Perception Exercise 1

1. Sobel filter

   1-D Gaussian G: $\begin{matrix}1\\2\\1\end{matrix}$

   1-D Derivative filter D: $\begin{matrix}1 & 0 & -1\end{matrix}$

   To cumpute the convulution G*D, we need to apply zero-padding.
   Zero padding is the process to add extra 0-value pixels surrounding the original image when there are no pixels to overlap the kernal pixels.

   With D as a convolution kernal, D(pad)=$\begin{matrix}0 & 0 & 0\\1 & 0 & -1\\0 & 0 & 0\end{matrix}$

   G(pad)=
   0 0 0 0 0
   0 0 1 0 0
   0 0 2 0 0
   0 0 1 0 0
   0 0 0 0 0
   Now we compute G(pad)*D(pad), with D(pad) flipped. By the formula of 2D convuliton:
   (I*G)(x,y)=$\sum_{i=-\infty}^{\infty} \quad \sum_{i=-\infty}^{\infty} I(i,j)G(x-i,y-j)$
   S(1,1)=0+0+0+⋯+1=1
   S(1,2)=0+0+0+⋯0=0
   S(1,3)=0+0+0+⋯-1=-1
   S(2,1)=0+0+0+⋯+2=2
   …
   …
   S(3,3)=0+0+0+⋯-1=-1

   S=$\begin{matrix}1 & 0 & -1\\2 & 0 & -2\\1 & 0 & -1\end{matrix}$

2. The Morphological Operations Erosion is the operation to replace the output pixels with the minimum value of all pixels in the neighborhood which decided by the structuring element S.
   Erosion is defined as

$$A \ominus B = \{x \mid x + b \in A \text{ for every } b \in B\}$$

   So for a structuring element S as [1 1 1; 1 1 1; 1 1 1]. The result from applying the erosion operation on a binary image is only those 1 pixels whose all the 8-connectivities neighbors are 1 remain 1, all the other pixels become 0.
   Dilation is defined as

$$A \oplus B = \{c \mid c = a + b \text{ for some } a \in A \text{ and } b \in B\}$$

Dilation is the opposite case which takes the maximum value. So it will make all the pixels as 1 except those 0 pixels whose all the 8-connectivities neighbors are 0.

Erosion operation is not commutative and dilation is commutative.
We can see it from the definition. If we swap A and B in dilation, the result is the same, but in erosion it is opposite direction.
Because dilation is like slide a kernel to each position in the image and apply union. While Erosion is like applying intersect.
It is like how addition is commutative while subtraction is not.