

Image Analysis, Assignment 1 Report

1 Image sampling

Sample the image evenly with 5*5 pixels, the lower left pixel is a sample from (0,0) and the upper right pixel is a sample from (1,1). So the 5*5 pixels' position are

(0,1)	(0.25,1)	(0.5,1)	(0.75,1)	(1,1)
(0, 0.75)	(0.25, 0.75)	(0.5, 0.75)	(0.75, 0.75)	(1, 0.75)
(0, 0.5)	(0.25, 0.5)	(0.5, 0.5)	(0.75, 0.5)	(1, 0.5)
(0, 0.25)	(0.25, 0.25)	(0.5, 0.25)	(0.75, 0.25)	(1, 0.25)
(0,0)	(0.25,0)	(0.5,0)	(0.75,0)	(1,0)

And to quantify with 32 different gray levels from 0 to 31, so if the intensity of a point is from 0 to 1/32, the gray level is 0, from 1/32 to 2/32, then the gray level is 1, ..., from 31/32 to 1, the gray level is 31. So we calculate the intensity of each points above.

Code:

```
x = [0 0.25 0.5 0.75 1
      0 0.25 0.5 0.75 1
      0 0.25 0.5 0.75 1
      0 0.25 0.5 0.75 1
      0 0.25 0.5 0.75 1]
y = [1 1 1 1 1
      0.75 0.75 0.75 0.75 0.75
      0.5 0.5 0.5 0.5 0.5
      0.25 0.25 0.25 0.25 0.25
      0 0 0 0 0]
f = x+y-2.*x.*y
g = floor(f/(1/32))
g(g==32) = 31
```

resulting matrix:

```
g = 5x5
    31    24    16     8     0
    24    20    16    12     8
    16    16    16    16    16
     8    12    16    20    24
     0     8    16    24    31
```

2 Histogram equalization

$$\int_0^s p_s(t) dt = \int_0^r p_r(t) dt$$

Take T so that $p_s(s) = 1, s \in [0,1]$

$$\int_0^r p_r(t) dt = \int_0^s 1 dt = s = T(r) = [t^{\frac{3}{2}}]_0^r = r^{\frac{3}{2}} - 0 = r^{\frac{3}{2}}, r \in [0,1]$$

3 Neighborhood of pixels

Use the MATLAB to do the threshold.

Code and resulting g:

```
m = size(A,1);
n = size(A,2);
B = zeros(m,n);
for i=1:m
    for j=1:n
        if A(i,j)>1
            B(i,j)=1;
        else
            B(i,j)=0;
        end
    end
end
```

1	1	1	1	0	1	1	1	0	1	1	1
0	0	1	0	0	0	0	0	0	1	1	1
0	1	0	0	0	0	0	1	0	0	1	1
1	0	0	0	0	1	1	0	1	1	1	1
1	1	0	0	0	1	0	1	1	0	1	1
0	1	1	0	0	0	0	0	0	1	1	1
1	0	1	0	0	0	0	0	1	1	0	1
1	0	1	0	1	1	1	0	1	0	1	1
1	0	0	0	1	1	1	0	1	1	1	1
1	1	1	1	0	0	1	0	0	0	1	0
0	1	1	0	0	0	0	0	0	1	1	1
0	1	1	0	1	1	1	0	1	0	1	1



There are 4 8-connected components for $g=1$.

4 Segmentation part of OCR

Code

```
function S = im2segment(im)
```

```

A = imgaussfilt(im,0.5);% gaussian filter
S = cell(1,5);% we have 5 numbers
m = size(im,1);
n = size(im,2);
for i=1:m
    for j=1:n
        if j==1||j==n||i==1||i==m
            A(i,j)=0;
        end
    end
end

for i=1:m % threshold
    for j=1:n
        if im(:,1)==[2 7 5 0 0 0 0 10 18 7 0 0 6 16 10 0 0 8 0 20 0 14 0 14 0
10 15 10]'
            if A(i,j)>=26
                A(i,j)=1;
            else
                A(i,j)=0;
            end
        else
            if A(i,j)>=40
                A(i,j)=1;
            else
                A(i,j)=0;
            end
        end
    end
end

for i=2:m-1
    for j=2:n-1
        if A(i+1,j)==0&&A(i,j+1)==0&&A(i-1,j)==0&&A(i,j-1)==0
            A(i,j)=0;
        end
    end
end

BW= logical(A);
B = bwlabel(BW,8);% use this function to find 8-connected components
S{1} = zeros(m,n);
S{2} = zeros(m,n);
S{3} = zeros(m,n);

```

```

S{4} = zeros(m,n);
S{5} = zeros(m,n);
for i=1:m
    for j=1:n
        if B(i,j)==1
            S{1}(i,j)=1;% s1 is the first component and the first number
        elseif B(i,j)==2
            S{2}(i,j)=2;
        elseif B(i,j)==3
            S{3}(i,j)=3;
        elseif B(i,j)==4
            S{4}(i,j)=4;
        elseif B(i,j)==5
            S{5}(i,j)=5;
        end
    end
end
end
S={S{1},S{2},S{3},S{4},S{5}};

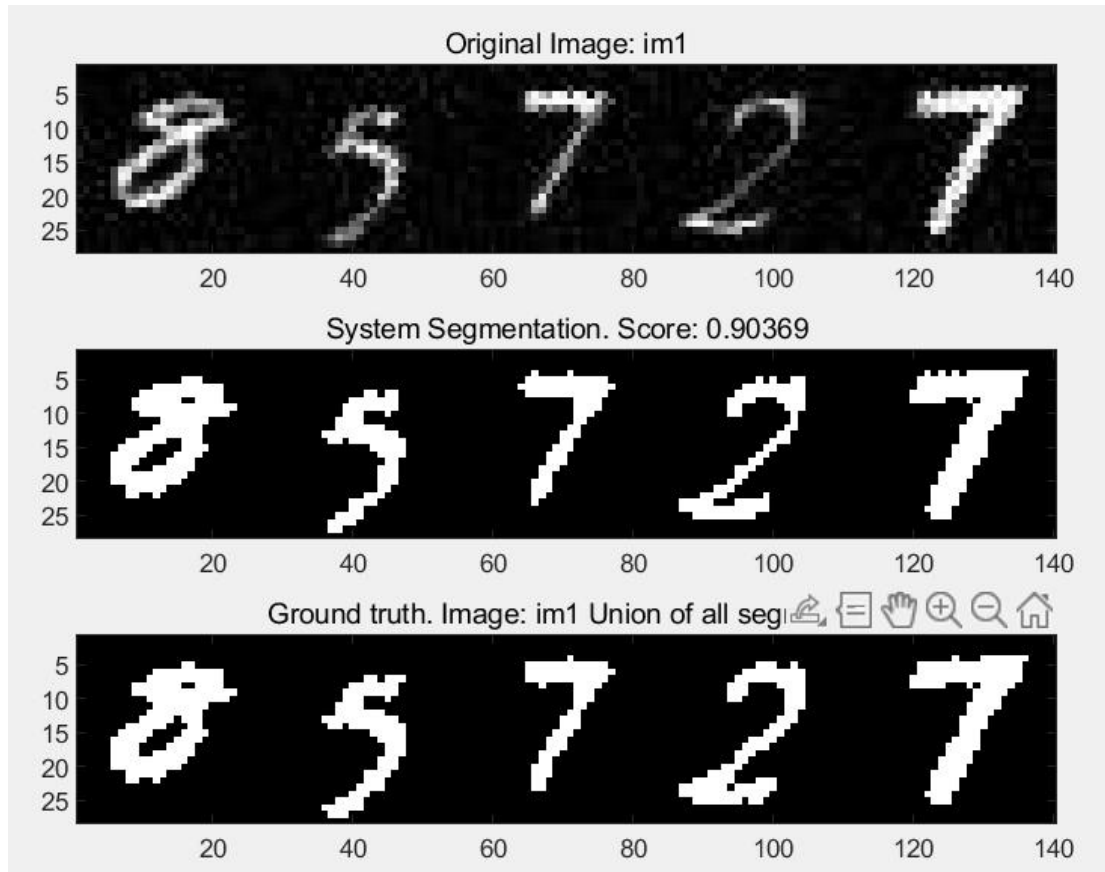
```

The jaccard scores for all segments in all images were

0.9512	0.8868	0.9302	0.7951	0.9379
0.9109	0.9424	0.8626	0.9658	0.9621
0.9310	0.9508	0.9380	0.9474	0.9333
0.7549	0.9137	0.9551	0.9478	0.9172
0.9187	0.9170	0.9424	0.8866	0.9448
0.9624	0.9298	0.9732	0.9527	0.9931
0.9461	0.8941	0.9204	0.9638	0.9250
0.9191	0.9598	0.7745	0.9188	0.8849
0.9268	0.9203	0.9432	0.9449	0.8901
0.8359	0.8750	0.8627	0.7574	0.8750

The mean of the jaccard scores were 0.91385

This is good!.



5 Dimensionality

In A the dimension $k=2*3=6$

Basis elements:

$$\begin{aligned}
 e_1 &= [1 \ 0 \ 0 \ 0 \ 0 \ 0] \\
 e_2 &= [0 \ 1 \ 0 \ 0 \ 0 \ 0] \\
 e_3 &= [0 \ 0 \ 1 \ 0 \ 0 \ 0] \\
 e_4 &= [0 \ 0 \ 0 \ 1 \ 0 \ 0] \\
 e_5 &= [0 \ 0 \ 0 \ 0 \ 1 \ 0] \\
 e_6 &= [0 \ 0 \ 0 \ 0 \ 0 \ 1]
 \end{aligned}$$

In B the dimension $k=1500*2000=3000000$

The basis elements can be chosen by making every single element is 1 and the others are 0 each time, and there are 3000000 elements in total.

6 Scalar products and norm on images

$$f \cdot g = \sum_{i=1}^M \sum_{j=1}^N \bar{f}(i, j) g(i, j)$$

The scalar product of two images is defined as

$$\|f\| = \sqrt{f \cdot f} = \sqrt{\sum_{i=1}^M \sum_{j=1}^N \bar{f}(i, j) f(i, j)}$$

The norm of an image f is defined as

$$\|u\| = \sqrt{3 * 3 + 7 * 7 + 1 + 4 * 4} = 5\sqrt{3}$$

$$\|v\| = 1/2\sqrt{1 + 1 + 1 + 1} = 1$$

$$\|w\| = 1/2\sqrt{1 + 1 + 1 + 1} = 1$$

$$u \cdot v = 2 + 5.5 = 7.5$$

$$u \cdot w = -1 - 1.5 = -2.5$$

$$v \cdot w = 0$$

$\{v, w\}$ is orthonormal because $v \cdot w = 0$, $v \cdot v = 1$, $w \cdot w = 1$

$$\text{Let } x_1 = u \cdot v = 7.5, x_2 = u \cdot w = -2.5$$

The orthogonal projection is $u_1 = 7.5v - 2.5w =$

$$\begin{bmatrix} 5 & -5 \\ -2.5 & 2.5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -5 \\ -2.5 & 2.5 \end{bmatrix}$$

$$|u - u_1|^2 = (u - u_1) \cdot (u - u_1) = 12.5$$

So the projection is not a good approximation, it is not close to u .

7 Image compression

$$\phi_1 \cdot \phi_2 = 0 \quad \phi_1 \cdot \phi_3 = 0 \quad \phi_1 \cdot \phi_4 = 0$$

$$\phi_2 \cdot \phi_3 = 0 \quad \phi_2 \cdot \phi_4 = 0 \quad \phi_3 \cdot \phi_4 = 0$$

$$\phi_1 \cdot \phi_1 = 1 \quad \phi_2 \cdot \phi_2 = 1 \quad \phi_3 \cdot \phi_3 = 1 \quad \phi_4 \cdot \phi_4 = 1$$

Let $f = f_a \perp$ the subspace spanned by $\{\phi_1, \phi_2, \phi_3, \phi_4\}$, so f_a is the orthogonal projection

and $|f - f_a|^2$ is as small as possible.

$$x_1 = f \cdot \phi_1 = 50/3 \quad x_2 = f \cdot \phi_2 = -4$$

$$x_3 = f \cdot \phi_3 = 3/2 \quad x_4 = f \cdot \phi_4 = 2$$

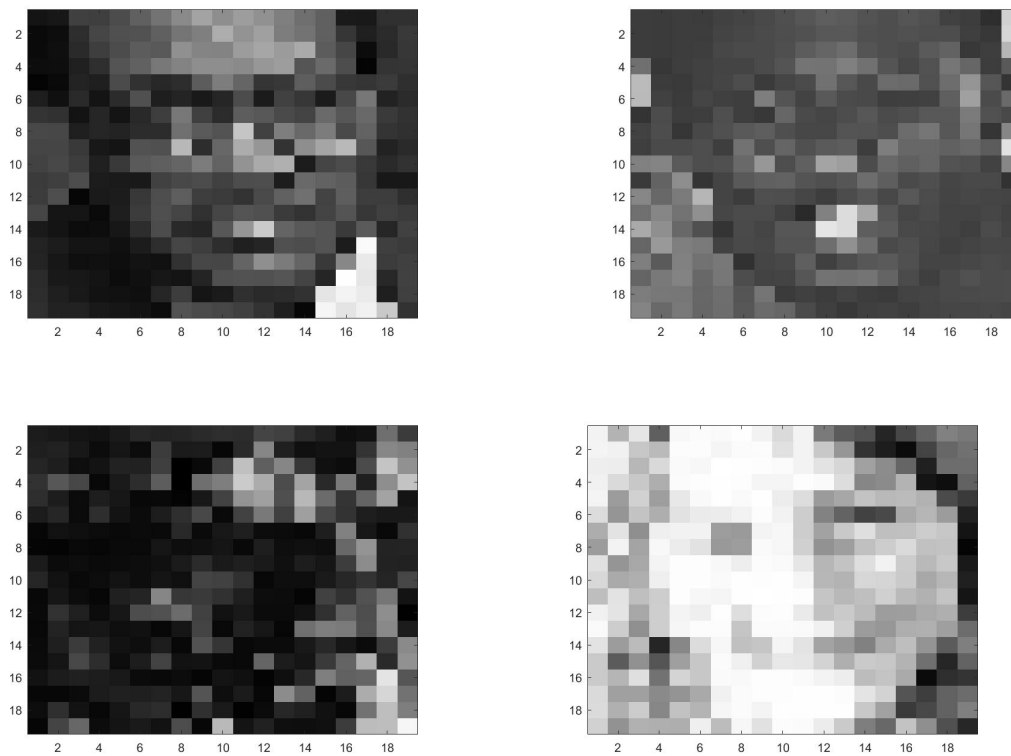
1.4167	6.2222	-0.0833
6.9722	5.5556	5.4722
2.8889	-0.6667	6.8889
3.5556	4.8889	7.5556

$$f_a = x_1\phi_1 + x_2\phi_2 + x_3\phi_3 + x_4\phi_4 =$$

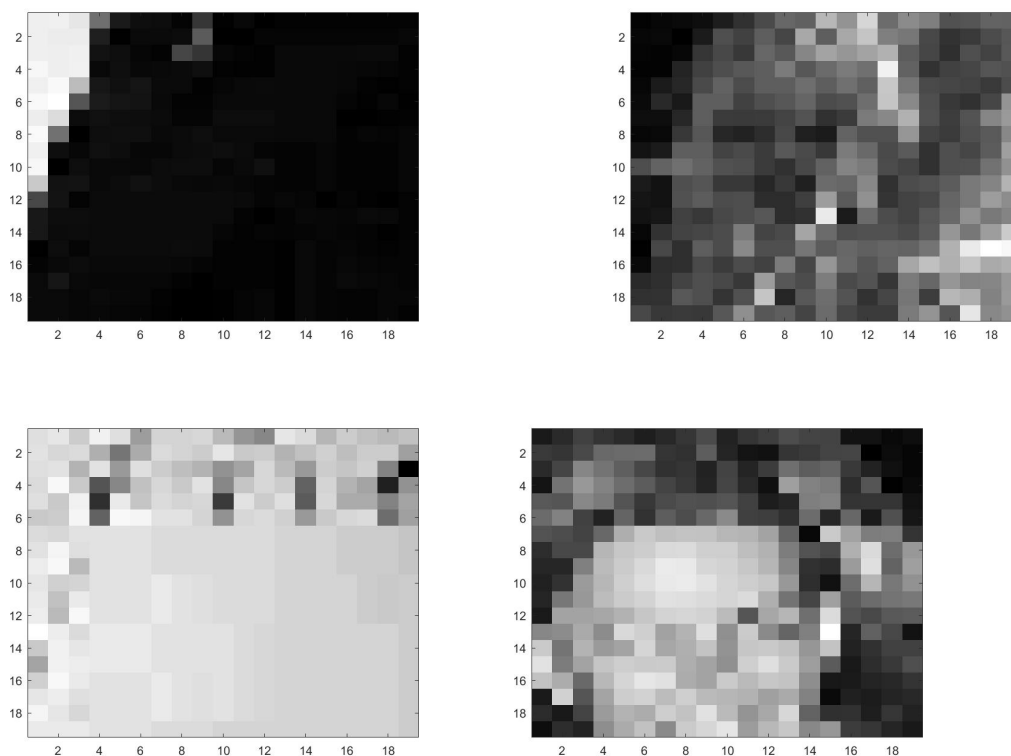
$$|f - f_a|^2 = (f - f_a) \cdot (f - f_a) = 139.9722$$

8 Image bases

```
function r = project(stacks,bases,k,i,j)% r is the norm of the difference
u = stacks{k}(:, :, j);% k is the number of stacks , j is number of images
e_1 = bases{i}(:, :, 1);% i is the number of basis , each basis has 4 images
e_2 = bases{i}(:, :, 2);
e_3 = bases{i}(:, :, 3);
e_4 = bases{i}(:, :, 4);
x_1 = sum(dot(u,e_1));% project the image onto a basis so x1 is scalar product
x_2 = sum(dot(u,e_2));% of u and each basis image
x_3 = sum(dot(u,e_3));
x_4 = sum(dot(u,e_4));
u_p = x_1 * e_1 + x_2 * e_2 + x_3 * e_3 + x_4 * e_4;% u_p is projection result
r = sqrt(sum(dot(u-u_p,u-u_p)));% r to the power of 2 =(u-up).(u-up)
end
```



The images in stack1 look like human faces but very vague and with very low resolution.

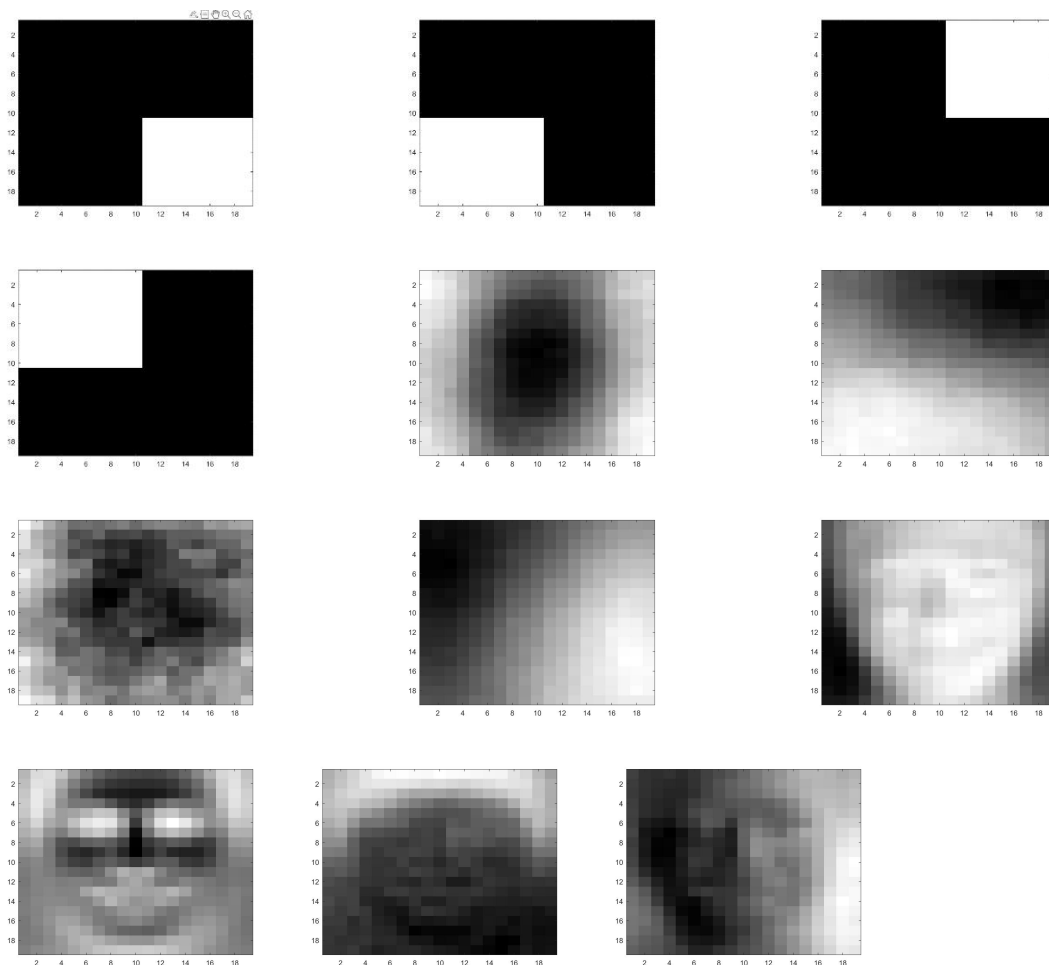


The images in stack2 have even lower resolution, sometimes they are almost black or white, I can't even tell they are human faces.

Plots of the four basis elements of each of the three bases:

Visual differences: bases 3 are most simple with only 4 squares.

Bases 1 and 2 are more complicated but the bases 1 have human face shape.



The mean of the error norms for the six combinations (two test sets against the three bases)

rm =

821.0271	795.1902
860.4754	649.2013
944.9009	697.3214

Basis 1 work best for test set 1 and basis 2 work best for test 2 , because they have the least mean of error norms.