, int r){
```

```

15     int k = logx[r - l + 1];
16     return __gcd(st[l][k], st[r - (1 << k) + 1][k]);
17 }

```

## 树状数组

```

1  template <typename T>
2  struct Fenwick {
3      const int n;
4      vector<T> a;
5      Fenwick(int n) : n(n), a(n + 1) {}
6      void add(int x, T v) {
7          while(x <= n){
8              a[x] += v;
9              x += x & -x;
10         }
11     }
12     T sum(int x) {
13         T ans = 0;
14         for (int i = x; i; i -= i & -i) {
15             ans += a[i];
16         }
17         return ans;
18     }
19     T rangeSum(int l, int r) {
20         return sum(r) - sum(l - 1);
21     }
22 };

```

## 主席树

```

1  #include <bits/stdc++.h>
2
3  using namespace std;
4  typedef long long ll;
5  typedef pair<int, int> pii;
6  const int N = 200010;
7  int root[N], tot = 0, num[N], len, a[N];
8  struct Info {
9      int sum, l, r;
10 } info[N << 5];
11
12 int getid(int x) {
13     return lower_bound(num + 1, num + len + 1, x) - num;
14 }
15
16 void build(int &x, int l, int r) { //创建空树
17     x = ++tot;
18     info[x].sum = 0;
19     if (l == r) return;
20     int mid = (l + r) / 2;
21     build(info[x].l, l, mid);
22     build(info[x].r, mid + 1, r);
23 }
24
25 void update(int pre, int &now, int l, int r, int q) { //更新
26     now = ++tot;
27     info[now] = info[pre];
28     info[now].sum++;
29     if (l == r) return;
30     int mid = (l + r) / 2;
31     if (mid >= q) update(info[pre].l, info[now].l, l, mid, q);
32     else update(info[pre].r, info[now].r, mid + 1, r, q);
33 }
34
35 int query(int pre, int now, int l, int r, int k) { //求第 k 小
36     if (l == r) return l;
37     int delta = info[info[now].l].sum - info[info[pre].l].sum;
38     int mid = (l + r) / 2;
39     if (delta >= k) return query(info[pre].l, info[now].l, l, mid, k);

```

```

40     else return query(info[pre].r, info[now].r, mid + 1, r, k - delta);
41 }
42
43 int query_sum(int pre, int now, int l, int r, int k) { // 求小于等于 k 的个数
44     if (l == r) return info[now].sum - info[pre].sum;
45     int mid = (l + r) >> 1;
46     if (k <= mid) return query_sum(info[pre].l, info[now].l, l, mid, k);
47     else return (info[info[now].l].sum - info[info[pre].l].sum) + query_sum(info[pre].r, info[now].r, mid + 1, r, k);
48 }
49 /*
50  先进行离散化
51  sort(num + 1, num + 1 + n);
52  len = unique(num + 1, num + 1 + n) - num - 1;
53  建空树
54  build(root[0], 1, len);
55  更新
56  update(root[i - 1], root[i], 1, len, getid(a[i]));
57  查询 [l, r]
58  query_sum(root[l - 1], root[r], 1, len, k)
59  query(root[l - 1], root[r], 1, len, k)
60  */

```

## 树链剖分

```

1  vector<int> e[N];
2
3  int n;
4  int sz[N], f[N], son[N];
5  int top[N], dfn[N], rk[N], tot, ru[N];
6  int a[N];
7
8  void dfs(int u, int fa){
9      sz[u] = 1;
10     f[u] = fa;
11     son[u] = -1;
12     for(int i : e[u]){
13         if(i == fa) continue;
14         dfs(i, u);
15         sz[u] += sz[i];
16         if(son[u] == -1 || sz[i] > sz[son[u]]) son[u] = i;
17     }
18 }
19
20 void dfs1(int u, int t){
21     top[u] = t;
22     dfn[u] = ++tot;
23     rk[tot] = u;
24     if(son[u] == -1){
25         ru[u] = dfn[u];
26         return;
27     }
28     dfs1(son[u], t);
29     for(int i : e[u]){
30         if(i == f[u] || i == son[u]) continue;
31         dfs1(i, i);
32     }
33     ru[u] = tot;
34 }
35
36 template<typename T>
37 struct SegmentTree{
38     T sum[N << 2], lz[N << 2];
39     void apply(int k, int l, int r, T x){
40         sum[k] += (r - l + 1) * x;
41         lz[k] += x;
42     }
43     void pd(int k, int l, int r){ // push down
44         int mid = (l + r) >> 1;
45         apply(k << 1, l, mid, lz[k]);
46         apply(k << 1 | 1, mid + 1, r, lz[k]);
47         lz[k] = 0;

```

```

48     }
49     void pu(int k){// push up
50         sum[k] = sum[k << 1] + sum[k << 1 | 1];
51     }
52     void build(int k, int l, int r){
53         if(l == r){
54             sum[k] = a[rk[l]];
55             lz[k] = 0;
56             return;
57         }
58         int mid = (l + r) >> 1;
59         build(k << 1, l, mid);
60         build(k << 1 | 1, mid + 1, r);
61         pu(k);
62     }
63     void mdf(int k, int l, int r, int ql, int qr, T x){// modify [ql, qr] add x
64         if(l > qr || r < ql) return;
65         if(l >= ql && r <= qr){
66             sum[k] += (r - l + 1) * x;
67             lz[k] += x;
68             return;
69         }
70         pd(k, l, r);
71         int mid = (l + r) >> 1;
72         mdf(k << 1, l, mid, ql, qr, x);
73         mdf(k << 1 | 1, mid + 1, r, ql, qr, x);
74         pu(k);
75     }
76     T query(int k, int l, int r, int ql, int qr){
77         if(l > qr || r < ql) return 0;
78         if(l >= ql && r <= qr){
79             return sum[k];
80         }
81         pd(k, l, r);
82         int mid = (l + r) >> 1;
83         return query(k << 1, l, mid, ql, qr) + query(k << 1 | 1, mid + 1, r, ql, qr);
84     }
85 };
86
87 SegmentTree<ll> seg;
88
89 int qrysum(int u, int v){
90     int fu = top[u], fv = top[v], ret = 0;
91     while(fu != fv){
92         if(dfn[fu] > dfn[fv]){
93             ret += seg.query(1, 1, n, dfn[fu], dfn[u]);
94             u = f[fu];
95         }else{
96             ret += seg.query(1, 1, n, dfn[fv], dfn[v]);
97             v = f[fv];
98         }
99         fu = top[u];
100        fv = top[v];
101    }
102    if(dfn[u] > dfn[v]) swap(u, v);
103    ret += seg.query(1, 1, n, dfn[u], dfn[v]);
104    return ret;
105 }
106
107 void solve() {
108     int m, rt;
109     cin >> n;
110     for(int i = 1; i <= n; i++) cin >> a[i];
111     for(int i = 0, u, v; i < n - 1; i++){
112         cin >> u >> v;
113         e[u].push_back(v);
114         e[v].push_back(u);
115     }
116     dfs(1, 0);
117     dfs1(1, 1);
118     seg.build(1, 1, n);

```

119 }

## 平衡树 Treap

### 普通平衡树

```
1 mt19937 mt(chrono::steady_clock::now().time_since_epoch().count());
2
3 int rng(int l, int r) {
4     uniform_int_distribution<int> uni(l, r);
5     return uni(mt);
6 }
7
8 struct Node{
9     Node *lt, *rt; // 左右子结点
10    int val, prio; // 值, 优先级
11    int cnt, sz; // 重复次数, 子树大小
12
13    Node(int _val) : val(_val), cnt(1), sz(1) {
14        lt = rt = nullptr;
15        prio = rng(1, 1e9);
16    }
17
18    void upd(){
19        sz = cnt;
20        if(lt != nullptr) sz += lt->sz;
21        if(rt != nullptr) sz += rt->sz;
22    }
23 };
24
25 struct Treap{
26     int siz(Node *p){
27         if(p == nullptr) return 0;
28         return p->sz;
29     }
30
31     Node *root;
32
33     pair<Node *, Node *> split(Node *cur, int key) { // 根据 val 分裂成 小于等于 key 和 大于 key 的两个 treap
34         if (cur == nullptr) return {nullptr, nullptr};
35         if (cur->val <= key) { // 当前属于第一个 treap
36             auto temp = split(cur->rt, key);
37             cur->rt = temp.first;
38             cur->upd();
39             return {cur, temp.second};
40         } else { // 当前属于第二个 treap
41             auto temp = split(cur->lt, key);
42             cur->lt = temp.second;
43             cur->upd();
44             return {temp.first, cur};
45         }
46     }
47
48     tuple<Node *, Node *, Node *> split_by_rk(Node *cur, int rk) { // 根据 rk 分裂成 小于 rk 和 等于 rk 和 大于 rk 的三个
49     ↪ treap, 其中第二个只有一个结点
50         if (cur == nullptr) return {nullptr, nullptr, nullptr};
51         int ls_siz = siz(cur->lt); // 左子树大小
52         if (rk <= ls_siz) { // 当前属于第三个 treap
53             Node *l, *mid, *r;
54             tie(l, mid, r) = split_by_rk(cur->lt, rk);
55             cur->lt = r;
56             cur->upd();
57             return {l, mid, cur};
58         } else if (rk <= ls_siz + cur->cnt) { // 当前属于第二个 treap
59             Node *lt = cur->lt;
60             Node *rt = cur->rt;
61             cur->lt = cur->rt = nullptr;
62             return {lt, cur, rt};
63         } else { // 当前属于第一个 treap
64             Node *l, *mid, *r;
65             tie(l, mid, r) = split_by_rk(cur->rt, rk - ls_siz - cur->cnt);
```

```

65         cur->rt = l;
66         cur->upd();
67         return {cur, mid, r};
68     }
69 }
70
71 Node *merge(Node *u, Node *v) { // 按照 prio 小根堆合并
72     if (u == nullptr && v == nullptr) return nullptr;
73     if (u != nullptr && v == nullptr) return u;
74     if (v != nullptr && u == nullptr) return v;
75     if (u->prio < v->prio) {
76         u->rt = merge(u->rt, v);
77         u->upd();
78         return u;
79     } else {
80         v->lt = merge(u, v->lt);
81         v->upd();
82         return v;
83     }
84 }
85
86 void insert(int val) { // 插入
87     auto temp = split(root, val);
88     auto l_tr = split(temp.first, val - 1);
89     Node *new_node;
90     if (l_tr.second == nullptr) {
91         new_node = new Node(val);
92     } else {
93         l_tr.second->cnt++;
94         l_tr.second->upd();
95     }
96     Node *l_tr_combined = merge(l_tr.first, l_tr.second == nullptr ? new_node : l_tr.second);
97     root = merge(l_tr_combined, temp.second);
98 }
99
100 void del(int val) { // 删除
101     auto temp = split(root, val);
102     auto l_tr = split(temp.first, val - 1);
103     if (l_tr.second == nullptr) {
104         root = merge(l_tr.first, temp.second);
105         return;
106     }
107     if (l_tr.second->cnt > 1) {
108         l_tr.second->cnt--;
109         l_tr.second->upd();
110         l_tr.first = merge(l_tr.first, l_tr.second);
111     } else {
112         if (temp.first == l_tr.second) {
113             temp.first = nullptr;
114         }
115         delete l_tr.second;
116         l_tr.second = nullptr;
117     }
118     root = merge(l_tr.first, temp.second);
119 }
120
121 int qrank_by_val(Node *cur, int val) { // 查询 val 的 rk
122     auto temp = split(cur, val - 1);
123     int ret = siz(temp.first) + 1;
124     root = merge(temp.first, temp.second);
125     return ret;
126 }
127
128 int qval_by_rank(Node *cur, int rk) { // 查询 rk 的 val 第 rk 大的值
129     Node *l, *mid, *r;
130     tie(l, mid, r) = split_by_rk(cur, rk);
131     int ret = (mid == nullptr ? -114514 : mid->val);
132     root = merge(merge(l, mid), r);
133     return ret;
134 }
135

```



```

136     int qprev(int val) { // 查询第一个比 val 小的值
137         auto temp = split(root, val - 1);
138         int ret = qval_by_rank(temp.first, temp.first->sz);
139         root = merge(temp.first, temp.second);
140         return ret;
141     }
142
143     int qnex(int val) { // 查询第一个比 val 大的值
144         auto temp = split(root, val);
145         int ret = qval_by_rank(temp.second, 1);
146         root = merge(temp.first, temp.second);
147         return ret;
148     }
149 };

```

## 区间翻转

```

1  mt19937 mt(chrono::steady_clock::now().time_since_epoch().count());
2
3  int rng(int l, int r) {
4      uniform_int_distribution<int> uni(l, r);
5      return uni(mt);
6  }
7
8  struct Node{
9      Node *lt, *rt; // 左右子结点
10     int val, prio; // 值, 优先级
11     int cnt, sz; // 重复次数, 子树大小
12     bool rev; // 是否翻转
13
14     Node(int _val) : val(_val), cnt(1), sz(1) {
15         lt = rt = nullptr;
16         rev = false;
17         prio = rng(1, 1e9);
18     }
19
20     void pu(){
21         sz = cnt;
22         if(lt != nullptr) sz += lt->sz;
23         if(rt != nullptr) sz += rt->sz;
24     }
25
26     void pd(){
27         if(rev){
28             swap(lt, rt);
29             if(lt != nullptr) lt->rev ^= 1;
30             if(rt != nullptr) rt->rev ^= 1;
31             rev = false;
32         }
33     }
34 };
35
36 struct Treap{
37     Node* root;
38     int siz(Node *p){
39         if(p == nullptr) return 0;
40         return p->sz;
41     }
42
43     pair<Node *, Node *> split(Node *cur, int sz){
44         if(cur == nullptr) return {nullptr, nullptr};
45         cur->pd();
46         int lc = siz(cur->lt);
47         if(sz <= lc){
48             auto temp = split(cur->lt, sz);
49             cur->lt = temp.second;
50             cur->pu();
51             return {temp.first, cur};
52         }else{
53             auto temp = split(cur->rt, sz - lc - cur->cnt);
54             cur->rt = temp.first;

```

```

55         cur->pu();
56         return {cur, temp.second};
57     }
58 }
59
60 Node* merge(Node* u, Node* v) { // u 小 v 大
61     if (u == nullptr && v == nullptr) return nullptr;
62     if (u != nullptr && v == nullptr) return u;
63     if (u == nullptr && v != nullptr) return v;
64     u->pd(), v->pd();
65     if (u->prio < v->prio) { // u 为根
66         u->rt = merge(u->rt, v);
67         u->pu();
68         return u;
69     } else {
70         v->lt = merge(u, v->lt);
71         v->pu();
72         return v;
73     }
74 }
75
76 void insert(int val){
77     root = merge(root, new Node(val));
78 }
79
80 void seg_rev(int l, int r) {
81     auto res = split(root, l - 1); // [1, l - 1] [l, n]
82     auto ans = split(res.second, r - l + 1); // [l, r] [r + 1, n]
83     ans.first->rev = true;
84     root = merge(res.first, merge(ans.first, ans.second));
85 }
86
87 void print(Node* cur) {
88     if (cur == nullptr) return;
89     cur->pd();
90     print(cur->lt);
91     printf("%d ", cur->val);
92     print(cur->rt);
93 }
94 };

```

## 数学

### 组合数预处理

```

1  ll f[N], inv[N];
2
3  ll qpow(ll a, ll b) {
4      ll res = 1;
5      while (b) {
6          if (b & 1) res = res * a % mod;
7          a = a * a % mod;
8          b /= 2;
9      }
10     return res;
11 }
12
13 ll C(ll n, ll m) {
14     return f[n] * inv[m] % mod * inv[n - m] % mod;
15 }
16
17 void init(int M) {
18     f[0] = 1;
19     for (int i = 1; i <= M; i++) f[i] = f[i - 1] * i % mod;
20     inv[M] = qpow(f[M], mod - 2);
21     for (int i = M - 1; i >= 0; i--) inv[i] = inv[i + 1] * (i + 1) % mod;
22 }

```

## Exgcd

求解  $xa + yb = c$

有解需满足  $\gcd(a, b) | c$

设解出的一组特解为  $x_0, y_0$  则通解为  $x = x_0 + tb, y = y_0 - ta$

```
1 ll exgcd(ll a, ll b, ll &x, ll &y) {
2     if (!b) {
3         x = 1;
4         y = 0;
5         return a;
6     } else {
7         ll g = exgcd(b, a % b, x, y);
8         ll t = x;
9         x = y;
10        y = t - a / b * y;
11        return g;
12    }
13 }
14
15 ll upper(ll m, ll n) { //向上取整
16     if (m <= 0) return m / n;
17     return (m - 1) / n + 1;
18 }
19
20 ll lower(ll m, ll n) { //向下取整
21     if (m >= 0) return m / n;
22     return (m + 1) / n - 1;
23 }
```

## Lucas 定理

适用于模数为小质数

$$C_n^m \bmod p = C_{n \bmod p}^{m \bmod p} \times C_{\lfloor \frac{n}{p} \rfloor}^{\lfloor \frac{m}{p} \rfloor} \bmod p$$

```
1 ll C(ll n, ll r, ll p) {
2     if (r > n || r < 0) return 0;
3     return f[n] * inv[r] % p * inv[n - r] % p;
4 }
5
6 ll Lucas(ll n, ll m, ll p) {
7     if (m == 0) return 1;
8     return (C(n % p, m % p, p) * Lucas(n / p, m / p, p)) % p;
9 }
```

## 欧拉筛

```
1 const int N = 1e4 + 10, M = 10000;
2 vector<int> p;
3 int vis[N];
4
5 void init() {
6     for (int i = 2; i <= M; i++) {
7         if (!vis[i]) {
8             p.push_back(i);
9         }
10        for (int j = 0; j < p.size() && p[j] * i <= M; j++) {
11            vis[p[j] * i] = 1;
12            if (i % p[j] == 0) {
13                break;
14            }
15        }
16    }
17 }
```

求欧拉函数:  $\phi(n) = n \prod (1 - \frac{1}{p_i})$

```

1  const int N = 1e4 + 10, M = 10000;
2  vector<int> p;
3  int phi[N], vis[N];
4
5  void rua() { //欧拉筛 以及 求欧拉函数
6      for (int i = 2; i <= M; i++) {
7          if (!vis[i]) {
8              p.push_back(i);
9              phi[i] = i - 1;
10         }
11         for (int j = 0; j < p.size() && p[j] * i <= M; j++) {
12             vis[p[j] * i] = 1;
13             if (i % p[j] == 0) {
14                 phi[i * p[j]] = phi[i] * p[j];
15                 break;
16             } else {
17                 phi[i * p[j]] = phi[i] * phi[p[j]];
18             }
19         }
20     }
21 }

```

## 线性基

线性基是一个数的集合，并且每个序列都拥有至少一个线性基，取线性基中若干个数异或起来可以得到原序列中的任何一个数。原序列里面的任意一个数都可以由线性基里面的一些数异或得到线性基里面的任意一些数异或起来都不能得到 0 线性基里面的数的个数唯一，并且在保持性质一的前提下，数的个数是最少的

```

1  ll d[Log + 5];
2
3  void add(ll x) { // 线性基插入
4      for (int i = Log; i >= 0; i--) {
5          if ((x >> i) & 1) {
6              if (d[i]) x ^= d[i];
7              else {
8                  d[i] = x; // 插入成功
9                  break;
10             }
11         }
12     }
13 }

```

## 欧拉降幂

$$a^b \pmod{m} \equiv a^{b \bmod \phi(m) + \phi(m)} \pmod{m} [b \geq \phi(m)]$$

以下代码以计算  $a_l^{a_{l+2} \dots a_r}$  为例

```

1  unordered_map<ll, ll> mp;
2  ll a[N];
3  ll MOD(ll x, ll mod) { return x < mod ? x : x % mod + mod; }
4  ll qpow(ll a, ll b, ll mod) {
5      ll res = 1;
6      while (b) {
7          if (b & 1) res = MOD(res * a, mod);
8          b /= 2;
9          a = MOD(a * a, mod);
10     }
11     return res;
12 }
13 ll phi(ll x) {
14     if (mp[x]) return mp[x];
15     ll res = x;
16     for (ll i = 2; i * i <= x; i++) {
17         if (x % i == 0) {
18             res -= res / i;
19             while (x % i == 0) x /= i;
20         }
21     }

```

```

22     if (x > 1) {
23         res -= res / x;
24     }
25     return mp[x] = res;
26 }
27 ll solve(int l, int r, ll p) {
28     if (p == 1) return MOD(a[l], p);
29     if (l == r) return MOD(a[l], p);
30
31     return qpow(a[l], solve(l + 1, r, phi(p)), p);
32 }

```

## 矩阵快速幂

```

1  const int MOD = 1e9 + 7;
2
3  struct mat {
4      int n;
5      vector<vector<int>> a;
6
7      mat(int n): n(n), a(n, vector<int>(n)){}
8
9      mat operator*(const mat& b) const {
10         mat res(n);
11         for (int i = 0; i < n; i++) {
12             for (int j = 0; j < n; j++) {
13                 for (int k = 0; k < n; k++) {
14                     (res.a[i][j] += 1ll * a[i][k] * b.a[k][j] % MOD) %= MOD;
15                 }
16             }
17         }
18         return res;
19     }
20
21     void print(){
22         for(int i = 0; i < n; i++){
23             for(int j = 0; j < n; j++){
24                 cout << a[i][j] << ' ';
25             }
26             cout << '\n';
27         }
28         cout << '\n';
29     }
30 };
31
32 mat qpow(mat a, ll b) {
33     mat res(a.n);
34     for (int i = 0; i < a.n; i++) {
35         res.a[i][i] = 1;
36     }
37     while (b) {
38         if (b & 1) res = res * a;
39         a = a * a, b >>= 1;
40     }
41     return res;
42 }

```

## 中国剩余定理

$$x = num_i(mod\ r_i)$$

```

1  ll CRT(int n) { //适用于 ri 两两互质
2      ll N = 1, res = 0;
3      for (int i = 1; i <= n; i++) N *= r[i];
4      for (int i = 1; i <= n; i++) {
5          ll m = N / r[i], x, y;
6          exgcd(m, r[i], x, y);
7          res = (res + num[i] * m % N * x % N) % N;
8      }

```

```

9     return (res + N) % N;
10 }

```

### 通解解法

$$x = a_1 (\text{mod } m_1)$$

$$x = a_2 (\text{mod } m_2)$$

$$x = k_1 \times m_1 + a_1 = k_2 \times m_2 + a_2$$

$$k_1 \times m_1 - k_2 \times m_2 = a_2 - a_1$$

运用 `exgcd` 可求得一组解  $(k_1, k_2)$  可将上述两方程化为

$$x = k_1 \times m_1 + a_1 (\text{mod } \text{lcm}(m_1, m_2))$$

若有多个方程依次两两合并即可

### 整除分块

$$\sum_{i=1}^n \lfloor \frac{n}{i} \rfloor$$

```

1  ans = 0;
2  for(int l = 1, r; l <= n; l = r + 1) {
3      r = n / (n / l);
4      ans += n / l * (r - l + 1);
5  }

```

### 拉格朗日插值

设要求的  $n$  次多项式为  $f(k)$ , 已知  $f(x_i)$  ( $1 \leq i \leq n+1$ )

$$f(k) = \sum_{i=1}^{n+1} f(x_i) \prod_{j \neq i} \frac{k - x_j}{x_i - x_j}$$

设要求的  $n$  次多项式为  $f(k)$ , 已知  $f(i)$  ( $1 \leq i \leq n+1$ )

$$f(k) = \sum_{i=1}^{n+1} f(i) \times \frac{\prod_{j=1}^{n+1} (x-j)}{(x-i) \times (-1)^{n+1-i} \times (i-1)! \times (n+1-i)!}$$

以下代码求  $\sum_{i=1}^n i^k$

```

1  ll f[N], inv[N];
2
3  ll qpow(ll a, ll b) {
4      ll res = 1;
5      while (b) {
6          if (b & 1) res = res * a % mod;
7          a = a * a % mod;
8          b /= 2;
9      }
10     return res;
11 }
12
13 ll C(ll n, ll m) {
14     return f[n] * inv[m] % mod * inv[n - m] % mod;
15 }
16
17 void init(int M) {
18     f[0] = 1;
19     for (int i = 1; i <= M; i++) f[i] = f[i - 1] * i % mod;
20     inv[M] = qpow(f[M], mod - 2);
21     for (int i = M - 1; i >= 0; i--) inv[i] = inv[i + 1] * (i + 1) % mod;
22 }
23
24 void solve() { // 对 k+1 次多项式插值, 且横坐标连续
25     int n, k;
26     cin >> n >> k;

```

```

27     vector<ll> y(k + 3);
28     for(int i = 1; i <= k + 2; i++){ // 前 k+2 项
29         y[i] = (y[i - 1] + qpow(i, k)) % mod;
30     }
31     if(n <= k + 2){
32         cout << y[n] << '\n';
33         return;
34     }
35     init(2e6);
36     vector<ll> p(k + 3);
37     ll sum = 1;
38     for(int i = 1; i <= k + 2; i++){
39         p[i] = qpow(n - i, mod - 2);
40         sum = sum * (n - i) % mod;
41     }
42     ll ans = 0;
43     for(int i = 1; i <= k + 2; i++){
44         ll tmp = y[i] * sum % mod * p[i] % mod * inv[i - 1] % mod * inv[k + 2 - i] % mod;
45         if((k + 2 - i) & 1) ans -= tmp;
46         else ans += tmp;
47         ans %= mod;
48         if(ans < 0) ans += mod;
49     }
50     cout << ans;
51 }

```

## FFT

```

1  typedef vector<int> vi;
2  typedef long long ll;
3  typedef pair<int, int> pii;
4
5  typedef complex<double> C;
6  typedef vector<double> vd;
7
8  void fft(vector<C> &a) {
9      int n = (int) a.size(), L = 31 - __builtin_clz(n);
10     static vector<complex<long double>> R(2, 1);
11     static vector<C> rt(2, 1);
12     for (static int k = 2; k < n; k *= 2) {
13         R.resize(n);
14         rt.resize(n);
15         auto x = polar(1.0L, acos(-1.0L) / k);
16         for (int i = k; i < 2 * k; i++) {
17             rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i / 2];
18         }
19     }
20     vi rev(n);
21     for (int i = 0; i < n; i++) {
22         rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
23     }
24     for (int i = 0; i < n; i++) {
25         if (i < rev[i]) swap(a[i], a[rev[i]]);
26     }
27     for (int k = 1; k < n; k *= 2) {
28         for (int i = 0; i < n; i += 2 * k) {
29             for (int j = 0; j < k; j++) {
30                 C z = rt[j + k] * a[i + j + k];
31                 a[i + j + k] = a[i + j] - z;
32                 a[i + j] += z;
33             }
34         }
35     }
36 }
37
38 vd conv(const vd &a, const vd &b) {
39     if (a.empty() || b.empty()) return {};
40     vd res((int) a.size() + (int) b.size() - 1);
41     int L = 32 - __builtin_clz((int) res.size()), n = 1 << L;
42     vector<C> in(n), out(n);
43     copy(a.begin(), a.end(), begin(in));

```

```

44     for (int i = 0; i < (int) b.size(); i++) {
45         in[i].imag(b[i]);
46     }
47     fft(in);
48     for (C &x: in) x *= x;
49     for (int i = 0; i < n; i++) {
50         out[i] = in[-i & (n - 1)] - conj(in[i]);
51     }
52     fft(out);
53     for (int i = 0; i < (int) res.size(); i++) {
54         res[i] = imag(out[i]) / (4 * n);
55     }
56     return res;
57 }
58
59 using vll = vector<ll>;
60
61 vll gao(const vi &a, const vi &b) { //a 和 b 的卷积
62     vd aa((int) a.size()), bb((int) b.size());
63     for (int i = 0; i < (int) a.size(); i++) aa[i] = a[i];
64     for (int j = 0; j < (int) b.size(); j++) bb[j] = b[j];
65
66     vd cc = conv(aa, bb);
67     vll c((int) cc.size());
68     for (int i = 0; i < (int) c.size(); i++) c[i] = round(cc[i]);
69     return c;
70 }

```

## NTT

```

1  typedef long long ll;
2  typedef __int128 i128;
3  const int N = 1e6 + 5;
4  const ll mod = 4179340454199820289, G = 3, Gi = 1393113484733273430;
5
6  ll qpow(ll a, ll b, ll p) {
7      ll res = 1;
8      while (b) {
9          if (b & 1) res = (i128) res * a % p;
10         b >>= 1;
11         a = (i128) a * a % p;
12     }
13     return res;
14 }
15
16 ll f[N], g[N];
17 int bit, tot, rev[N];
18
19 void NTT(ll a[], int type) {
20     for (int i = 0; i < tot; i++)
21         if (i > rev[i])
22             swap(a[i], a[rev[i]]);
23     for (int mid = 1; mid < tot; mid <= 1) {
24         ll w1 = qpow(type == 1 ? G : Gi, (mod - 1) / (mid * 2), mod);
25         for (int i = 0; i < tot; i += mid * 2) {
26             ll wk = 1;
27             for (int j = 0; j < mid; j++, wk = (i128) wk * w1 % mod) {
28                 ll x = a[i + j], y = (i128) wk * a[i + j + mid] % mod;
29                 a[i + j] = (x + y) % mod, a[i + j + mid] = (x - y + mod) % mod;
30             }
31         }
32     }
33     if (type == -1) {
34         ll inv = qpow(tot, mod - 2, mod);
35         for (int i = 0; i < tot; i++)
36             a[i] = (i128) a[i] * inv % mod;
37     }
38 }
39
40 void gao(){
41     int n = 0, m = 0; // f, g 长度

```



```

42     for(int i = 0; i < n; i++) f[i] = 0;
43     for (int i = 0; i < m; i++) g[i] = 0;
44     while ((1 << bit) <= n + m) bit++;
45     tot = 1 << bit;
46     for (int i = 0; i < tot; i++)
47         rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (bit - 1));
48     for (int i = n; i < tot; i++) f[i] = 0;
49     for (int i = m; i < tot; i++) g[i] = 0;
50     NTT(f, 1), NTT(g, 1);
51     for (int i = 0; i < tot; i++) f[i] = (i128) f[i] * g[i] % mod;
52     NTT(f, -1);
53 }

```

## 高斯消元

```

1  const double eps = 1e-8;
2
3  int sgn(double x) {
4      if (fabs(x) < eps) return 0;
5      if (x < 0) return -1;
6      return 1;
7  }
8
9  double a[4][4], x[4], b[4][4], y[4];
10 int equ, var;
11
12 int Gauss() {
13     int i, j, k, col, max_r;
14     for (k = 0, col = 0; k < equ && col < var; ++k, ++col) {
15         max_r = k;
16         for (i = k + 1; i < equ; ++i) if (fabs(a[i][col]) > fabs(a[max_r][col])) max_r = i;
17         if (fabs(a[max_r][col]) < eps) return 0;
18         if (k != max_r) {
19             for (j = col; j < var; ++j) swap(a[k][j], a[max_r][j]);
20             swap(x[k], x[max_r]);
21         }
22         x[k] /= a[k][col];
23         for (j = col + 1; j < var; ++j) a[k][j] /= a[k][col];
24         a[k][col] = 1;
25         for (i = 0; i < equ; ++i) {
26             if (i != k) {
27                 x[i] -= x[k] * a[i][col];
28                 for (j = col + 1; j < var; ++j) a[i][j] -= a[k][j] * a[i][col];
29                 a[i][col] = 0;
30             }
31         }
32     }
33     return 1;
34 }
35
36 int Gauss(int n, int m){ // equ var
37     equ = n;
38     var = m;
39     for(int i = 0; i < n; i++){
40         for(int j = 0; j < m; j++){
41             a[i][j] = b[i][j];
42         }
43         x[i] = y[i];
44     }
45     if(!Gauss()) return 0;
46     for(int i = 0; i < n; i++){
47         double res = 0;
48         for(int j = 0; j < m; j++){
49             res += x[j] * b[i][j];
50         }
51         if(sgn(res - y[i])) return 0;
52     }
53     return 1;
54 }

```

## 图论

### LCA

#### 倍增求法

```
1  const int N = 100010, Log = 20;
2  int anc[N][Log + 5], depth[N];
3  vector<int> e[N];
4
5  void dfs(int u, int fa) {
6      anc[u][0] = fa;
7      depth[u] = depth[fa] + 1;
8      for (int i : e[u]) {
9          if(i == fa) continue;
10         dfs(i, u);
11     }
12 }
13
14 void init(int root, int n) { //初始化
15     depth[0] = 0;
16     dfs(root, 0);
17     for (int j = 1; j <= Log; j++) {
18         for (int i = 1; i <= n; i++) {
19             anc[i][j] = anc[anc[i][j - 1]][j - 1];
20         }
21     }
22 }
23
24 int rush(int u, int h) {
25     for (int i = 0; i <= Log; i++) {
26         if (h >> i & 1) u = anc[u][i];
27     }
28     return u;
29 }
30
31 int qry(int x, int y) { // 求 x 和 y 的 lca
32     if (depth[x] < depth[y]) swap(x, y);
33     x = rush(x, depth[x] - depth[y]);
34     if (x == y) return x;
35     for (int i = Log; i >= 0; i--) {
36         if (anc[x][i] != anc[y][i]) {
37             x = anc[x][i];
38             y = anc[y][i];
39         }
40     }
41     return anc[x][0];
42 }
```

## 计算几何

### 二维几何

```
1  namespace Geometry {
2  using T = ll;
3  constexpr T eps = 0;
4
5  bool eq(const T &x, const T &y) { return abs(x - y) <= eps; }
6  inline constexpr int type(T x, T y) {
7      if(x == 0 and y == 0) return 0;
8      if(y < 0 or (y == 0 and x > 0)) return -1;
9      return 1;
10 }
11 struct Point {
12     T x, y;
13     constexpr Point(T _x = 0, T _y = 0) : x(_x), y(_y) {}
14     constexpr Point operator+() const noexcept { return *this; }
15     constexpr Point operator-() const noexcept { return Point(-x, -y); }
16     constexpr Point operator+(const Point &p) const { return Point(x + p.x, y + p.y); }
```

```

17 constexpr Point operator-(const Point &p) const { return Point(x - p.x, y - p.y); }
18 constexpr Point &operator+=(const Point &p) { return x += p.x, y += p.y, *this; }
19 constexpr Point &operator-=(const Point &p) { return x -= p.x, y -= p.y, *this; }
20 constexpr T operator*(const Point &p) const { return x * p.x + y * p.y; }
21 constexpr Point &operator*=(const T &k) { return x *= k, y *= k, *this; }
22 constexpr Point operator*(const T &k) { return Point(x * k, y * k); }
23 constexpr bool operator==(const Point &r) const noexcept { return r.x == x and r.y == y; }
24 constexpr T cross(const Point &r) const { return x * r.y - y * r.x; }
25
26 constexpr bool operator<(const Point &r) const { return pair(x, y) < pair(r.x, r.y); }
27
28 // 1 : left, 0 : same, -1 : right
29 constexpr int toleft(const Point &r) const {
30     auto t = cross(r);
31     return t > eps ? 1 : t < -eps ? -1 : 0;
32 }
33
34 constexpr bool arg_cmp(const Point &r) const {
35     int L = type(x, y), R = type(r.x, r.y);
36     if(L != R) return L < R;
37
38     T X = x * r.y, Y = r.x * y;
39     if(X != Y) return X > Y;
40     return x < r.x;
41 }
42 };
43 bool arg_cmp(const Point &l, const Point &r) { return l.arg_cmp(r); }
44 ostream &operator<<(ostream &os, const Point &p) { return os << p.x << " " << p.y; }
45 istream &operator>>(istream &is, Point &p) {
46     is >> p.x >> p.y;
47     return is;
48 }
49
50 struct Line {
51     Point a, b;
52     Line() = default;
53     Line(Point a, Point b) : a(a), b(b) {}
54     // ax + by = c
55     Line(T A, T B, T C) {
56         if(A == 0) {
57             a = Point(0, C / B), b = Point(1, C / B);
58         } else if(B == 0) {
59             a = Point(C / A, 0), b = Point(C / A, 1);
60         } else {
61             a = Point(0, C / B), b = Point(C / A, 0);
62         }
63     }
64     // 1 : left, 0 : same, -1 : right
65     constexpr int toleft(const Point &r) const {
66         auto t = (b - a).cross(r - a);
67         return t > eps ? 1 : t < -eps ? -1 : 0;
68     }
69
70     friend std::ostream &operator<<(std::ostream &os, Line &ls) {
71         return os << "{"
72             << "(" << ls.a.x << ", " << ls.a.y << "), (" << ls.b.x << ", " << ls.b.y << ")}";
73     }
74 };
75 istream &operator>>(istream &is, Line &p) { return is >> p.a >> p.b; }
76
77 struct Segment : Line {
78     Segment() = default;
79     Segment(Point a, Point b) : Line(a, b) {}
80 };
81
82 ostream &operator<<(ostream &os, Segment &p) { return os << p.a << " to " << p.b; }
83 istream &operator>>(istream &is, Segment &p) {
84     is >> p.a >> p.b;
85     return is;
86 }
87

```

```

88 struct Circle {
89     Point p;
90     T r;
91     Circle() = default;
92     Circle(Point p, T r) : p(p), r(r) {}
93 };
94
95 using pt = Point;
96 using Points = vector<pt>;
97 using Polygon = Points;
98 T cross(const pt &x, const pt &y) { return x.x * y.y - x.y * y.x; }
99 T dot(const pt &x, const pt &y) { return x.x * y.x + x.y * y.y; }
100
101 T abs2(const pt &x) { return dot(x, x); }
102 // http://judge.u-aizu.ac.jp/onlinejudge/description.jsp?id=CGL_1_C
103 // 点の回転方向
104 int ccw(const Point &a, Point b, Point c) {
105     b = b - a, c = c - a;
106     if(cross(b, c) > 0) return +1; // "COUNTER_CLOCKWISE"
107     if(cross(b, c) < 0) return -1; // "CLOCKWISE"
108     if(dot(b, c) < 0) return +2; // "ONLINE_BACK"
109     if(abs2(b) < abs2(c)) return -2; // "ONLINE_FRONT"
110     return 0; // "ON_SEGMENT"
111 }
112
113 // http://judge.u-aizu.ac.jp/onlinejudge/description.jsp?id=CGL_2_A
114 // 平行判定
115 bool parallel(const Line &a, const Line &b) { return (cross(a.b - a.a, b.b - b.a) == 0); }
116
117 // http://judge.u-aizu.ac.jp/onlinejudge/description.jsp?id=CGL_2_A
118 // 垂直判定
119 bool orthogonal(const Line &a, const Line &b) { return (dot(a.a - a.b, b.a - b.b) == 0); }
120
121 bool intersect(const Line &l, const Point &p) { return abs(ccw(l.a, l.b, p)) != 1; }
122
123 bool intersect(const Line &l, const Line &m) { return !parallel(l, m); }
124
125 bool intersect(const Segment &s, const Point &p) { return ccw(s.a, s.b, p) == 0; }
126
127 bool intersect(const Line &l, const Segment &s) { return cross(l.b - l.a, s.a - l.a) * cross(l.b - l.a, s.b - l.a) <=
    ↪ 0; }
128
129 // http://judge.u-aizu.ac.jp/onlinejudge/description.jsp?id=CGL_2_B
130 bool intersect(const Segment &s, const Segment &t) { return ccw(s.a, s.b, t.a) * ccw(s.a, s.b, t.b) <= 0 && ccw(t.a,
    ↪ t.b, s.a) * ccw(t.a, t.b, s.b) <= 0; }
131
132 bool intersect(const Polygon &ps, const Polygon &qs) {
133     int pl = si(ps), ql = si(qs), i = 0, j = 0;
134     while((i < pl or j < ql) and (i < 2 * pl) and (j < 2 * ql)) {
135         auto ps0 = ps[(i + pl - 1) % pl], ps1 = ps[i % pl];
136         auto qs0 = qs[(j + ql - 1) % ql], qs1 = qs[j % ql];
137         if(intersect(Segment(ps0, ps1), Segment(qs0, qs1))) return true;
138         Point a = ps1 - ps0;
139         Point b = qs1 - qs0;
140         T v = cross(a, b);
141         T va = cross(qs1 - qs0, ps1 - ps0);
142         T vb = cross(ps1 - ps0, qs1 - ps0);
143
144         if(!v and va < 0 and vb < 0) return false;
145         if(!v and !va and !vb) {
146             i += 1;
147         } else if(v >= 0) {
148             if(vb > 0)
149                 i += 1;
150             else
151                 j += 1;
152         } else {
153             if(va > 0)
154                 j += 1;
155             else
156                 i += 1;

```

```

157     }
158 }
159 return false;
160 }
161
162 T norm(const Point &p) { return p.x * p.x + p.y * p.y; }
163 Point projection(const Segment &l, const Point &p) {
164     T t = dot(p - l.a, l.a - l.b) / norm(l.a - l.b);
165     return l.a + (l.a - l.b) * t;
166 }
167
168 Point crosspoint(const Line &l, const Line &m) {
169     T A = cross(l.b - l.a, m.b - m.a);
170     T B = cross(l.b - l.a, l.b - m.a);
171     if(A == 0 and B == 0) return m.a;
172     return m.a + (m.b - m.a) * (B / A);
173 }
174
175 // http://judge.u-aizu.ac.jp/onlinejudge/description.jsp?id=CGL_2_C
176 Point crosspoint(const Segment &l, const Segment &m) { return crosspoint(Line(l), Line(m)); }
177
178 // http://judge.u-aizu.ac.jp/onlinejudge/description.jsp?id=CGL_3_B
179 // 凸性判定
180 bool is_convex(const Points &p) {
181     int n = (int)p.size();
182     for(int i = 0; i < n; i++) {
183         if(ccw(p[(i + n - 1) % n], p[i], p[(i + 1) % n]) == -1) return false;
184     }
185     return true;
186 }
187
188 Points convex_hull(Points p) {
189     int n = p.size(), k = 0;
190     if(n <= 2) return p;
191     sort(begin(p), end(p), [](pt x, pt y) { return (x.x != y.x ? x.x < y.x : x.y < y.y); });
192     Points ch(2 * n);
193     for(int i = 0; i < n; ch[k++] = p[i++]) {
194         while(k >= 2 && cross(ch[k - 1] - ch[k - 2], p[i] - ch[k - 1]) <= 0) --k;
195     }
196     for(int i = n - 2, t = k + 1; i >= 0; ch[k++] = p[i--]) {
197         while(k >= t && cross(ch[k - 1] - ch[k - 2], p[i] - ch[k - 1]) <= 0) --k;
198     }
199     ch.resize(k - 1);
200     return ch;
201 }
202
203 // 面積の 2 倍
204 T area2(const Points &p) {
205     T res = 0;
206     rep(i, si(p)) { res += cross(p[i], p[(i == si(p) - 1 ? 0 : i + 1)]); }
207     return res;
208 }
209
210 enum { _OUT, _ON, _IN };
211
212 int contains(const Polygon &Q, const Point &p) {
213     bool in = false;
214     for(int i = 0; i < Q.size(); i++) {
215         Point a = Q[i] - p, b = Q[(i + 1) % Q.size()] - p;
216         if(a.y > b.y) swap(a, b);
217         if(a.y <= 0 && 0 < b.y && cross(a, b) < 0) in = !in;
218         if(cross(a, b) == 0 && dot(a, b) <= 0) return _ON;
219     }
220     return in ? _IN : _OUT;
221 }
222
223 Polygon Minkowski_sum(const Polygon &P, const Polygon &Q) {
224     vector<Segment> e1(P.size()), e2(Q.size()), ed(P.size() + Q.size());
225     const auto cmp = [](const Segment &u, const Segment &v) { return (u.b - u.a).arg_cmp(v.b - v.a); };
226     rep(i, P.size()) e1[i] = {P[i], P[(i + 1) % P.size()]};
227     rep(i, Q.size()) e2[i] = {Q[i], Q[(i + 1) % Q.size()]};

```

```

228 rotate(begin(e1), min_element(all(e1), cmp), end(e1));
229 rotate(begin(e2), min_element(all(e2), cmp), end(e2));
230 merge(all(e1), all(e2), begin(ed), cmp);
231 const auto check = [] (const Points &res, const Point &u) {
232     const auto back1 = res.back(), back2 = *prev(end(res), 2);
233     return eq(cross(back1 - back2, u - back2), eps) and dot(back1 - back2, u - back1) >= -eps;
234 };
235 auto u = e1[0].a + e2[0].a;
236 Points res{u};
237 res.reserve(P.size() + Q.size());
238 for(const auto &v : ed) {
239     u = u + v.b - v.a;
240     while(si(res) >= 2 and check(res, u)) res.pop_back();
241     res.eb(u);
242 }
243 if(res.size() and check(res, res[0])) res.pop_back();
244 return res;
245 }
246
247 // -1 : on, 0 : out, 1 : in
248 // O(log(n))
249 int is_in(const Polygon &p, const Point &a) {
250     if(p.size() == 1) return a == p[0] ? -1 : 0;
251     if(p.size() == 2) return intersect(Segment(p[0], p[1]), a);
252     if(a == p[0]) return -1;
253     if((p[1] - p[0]).toleft(a - p[0]) == -1 || (p.back() - p[0]).toleft(a - p[0]) == 1) return 0;
254     const auto cmp = [&](const Point &u, const Point &v) { return (u - p[0]).toleft(v - p[0]) == 1; };
255     const size_t i = lower_bound(p.begin() + 1, p.end(), a, cmp) - p.begin();
256     if(i == 1) return intersect(Segment(p[0], p[i]), a) ? -1 : 0;
257     if(i == p.size() - 1 && intersect(Segment(p[0], p[i]), a)) return -1;
258     if(intersect(Segment(p[i - 1], p[i]), a)) return -1;
259     return (p[i] - p[i - 1]).toleft(a - p[i - 1]) > 0;
260 }
261
262 Points halfplane_intersection(vector<Line> L, const T inf = 1e9) {
263     Point box[4] = {Point(inf, inf), Point(-inf, inf), Point(-inf, -inf), Point(inf, -inf)};
264     rep(i, 4) { L.emplace_back(box[i], box[(i + 1) % 4]); }
265     sort(all(L), [](const Line &l, const Line &r) { return (l.b - l.a).arg_cmp(r.b - r.a); });
266     deque<Line> dq;
267     int len = 0;
268     auto check = [](const Line &a, const Line &b, const Line &c) { return a.toleft(crosspoint(b, c)) == -1; };
269     rep(i, L.size()) {
270         while(dq.size() > 1 and check(L[i], *(end(dq) - 2), *(end(dq) - 1))) dq.pop_back();
271         while(dq.size() > 1 and check(L[i], dq[0], dq[1])) dq.pop_front();
272         // dump(L[i], si(dq));
273
274         if(dq.size() and eq(cross(L[i].b - L[i].a, dq.back().b - dq.back().a), 0)) {
275             if(dot(L[i].b - L[i].a, dq.back().b - dq.back().a) < eps) return {};
276             if(L[i].toleft(dq.back().a) == -1)
277                 dq.pop_back();
278             else
279                 continue;
280         }
281         dq.emplace_back(L[i]);
282     }
283
284     while(dq.size() > 2 and check(dq[0], *(end(dq) - 2), *(end(dq) - 1))) dq.pop_back();
285     while(dq.size() > 2 and check(dq.back(), dq[0], dq[1])) dq.pop_front();
286     if(si(dq) < 3) return {};
287     Polygon ret(dq.size());
288     rep(i, dq.size()) ret[i] = crosspoint(dq[i], dq[(i + 1) % dq.size()]);
289     return ret;
290 }
291 } // namespace Geometry
292
293 using namespace Geometry;

```

Andrew

```

1 const double eps = 1e-9, pi = acos(-1.0);
2 const int N = 1e5 + 10;

```

```

3
4  int n, cnt, m;
5
6  int sgn(double x) {
7      if(fabs(x) < eps) return 0;
8      if(x > 0) return 1;
9      return -1;
10 }
11
12 struct point {
13     double x, y;
14     point(double a = 0.0, double b = 0.0) : x(a), y(b) {}
15     bool operator < (point t) {
16         if(sgn(x - t.x) == 0) return y < t.y;
17         return x < t.x;
18     }
19     point operator - (point p){
20         return {x - p.x, y - p.y};
21     }
22     double operator ^ (point p){
23         return x * p.y - y * p.x;
24     }
25 }p[N], ans[N];
26
27 double dis(point a, point b) {
28     a = a - b;
29     return sqrt(a.x * a.x + a.y * a.y);
30 }
31
32 void Andrew() {
33     sort(p, p + n);
34     int p1 = 0, p2;
35     for(int i = 0; i < n; i++) {
36         while(p1 > 1 && sgn((ans[p1] - ans[p1 - 1]) ^ (p[i] - ans[p1 - 1])) <= 0) p1--;
37         ans[++p1] = p[i];
38     }
39     p2 = p1;
40     for(int i = n - 2; i >= 0; i--) {
41         while(p2 > p1 && sgn((ans[p2] - ans[p2 - 1]) ^ (p[i] - ans[p2 - 1])) <= 0) p2--;
42         ans[++p2] = p[i];
43     }
44     double target = 0.0;
45     for(int i = 1; i < p2; i++){
46         target += dis(ans[i], ans[i + 1]);
47     }
48     printf("%.2f\n", target);
49 }
50 /*
51 usage:
52 scanf("%d", &n);
53 for(int i = 0; i < n; i++)
54     scanf("%lf%lf", &p[i].x, &p[i].y);
55 Andrew();
56 */

```

## CHT

```

1 // 维护上凸壳
2 struct Line {
3     ll k, b;
4     double intersect(Line l) {
5         //交点 x 坐标
6         double db = l.b - b;
7         double dk = k - l.k;
8         return db / dk;
9     }
10
11     ll calc (int x) {
12         return k * x + b;
13     }
14 };

```

```

15
16 struct CHT {
17     vector<double> x; // 相邻线交点
18     vector<Line> line; // 线
19
20     void init(Line l) {
21         x.push_back(-inf);
22         line.push_back(l);
23     }
24
25     void addLine(Line l) {
26         while (line.size() >= 2 && l.intersect(line[line.size() - 2]) <= x.back()) {
27             x.pop_back();
28             line.pop_back();
29         }
30         x.push_back(l.intersect(line.back()));
31         line.push_back(l);
32     }
33
34     ll query(int qx) {
35         int id = upper_bound(x.begin(), x.end(), qx) - x.begin() - 1; // 计算点属于的线 id
36         return line[id].calc(qx);
37     }
38 };

```

## 字符串

### KMP

```

1 int nxt[N];
2 string a, b;
3 //a 为模式串 b 为匹配串
4
5 int kmp(int n, int m){
6     int res = 0;
7     nxt[0] = -1;
8     for(int j = -1, i = 0; i < n;){
9         if(j == -1 || a[j] == a[i]){
10             i++;j++;
11             nxt[i] = j;
12         }else{
13             j = nxt[j];
14         }
15     }
16     //i 模式串 j 匹配串
17     for(int i = 0, j = 0; j < m; ){
18         if(i == -1 || a[i] == b[j]){
19             i++;j++;
20         }else i = nxt[i];
21         if(i == n){
22             res += 1;
23             // position:: j - n + 1
24             i = nxt[i];
25         }
26     }
27     return res;
28 }

```

### 序列自动机

```

1 构建:
2 for(int i = n; i >= 1; i--){
3     for(int j = 0; j < 26; j++) ne[i - 1][j] = ne[i][j];
4     ne[i - 1][s[i - 1] - 'a'] = i;
5 }
6
7 求三 (或多个) 个串的公共子序列个数:
8 int dfs(int p1, int p2, int p3){
9     if(f[p1][p2][p3]) return f[p1][p2][p3];

```



```

10     for(int i = 0; i < 26; i++){
11         if(ne[0][p1][i] && ne[1][p2][i] && ne[2][p3][i]){
12             f[p1][p2][p3] = (f[p1][p2][p3] + dfs(ne[0][p1][i], ne[1][p2][i], ne[2][p3][i])) % mod;
13         }
14     }
15     f[p1][p2][p3] = (f[p1][p2][p3] + 1) % mod;
16     return f[p1][p2][p3];
17 }

```

## 字典树

```

1 //对数排序 查找排序后第 k 个数 每个数 <= 1e9
2 const int N = 5000010; // 总长度
3 int trie[N][10], tot, sum[N][10], ssum = 0;
4 int color[N];
5
6 void insert(int x){
7     int r = 1e9, p = 0;
8     ssum++;
9     for(int i = 0; i < 10; i++, r /= 10){
10         int c = x / r;
11         c %= 10;
12         sum[p][c]++;
13         if(!trie[p][c]) trie[p][c] = ++tot;
14         p = trie[p][c];
15     }
16     color[p]++;
17 }
18
19 int find(int k){
20     int res = 0, p = 0;
21     while(k > 0){
22         for(int i = 0; i < 10; i++){
23             if(sum[p][i] < k) k -= sum[p][i];
24             else{
25                 res = res * 10 + i;
26                 p = trie[p][i];
27                 k -= color[p];
28                 break;
29             }
30         }
31     }
32     return res;
33 }

```

## 字符串双哈希

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4 typedef long long ll;
5 typedef pair<int, int> pii;
6 typedef pair<ll, ll> pll;
7
8 const int N = 1e5 + 10;
9 const pii mod = {1e9 + 7, 1e9 + 9};
10 const pii base = {131, 251};
11 pll pw[N];
12
13 pll operator*(const pll &p1, const pll &p2) {
14     return {p1.first * p2.first % mod.first, p1.second * p2.second % mod.second};
15 }
16
17 pll operator+(const pll &p1, const pll &p2) {
18     return {(p1.first + p2.first) % mod.first, (p1.second + p2.second) % mod.second};
19 }
20
21 pll operator-(const pll &p1, const pll &p2) {
22     return {(p1.first - p2.first + mod.first) % mod.first, (p1.second - p2.second + mod.second) % mod.second};
23 }

```

```

24
25 struct Hash {
26     vector<pll> f;
27     int n{};
28
29     void init(ll ss[], int _n) {
30         n = _n;
31         f.resize(n + 1, {0, 0});
32         for (int i = 1; i <= n; i++) {
33             ll ch = ss[i];
34             f[i] = f[i - 1] * base + pll{ch, ch};
35         }
36     }
37
38     pll ask(int l, int r) { //[l, r]
39         return f[r] - f[l - 1] * pw[r - l + 1];
40     }
41 };
42 //记得初始化 pw
43 //pw[0] = {1, 1};
44 //for (int i = 1; i <= n; i++) pw[i] = pw[i - 1] * base;

```

## 杂项

### 莫队

时间复杂度  $O(\frac{n^2}{S} + mS)$ ,  $n$  为长度,  $m$  个询问, 块长为  $S$  (一般取  $\sqrt{n}$  或  $\frac{n}{\sqrt{m}}$ )

```

1  int unit;
2  int a[N];
3
4  struct node {
5      int l, r, id;
6
7      bool operator < (const node &k) const {
8          if (l / unit != k.l / unit) return l / unit < k.l / unit;
9          return r < k.r;
10     }
11 } q[N];
12 void add(int i) {
13
14 }
15
16 void sub(int i) {
17
18 }
19 void solve(){
20     unit = (int)sqrt(m); // m 个区间
21     sort(q + 1, q + 1 + m);
22     int L = 1, R = 0;
23     for (int i = 1; i <= m; i++) {
24         while (R < q[i].r) {
25             R++;
26             add(R);
27         }
28         while (R > q[i].r) {
29             sub(R);
30             R--;
31         }
32         while (L > q[i].l) {
33             L--;
34             add(L);
35         }
36         while (L < q[i].l) {
37             sub(L);
38             L++;
39         }
40     }
41 }

```

## unordered\_map

```
1 struct HashFunc{
2     static uint64_t splitmix64(uint64_t x) {
3         // http://xorshift.di.unimi.it/splitmix64.c
4         x += 0x9e3779b97f4a7c15;
5         x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
6         x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
7         return x ^ (x >> 31);
8     }
9     template<typename T, typename U>
10    size_t operator()(const std::pair<T, U>& p) const {
11        static const uint64_t FIXED_RANDOM = chrono::steady_clock::now().time_since_epoch().count();
12        return splitmix64(p.first + FIXED_RANDOM) ^ splitmix64(p.second + FIXED_RANDOM);
13    }
14 };
15
16 // 键值比较, 哈希碰撞的比较定义, 需要直到两个自定义对象是否相等
17 struct EqualKey {
18     template<typename T, typename U>
19     bool operator ()(const std::pair<T, U>& p1, const std::pair<T, U>& p2) const {
20         return p1.first == p2.first && p1.second == p2.second;
21     }
22 };
23 unordered_map<pii, int, HashFunc, EqualKey> mp;
```

## DSU

```
1 struct DSU{
2     int f[N];
3     void init(int n){
4         for(int i = 0; i <= n; i++) f[i] = -1;
5     }
6     int find(int x){
7         return f[x] < 0 ? x : f[x] = find(f[x]);
8     }
9     void merge(int x, int y){
10        int fx = find(x), fy = find(y);
11        if(fx == fy) return;
12        if(f[fx] > f[fy]) swap(fx, fy);
13        f[fx] += f[fy];
14        f[fy] = fx;
15    }
16 }dsu;
```

## Floyd 判圈

```
1 //适用于每个点出度唯一的图找环
2 const ll mod = 1099511627776;
3
4 ll calc(ll x){
5     return (x + (x >> 20) + 12345) % mod;
6 }
7
8 void Floyd_Cycle_Detection_Algorithm(){
9     ll p1 = 1611516670, p2 = 1611516670; // 起始点
10    do{
11        p1 = calc(p1); // 移动一次
12        p2 = calc(calc(p2)); // 移动两次
13    }while(p1 != p2);
14    // 存在环
15    ll len = 0; // 环长
16    do{
17        p2 = calc(p2);
18        len++;
19    }while(p1 != p2);
20    p1 = 1611516670; // 寻找环起点
21    ll c1 = 0; // 起点到环起点的距离
22    while(p1 != p2){
23        p1 = calc(p1);
```

```

24         p2 = calc(p2);
25         c1++;
26     }
27     cout << p1 << ' ' << len << ' ' << c1 << '\n';
28 }

```

### 三分搜索

```

1  auto ternary_search = [&](ld l, ld r) {
2      int it = 300;           //set the error limit here
3      while (it--) {
4          ld m1 = l + (r - l) / 3;
5          ld m2 = r - (r - l) / 3;
6          ld f1 = f(m1);      //evaluates the function at m1
7          ld f2 = f(m2);      //evaluates the function at m2
8          if (f1 < f2)
9              l = m1;
10         else
11             r = m2;
12     }
13     return l;                //return the maximum of f(x) in [l, r]
14 };

1  auto ternary_search = [&](int l, int r) {
2      int it = 300;           //set the error limit here
3      while (it--) {
4          int m1 = (l + r) >> 1;
5          int m2 = m1 + 1;
6          ld f1 = f(m1);      //evaluates the function at m1
7          ld f2 = f(m2);      //evaluates the function at m2
8          if (f1 < f2)
9              l = m1;
10         else
11             r = m2;
12     }
13     return l;                //return the maximum of f(x) in [l, r]
14 };

```

### 随机器

```

1  // random shuffle
2  random_device rd;
3  mt19937 rng(rd());
4  shuffle(a + 1, a + 1 + n, rng);

1  // 区间随机
2  mt19937 mt(chrono::steady_clock::now().time_since_epoch().count());
3
4  int rng(int l, int r) {
5      uniform_int_distribution<int> uni(l, r);
6      return uni(mt);
7  }

```