Elements of Machine Learning & Data Science

Winter semester 2024/25

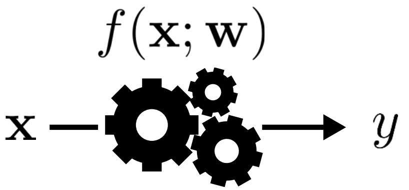
Lecture 8 – Introduction to ML

18.11.2025

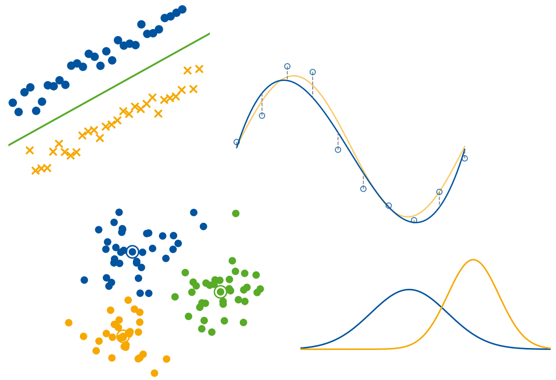
Prof. Bastian Leibe

Machine Learning Topics

8. **Introduction to ML**
9. Probability Density Estimation
10. Linear Discriminants
11. Linear Regression
12. Logistic Regression
13. Support Vector Machines
14. Neural Network Basics



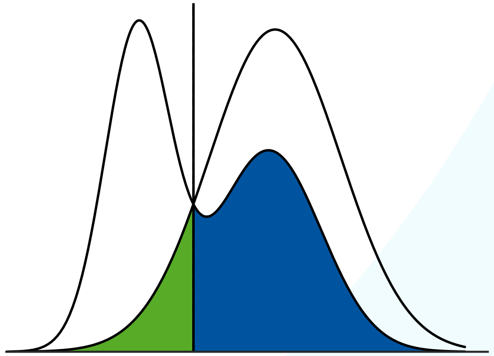
Machine Learning Concepts



Forms of Machine Learning

$$p(\mathcal{C}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{C})p(\mathcal{C})}{p(\mathbf{x})}$$

Bayes Decision Theory



Bayes Optimal Classification

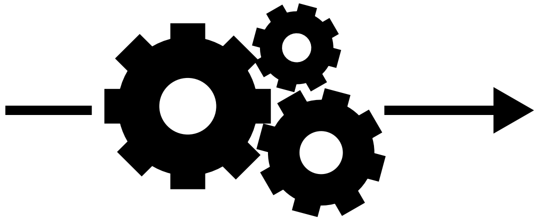
Topics for Today

1. **Motivation**

2. Forms of Learning

3. Terms, Concepts, and Notation

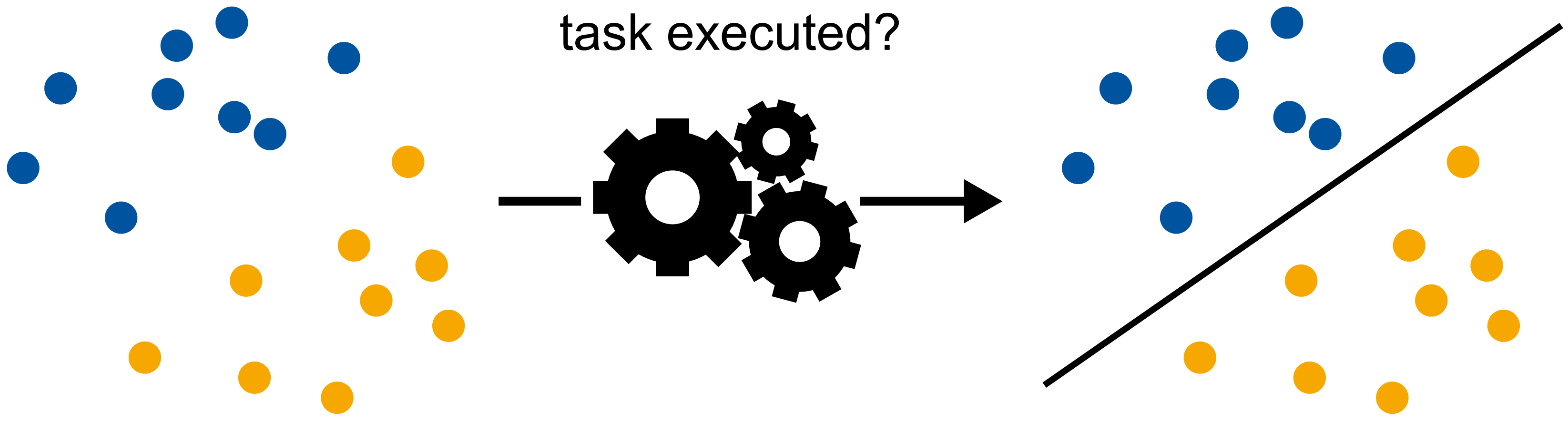
4. Bayes Decision Theory



What is Machine Learning?

Machines that learn to perform a task from experience

Performance:
How well is the task executed?



Experience:
training data

Learning: discover relevant criteria & adapt to given situation

Task: problem to solve (e.g., classification)

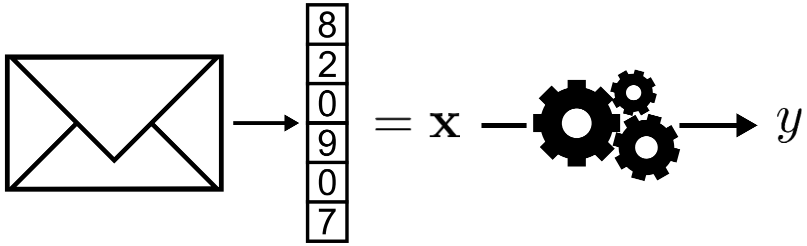
Mathematical Formulation

Machines that learn to perform a task from experience

Often described through a mathematical function:

$$y = f(\mathbf{x}; \mathbf{w})$$

Output y Input $\mathbf{x}$ Parameters $\mathbf{w}$ (learned!)



Discrete targets: Classification

$$y \in \{\text{important}, \text{spam}\}$$

Continuous targets: Regression

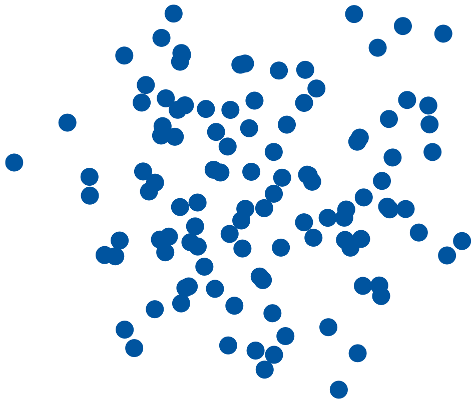
$$y = p(\text{spam}) \in [0, 1]$$

Learning from Data

Machines that learn to perform a task from experience

Learning from collected samples:

$$\mathcal{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\}$$



Labels available?

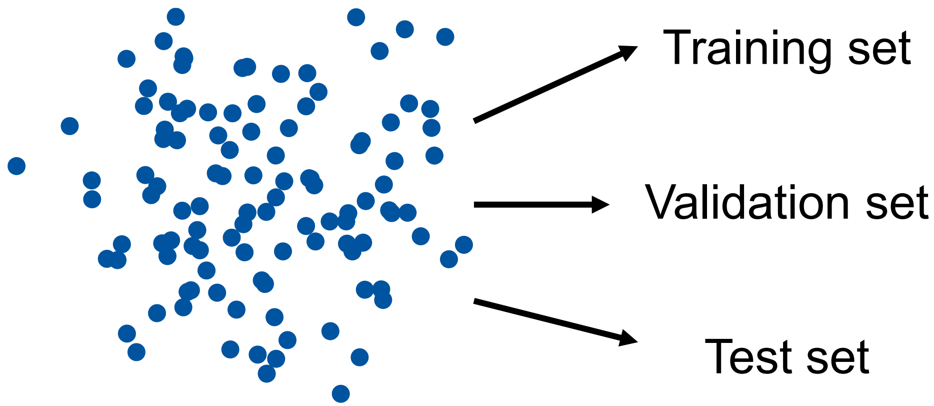
- Yes → Supervised learning
- Some → Semi-supervised learning
- No → Unsupervised learning

Learning via sparse feedback: Reinforcement learning

Measuring Success

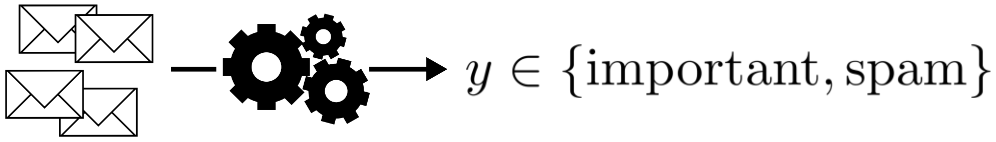
Machines that learn to perform a task from experience

- Performance measure: typically a single number.
  - Calculate with a suitable metric.
- Divide data into disjoint subsets:

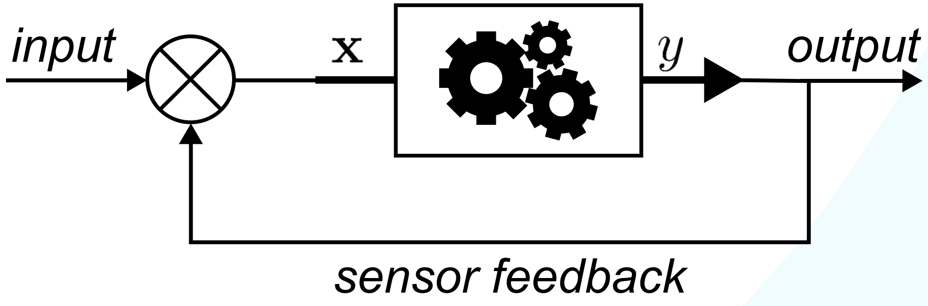


- Measure generalization performance on test set.

E.g., % correctly recognized spam mails



E.g., average distance to desired endpoint

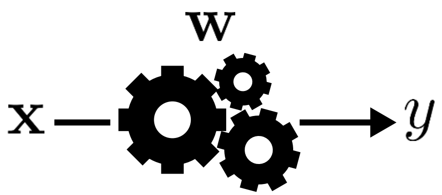


Learning as Optimization

Machines that learn to perform a task from experience

Learning = optimizing $f(\mathbf{x}; \mathbf{w})$

$\mathbf{w}$ *describes the type of model that we use.*



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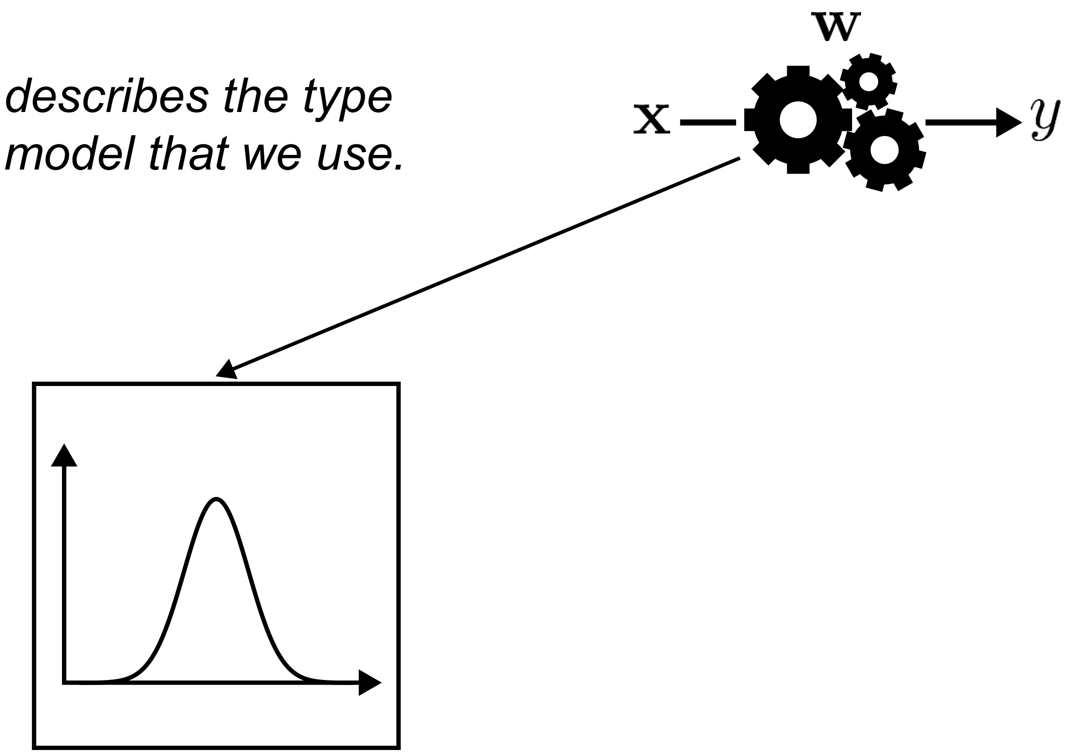

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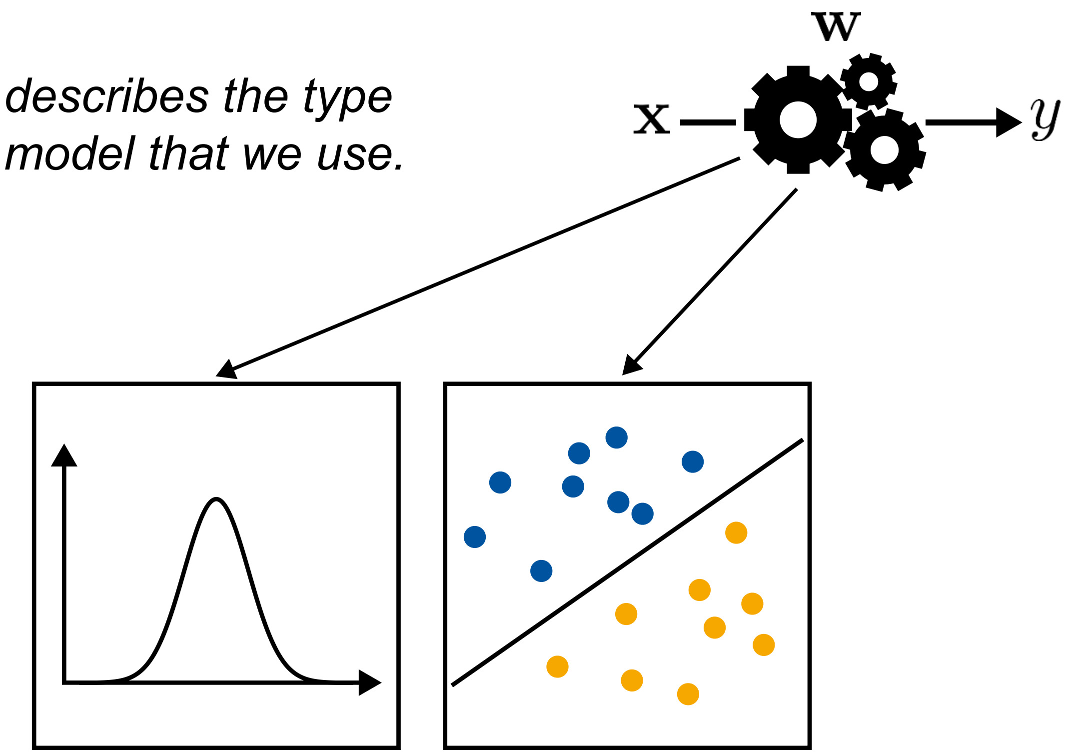

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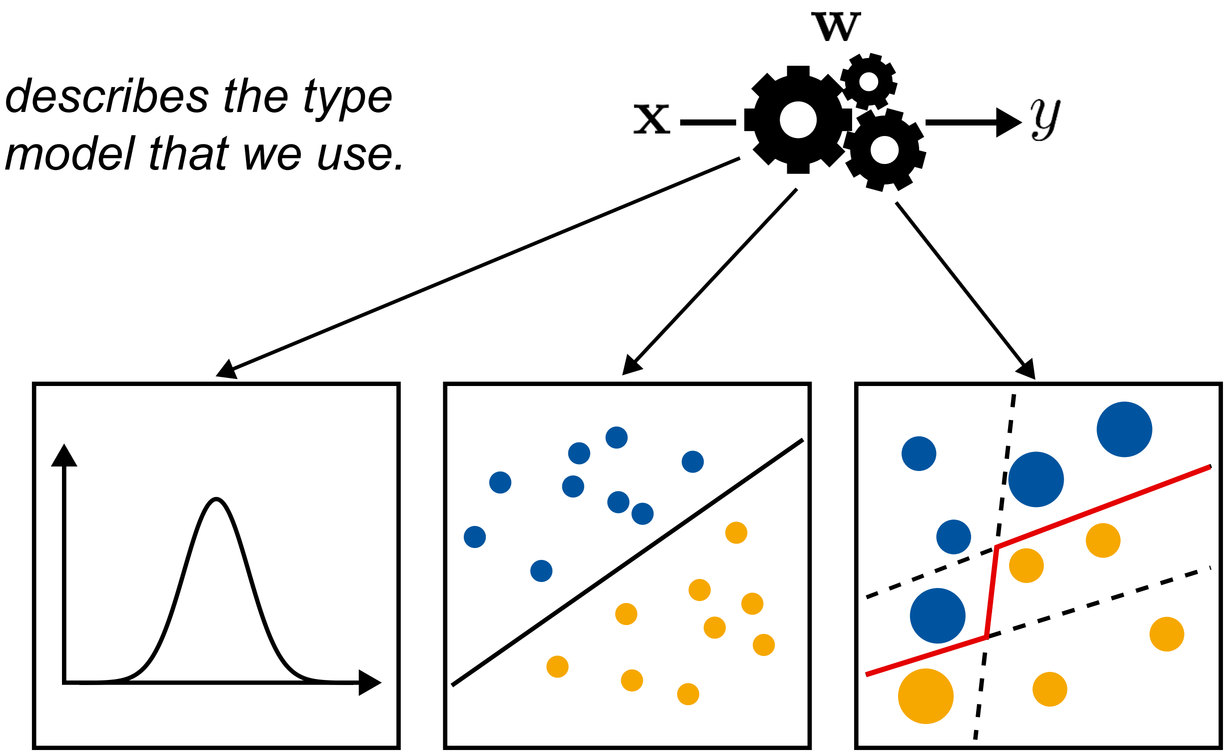

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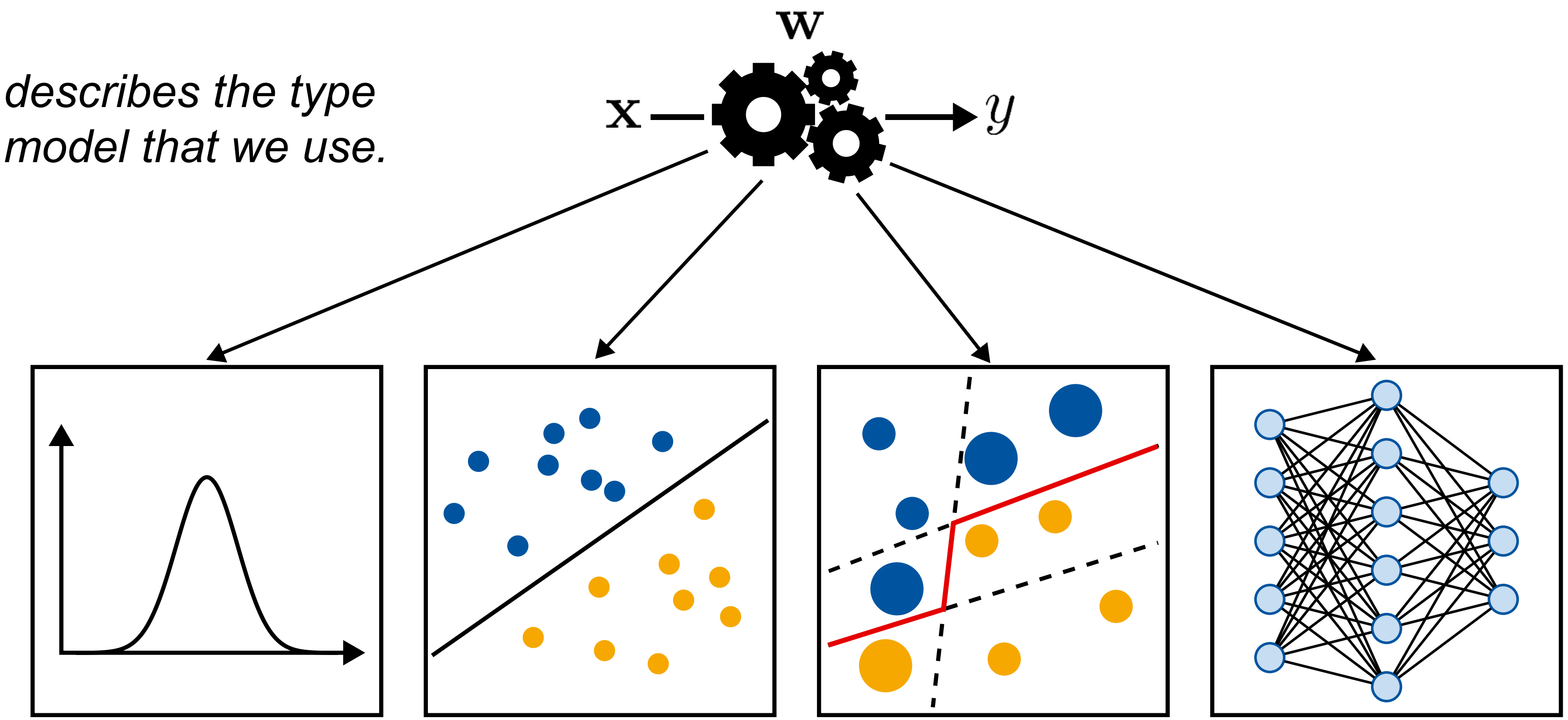

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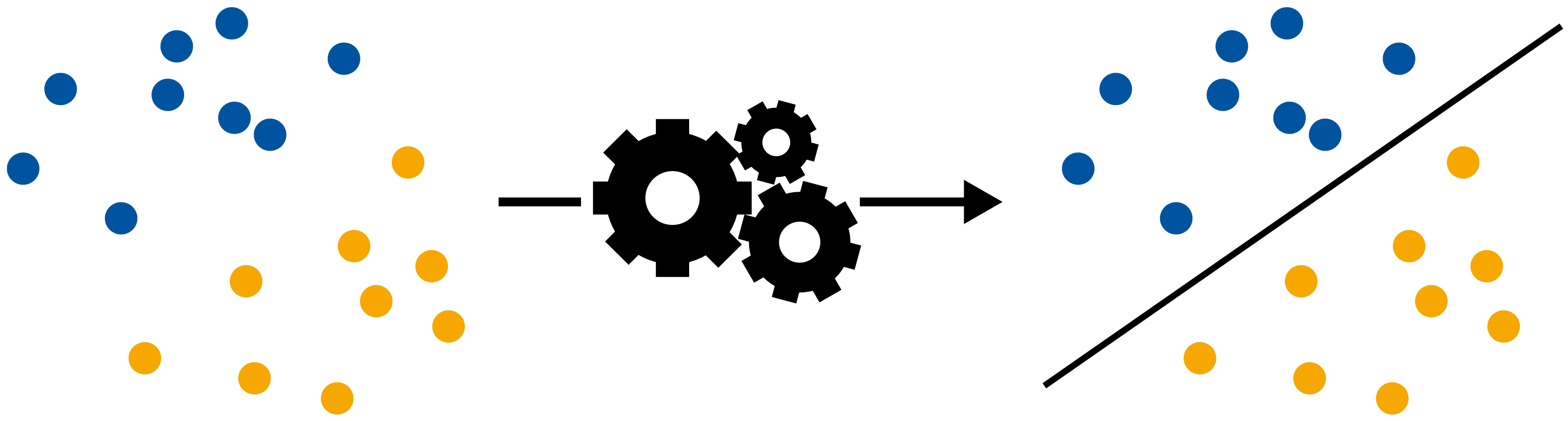
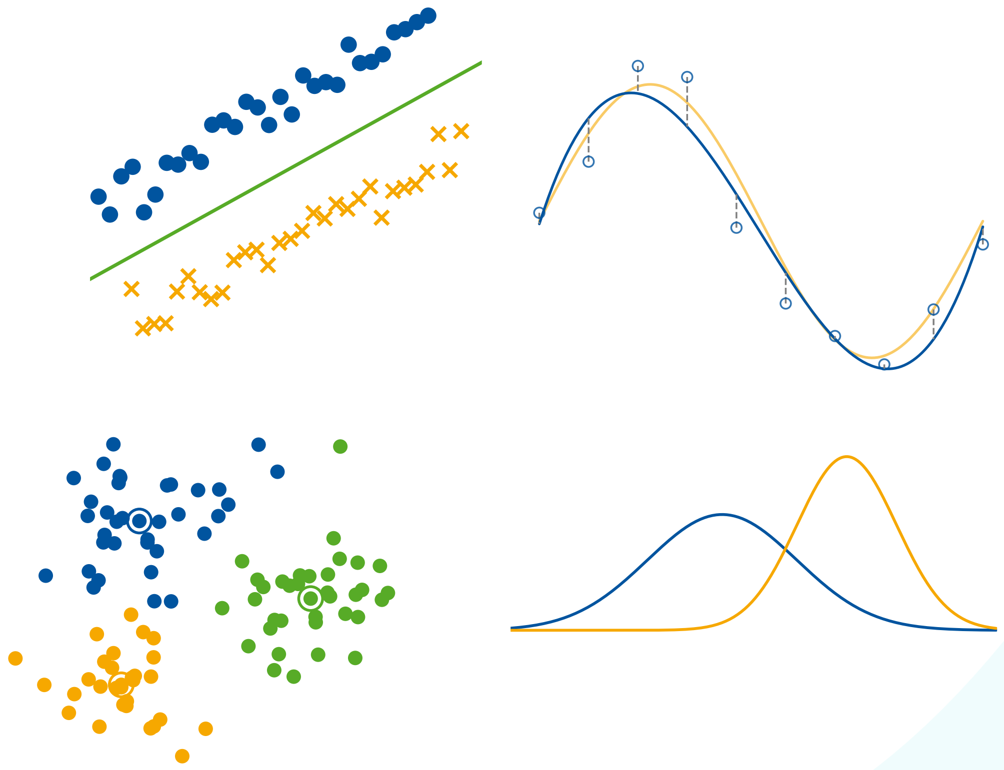


We will focus on statistical Machine Learning.

Topics for Today

1. Motivation
2. **Forms of Learning**
3. Terms, Concepts, and Notation
4. Bayes Decision Theory

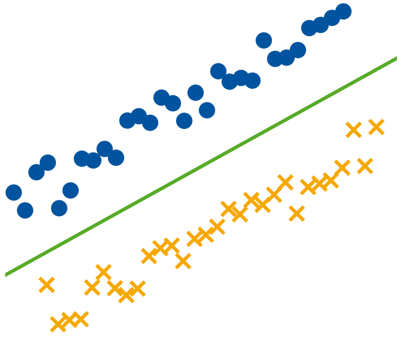


Supervised vs. Unsupervised Learning

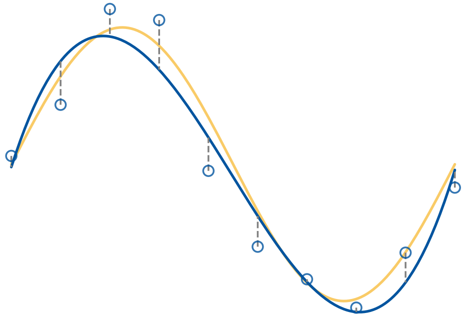
Discrete targets

Continuous targets

Known targets



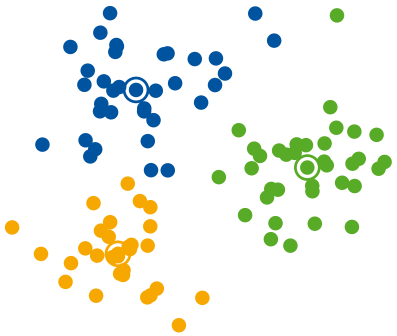
Classification



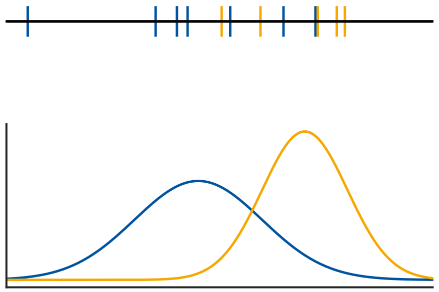
Regression

Supervised learning

Unknown targets



Clustering



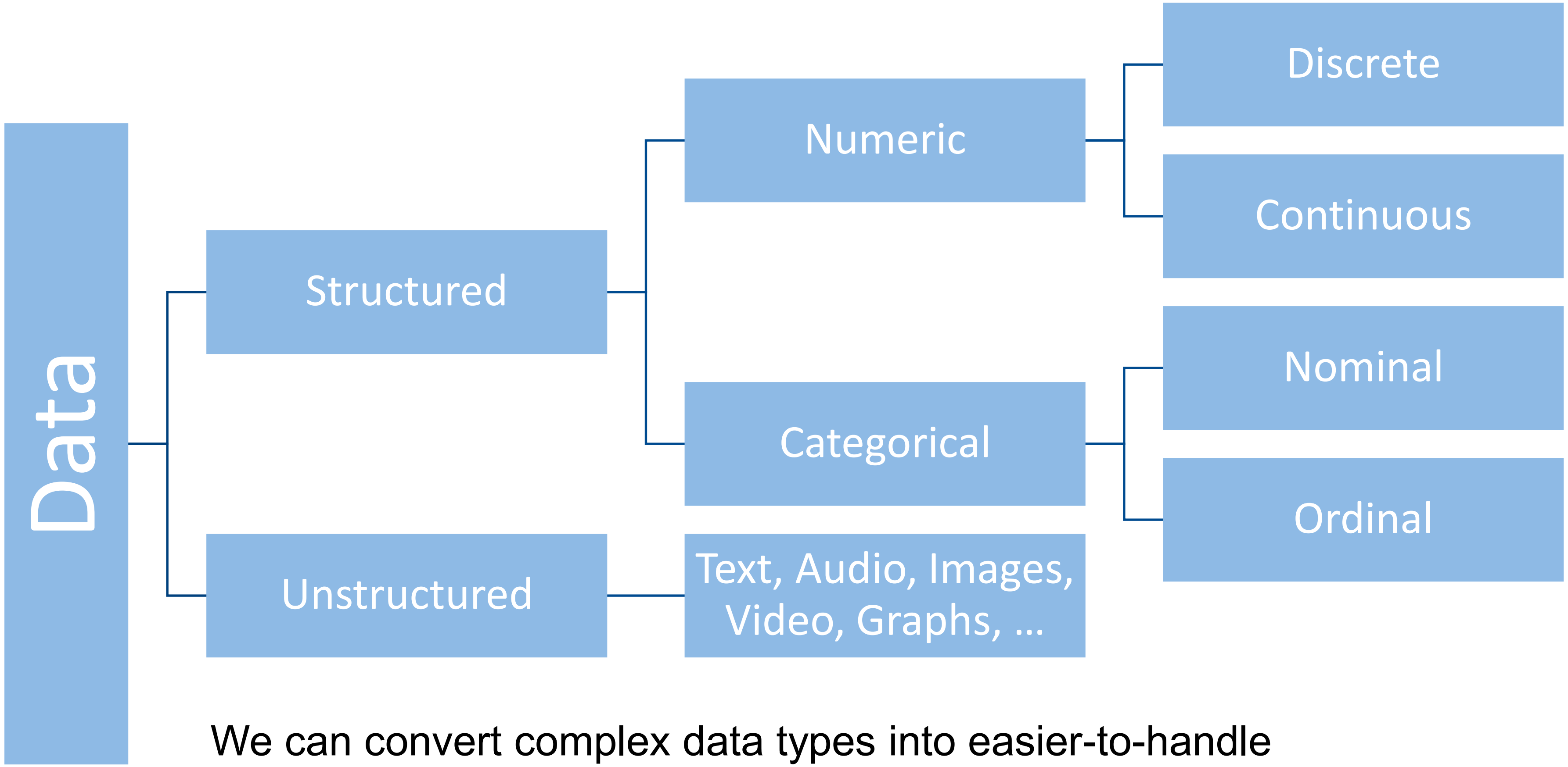
Density estimation

Unsupervised learning

Supervised Learning

- We will mostly focus on supervised learning.
- Given training data with labels: $\mathcal{D} = \{(\mathbf{x}_1, t_1), \dots, (\mathbf{x}_N, t_N)\}$
- The goal is to learn a predictive function $y(\mathbf{x}; \mathbf{w})$ that yields good performance on unseen test data.
- In real-world scenarios, we also need to preprocess our data to handle, e.g.,
  - Missing or wrong values
  - Outliers
  - Inconsistencies

Data Types - Overview

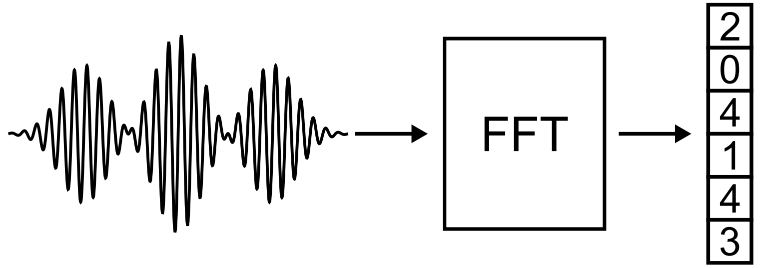


We can convert complex data types into easier-to-handle continuous vector-space data via feature extraction.

Features

- Feature extraction is the process that creates descriptive vectors from samples.
  - Features should be invariant to irrelevant input variations.
  - Selecting the “right” features is crucial.
  - Usually encode some domain knowledge.
  - Higher-dimensional features are more discriminative.
- Curse of dimensionality: complexity increases exponentially with number of dimensions.

Example: convert audio snippet to feature vector with Fast Fourier Transform (FFT).



Introduction

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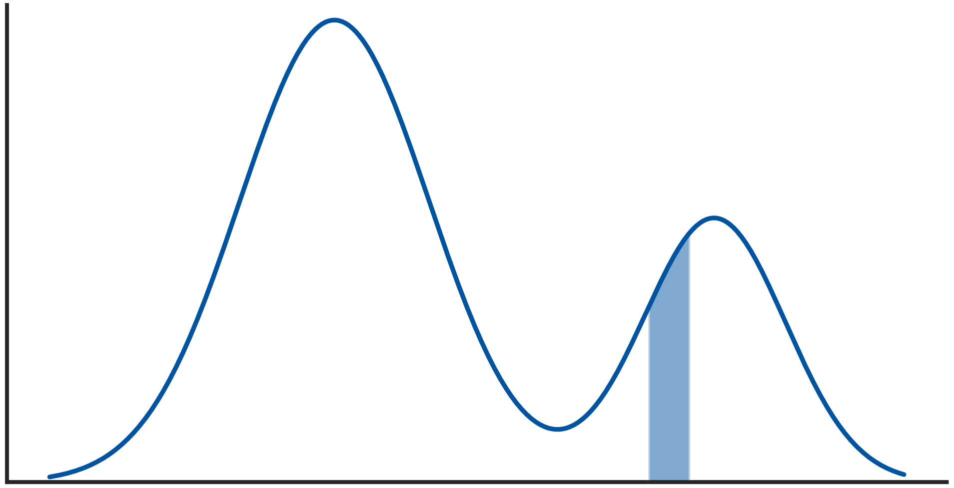
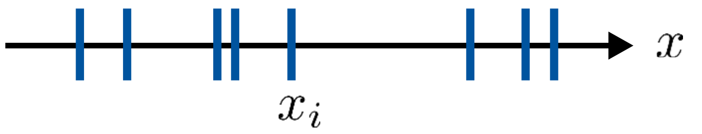
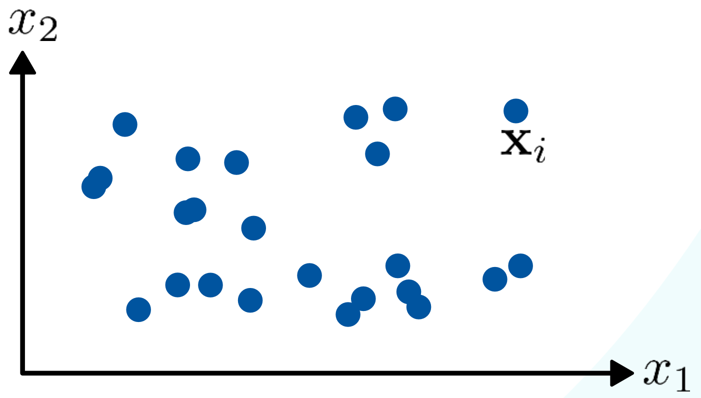
2. Forms of learning
3. **Terms, Concepts, and Notation**
4. Bayes Decision Theory



Terms, Concepts, and Notation

- Most of our tools will be based on statistics and probability theory.
- We will review the most important concepts here.
- Some Notation:
  - Scalar data $x \in \mathbb{R}$
  - Vector-valued data $\mathbf{x} \in \mathbb{R}^D$
  - Datasets $\mathcal{X} = \{x_1, \dots, x_N\}$





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- Some Notation:
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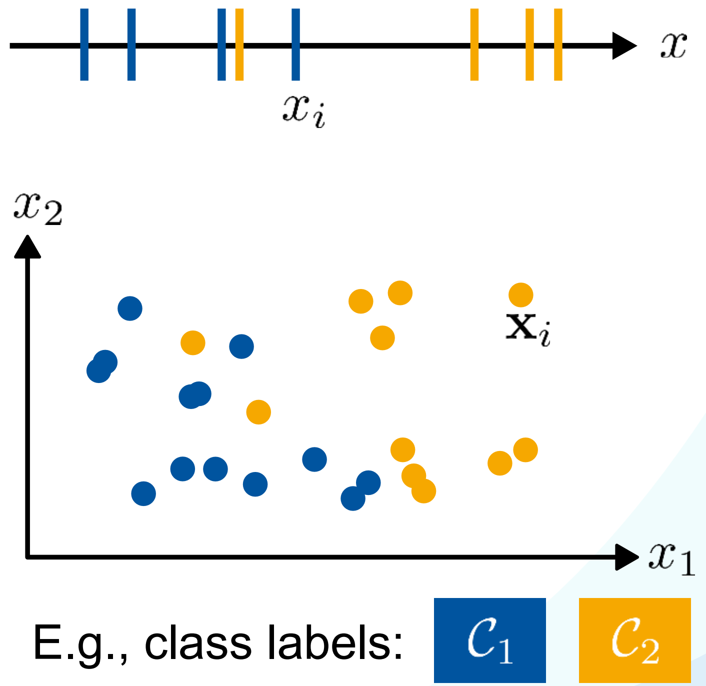
  - Datasets $\mathcal{X} = \{x_1, \ldots, x_N\}$
  - Labelled datasets $\mathcal{D} = \{(x_1, t_1), \ldots, (x_N, t_N)\}$
  - Matrices $\mathbf{M} \in \mathbb{R}^{m \times n}$
  - Dot product $\mathbf{w}^\mathsf{T}\mathbf{x} = \sum_{j=1}^{D} w_j x_j$



Probability Basics

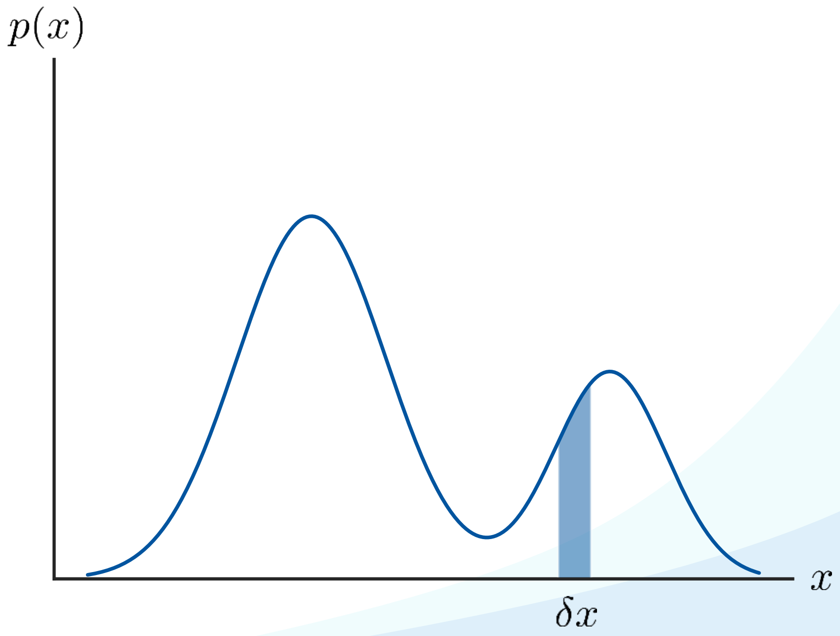
- Probabilities are defined over random variables:
  - Discrete case:

$$p(X = x_j) = \frac{n_j}{N}$$

- Continuous case:

$$p(X \in (x_1, x_2)) = \int_{x_1}^{x_2} p(x)\,\mathrm{d}x$$

Where $p(x)$ is the probability density function (pdf) of x .



Probability Basics

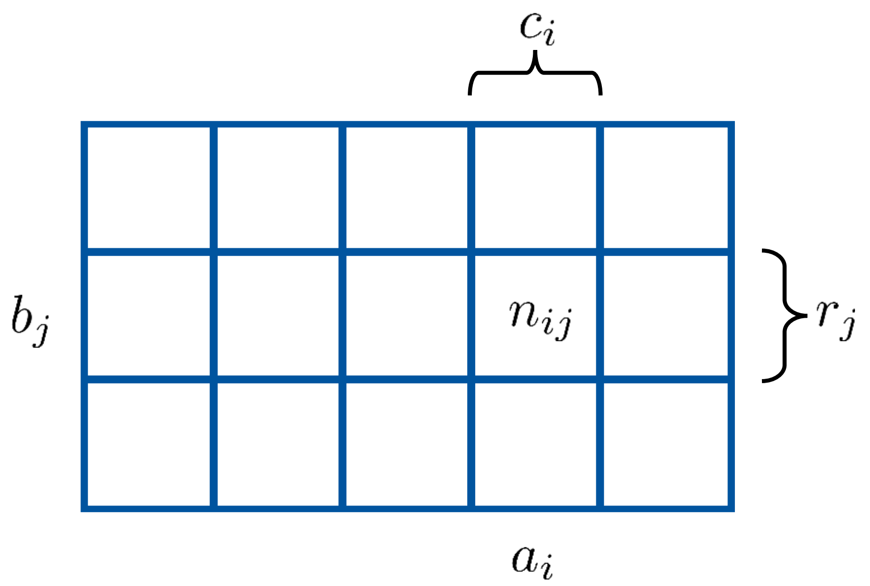
- Random variables $A \in \{a_i\}, B \in \{b_j\}$
- Consider N trials:

$$n_{ij} = \#\{A = a_i \wedge B = b_j\}$$

$$c_i = \#\{A = a_i\}$$

$$r_j = \#\{B = b_j\}$$

- Derive from this:
  - Joint probability $p(A = a_i, B = b_j) = \frac{n_{ij}}{N}$
  - Marginal probability $p(A = a_i) = \frac{c_i}{N}$
  - Conditional probability $p(B = b_j | A = a_i) = \frac{n_{ij}}{c_i}$

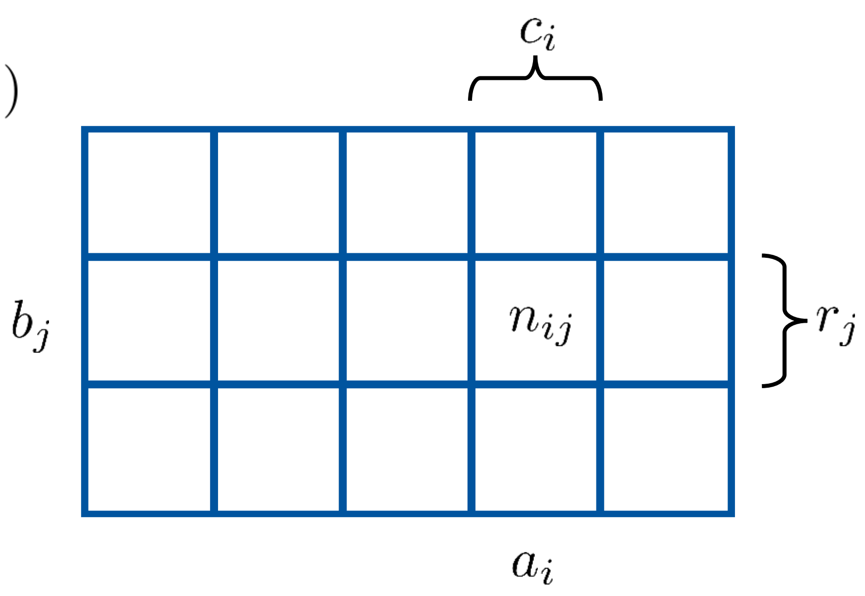


- **Sum rule**:

$$p(A = a_i) = \frac{c_i}{N} = \frac{1}{N}\sum_j n_{ij} = \sum_{b_j} p(A = a_i, B = b_j)$$

- **Product rule**:

$$p(A = a_i, B = b_j) = \frac{n_{ij}}{N} = \frac{n_{ij}}{c_i} \cdot \frac{c_i}{N}$$
$$= p(B = b_j | A = a_i)p(A = a_i)$$



Rules of Probability - Summary

- Sum rule:

$$p(A) = \sum_B p(A, B)$$

- Product rule:

$$p(A, B) = p(B|A)p(A)$$

- Combine into Bayes' Theorem:

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)} = \frac{p(B|A)p(A)}{\sum_A p(B|A)p(A)}$$

This is the most important equation in this course!

Introduction

1. Motivation
2. Forms of learning
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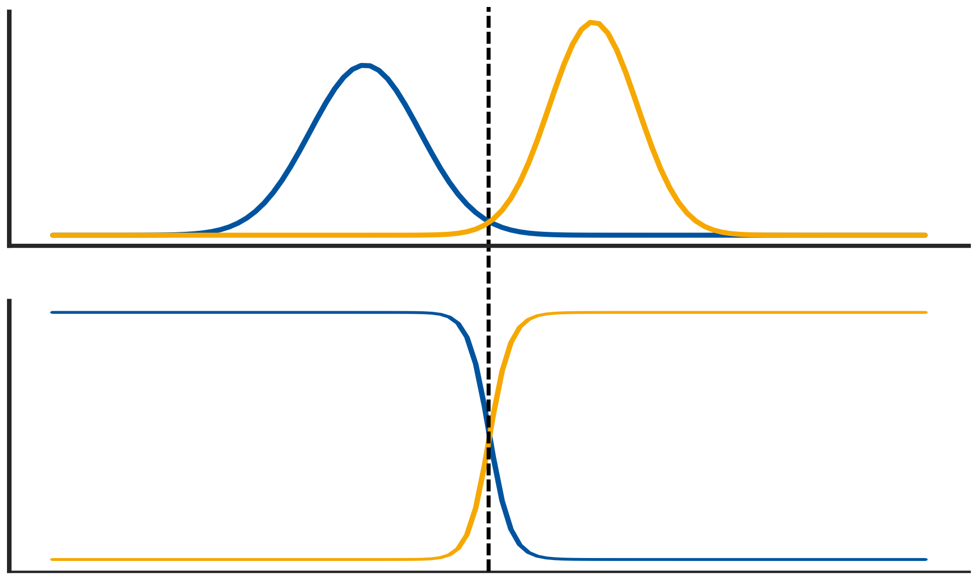
4. **Bayes Decision Theory**



Bayes Decision Theory

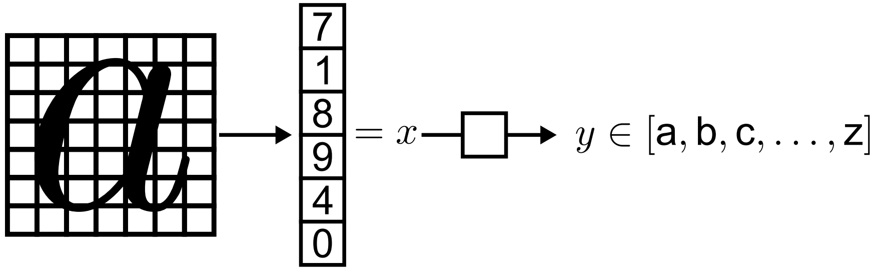
- Goal: predict an output class $\mathcal{C}$ from measurements $\mathbf{x}$, by minimizing the probability of misclassification.
- *How can we make such decisions optimally?*

- Bayes Decision Theory gives us the tools for this
  - Based on Bayes' Theorem:

$$p(Y|X) = \frac{p(X|Y)p(Y)}{p(X)}$$

  - In the following, we will introduce its basic concepts…

Example: handwritten character recognition



$\mathbf{x}$: e.g., pixel values

Core Concept: Priors

- What can we tell about the outcome of an experiment *before* making any measurements?
- The a-priori probability $p(\mathcal{C})$ captures the probability distribution over the different class outcomes
  - Based on previously observed data
  - i.e., independent of the actual measurement
- The prior probabilities over all possible class outcomes sum to one.

Example: in English text, the letter "e" makes up ~13% of all letters:

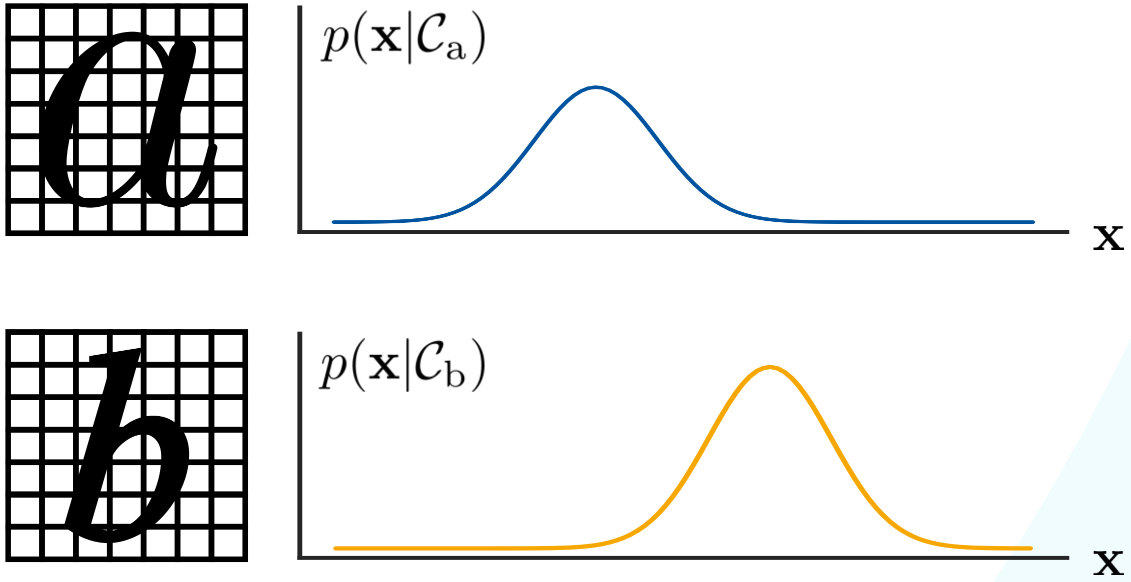
$$p(\mathcal{C}_{\mathrm{e}}) = 0.13$$

And there are 26 letters in the English alphabet:

$$\sum_{\alpha \in \{\mathrm{a},\ldots,\mathrm{z}\}} p(\mathcal{C}_\alpha) = 1$$

Core Concept: Likelihood

- How *likely* is it that we *observe* a certain measurement $\mathbf{x}$ *given* an example of class $\mathcal{C}$?
- This is expressed by the likelihood $p(\mathbf{x}|\mathcal{C})$
  - It is called a *class-conditional distribution*, since it specifies the distribution of $\mathbf{x}$ conditioned on the class $\mathcal{C}$.
  - We can estimate the likelihood from the distribution of measurements $\mathbf{x}$ observed on the given training data.



- Here, $\mathbf{x}$ measures certain properties of the input data.
  - E.g., the fraction of black pixels
  - We simply treat it as a vector $\mathbf{x} \in \mathbb{R}^D$.

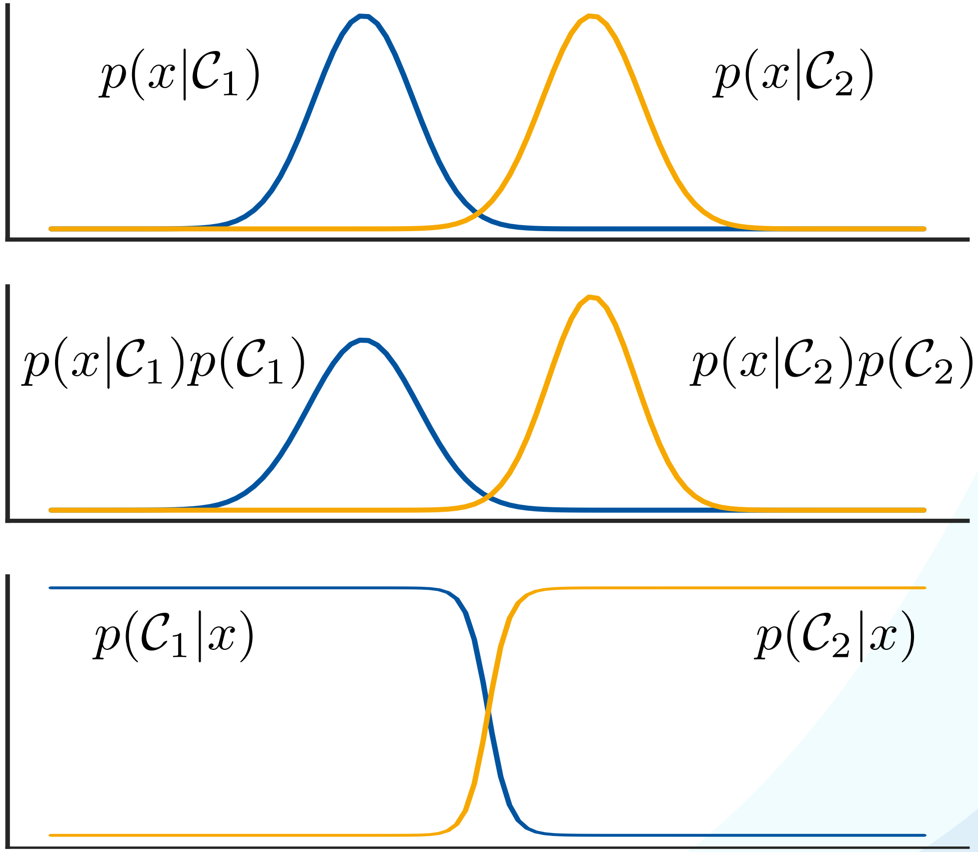
Core Concept: Posterior

- What is the probability for class $\mathcal{C}_k$ if we made a measurement $\mathbf{x}$?
- This a-posteriori probability $p(\mathcal{C}_k|\mathbf{x})$ can be computed via Bayes' Theorem after we observed $\mathbf{x}$:

$$p(\mathcal{C}_k|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{C}_k)p(\mathcal{C}_k)}{p(\mathbf{x})} = \frac{p(\mathbf{x}|\mathcal{C}_k)p(\mathcal{C}_k)}{\sum_j p(\mathbf{x}|\mathcal{C}_j)p(\mathcal{C}_j)}$$

- *This is usually what we're interested in!*
- Interpretation

$$posterior = \frac{likelihood \cdot prior}{normalization\ factor}$$



Making Optimal Decisions

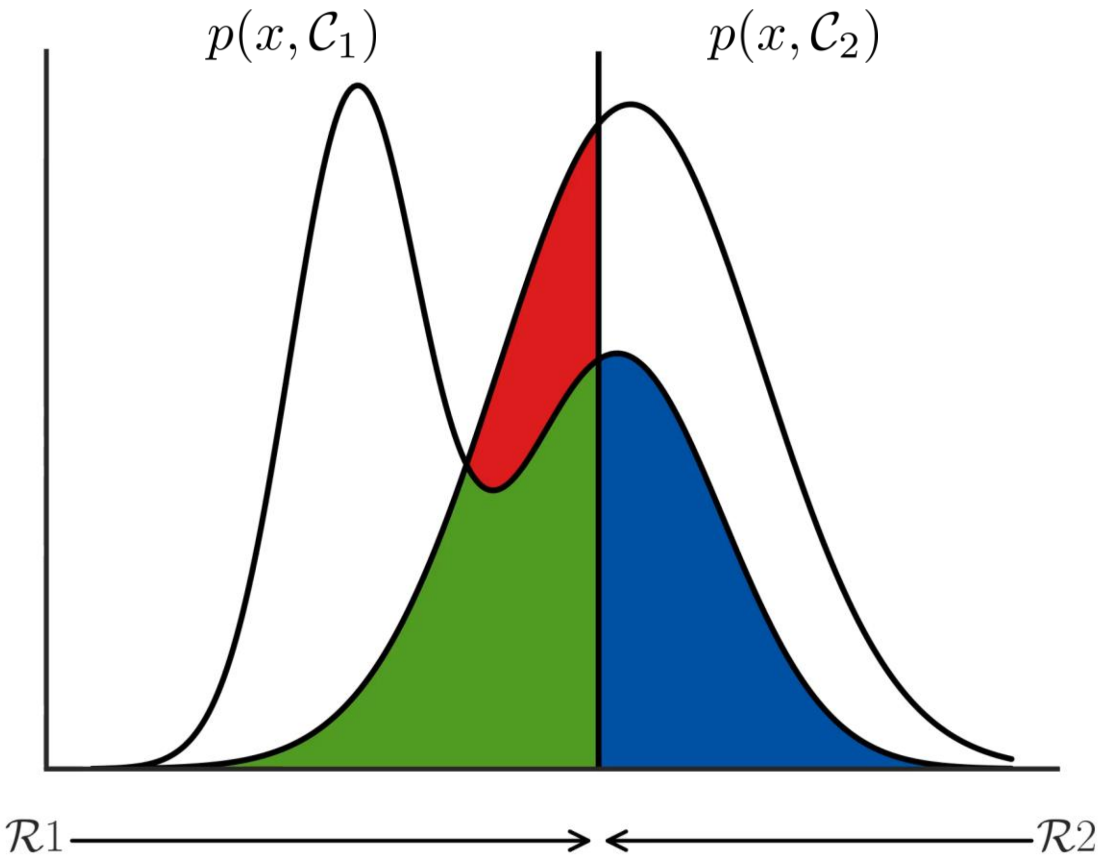
- Goal: minimize the probability of misclassification.

$$p(\text{mistake}) = p(x \in \mathcal{R}_1, \mathcal{C}_2) + p(x \in \mathcal{R}_2, \mathcal{C}_1)$$
$$= \int_{\mathcal{R}_1} p(x, \mathcal{C}_2)\,\mathrm{d}x + \int_{\mathcal{R}_2} p(x, \mathcal{C}_1)\,\mathrm{d}x$$
$$= \int_{\mathcal{R}_1} p(\mathcal{C}_2|x)p(x)\,\mathrm{d}x + \int_{\mathcal{R}_2} p(\mathcal{C}_1|x)p(x)\,\mathrm{d}x$$

- Note:

+ = constant

We can only reduce



$\mathcal{R}_1$ and $\mathcal{R}_2$ are the decision regions after setting a decision threshold.

Making Optimal Decisions

- Goal: minimize the probability of misclassification.

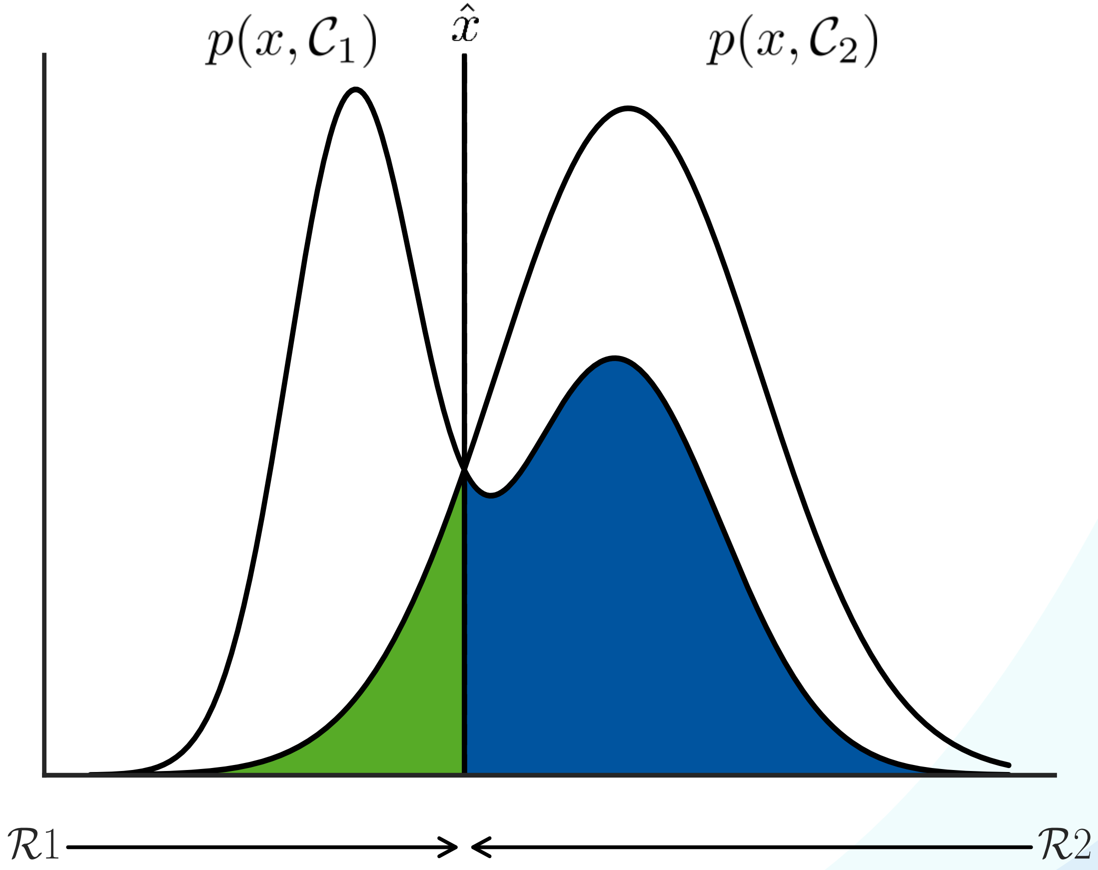
$$
\begin{aligned}
p(\text{mistake}) &= p(x \in \mathcal{R}_1, \mathcal{C}_2) + p(x \in \mathcal{R}_2, \mathcal{C}_1) \\
&= \int_{\mathcal{R}_1} p(x, \mathcal{C}_2)\,\mathrm{d}x + \int_{\mathcal{R}_2} p(x, \mathcal{C}_1)\,\mathrm{d}x \\
&= \int_{\mathcal{R}_1} p(\mathcal{C}_2|x)p(x)\,\mathrm{d}x + \int_{\mathcal{R}_2} p(\mathcal{C}_1|x)p(x)\,\mathrm{d}x
\end{aligned}
$$

- Note:

+ = constant

We can only reduce

- *Minimal error at the intersection* $\hat{x}$



$\mathcal{R}_1$ and $\mathcal{R}_2$ are the decision regions after setting a decision threshold.

Making Optimal Decisions

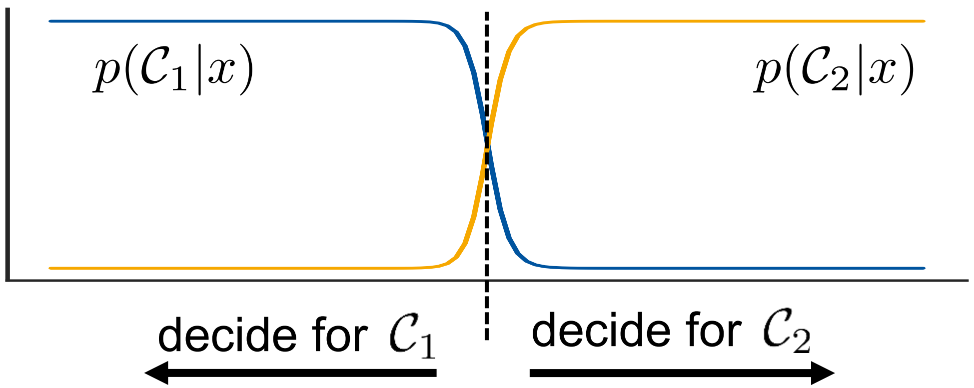
- Our goal is to minimize the probability of a misclassification.
- The optimal decision rule is: decide for $\mathcal{C}_1$ iff

$$p(\mathcal{C}_1|\mathbf{x}) > p(\mathcal{C}_2|\mathbf{x})$$

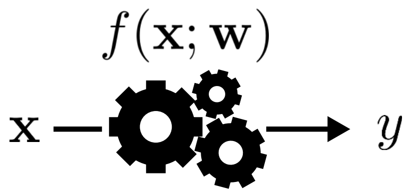
- Or for multiple classes: decide for $\mathcal{C}_k$ iff

$$p(\mathcal{C}_k|\mathbf{x}) > p(\mathcal{C}_j|\mathbf{x}) \; \forall j \neq k$$

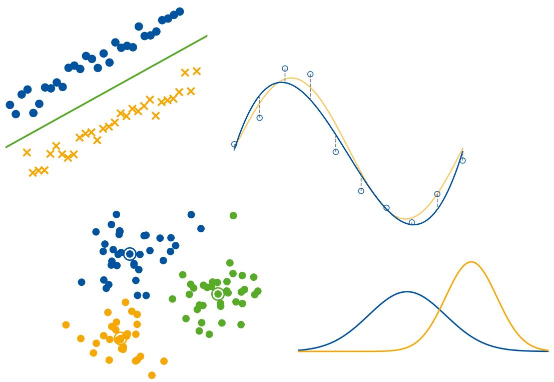
- *Once we can estimate posterior probabilities, we can use this rule to build classifiers.*



Summary: Introduction to ML



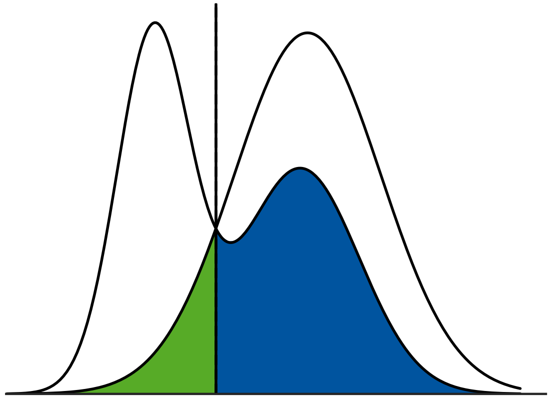
Machine Learning



Forms of Machine Learning

$$p(\mathcal{C}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{C})p(\mathcal{C})}{p(\mathbf{x})}$$

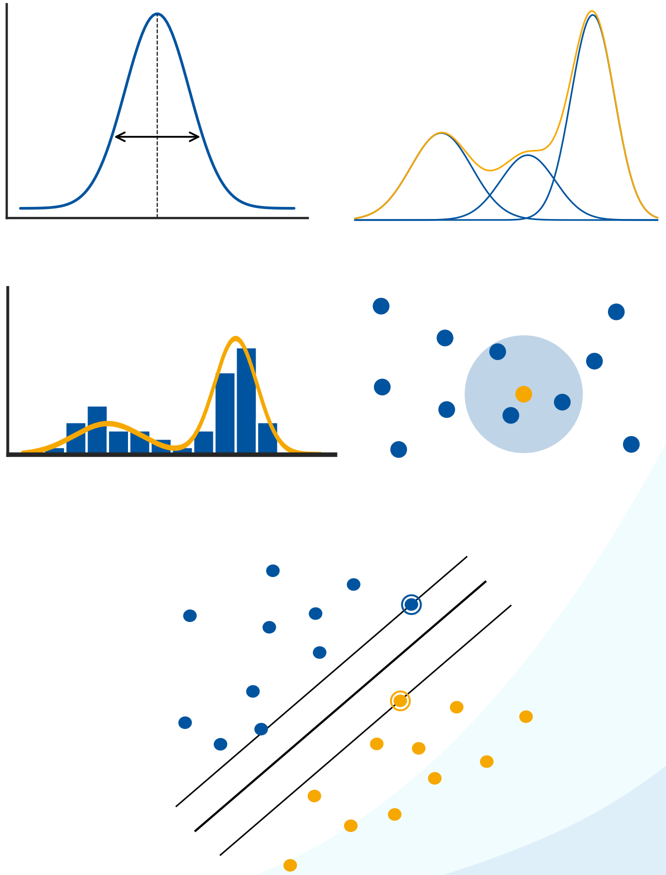
Bayes Theorem



Bayes Optimal Classification

Next Lectures…

- Ways how to estimate the probability densities $p(\mathbf{x}|\mathcal{C}_k)$
  - Parametric methods
    - Gaussian distribution
    - Mixtures of Gaussians
  - Non-parametric methods
    - Histograms
    - k-Nearest Neighbor
    - Kernel Density Estimation



- Ways to directly model the posteriors $p(\mathcal{C}_k|\mathbf{x})$
  - Linear discriminants
  - Logistic regression, SVMs, Neural Networks, …

Machine Learning Topics

8. Introduction to ML
9. **Probability Density Estimation**
10. Linear Discriminants
11. Linear Regression
12. Logistic Regression
13. Support Vector Machines
14. Neural Network Basics

References and Further Reading

- More information, including a short review of Probability theory and a good introduction in Bayes Decision Theory can be found in Chapters 1.1, 1.2 and 1.5 of

Christopher M. Bishop
Pattern Recognition and Machine Learning
Springer, 2006