FMAN95 Computer Vision
Assignment 2
Calibration and DLT

Exercise 1

Ex 1

suppose we have found camera P and 3D-points X such that $\lambda x = PX$

we can take an unknown projective transformation T

let $\tilde{P} = PT$ and $\tilde{X} = T^{-1}X$

The new cameras and scene points also solve the problem. since

$$\lambda x = PX = PTT^{-1}X = \tilde{P}\tilde{X}$$

Computer Exercise 1

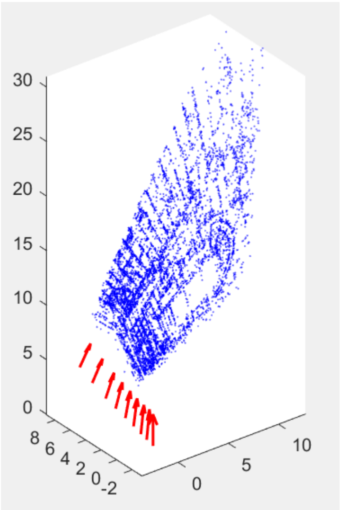

Figure: Plot of reconstruction

It is not a reasonable reconstruction due to obvious distortions.

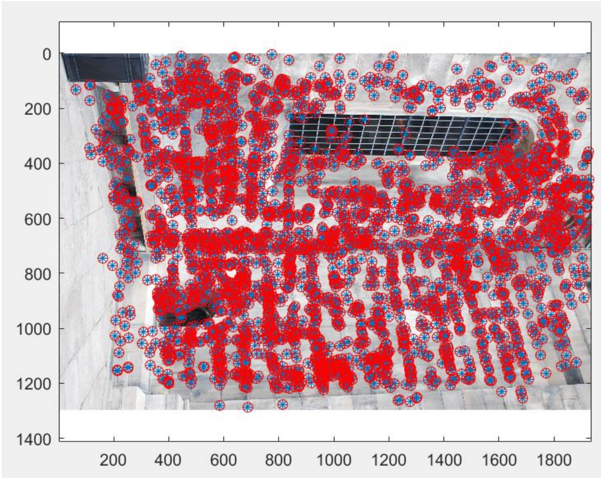

Figure: Plot of projections of the 3D points and the corresponding image points

The projections appear to be close to the corresponding image points.

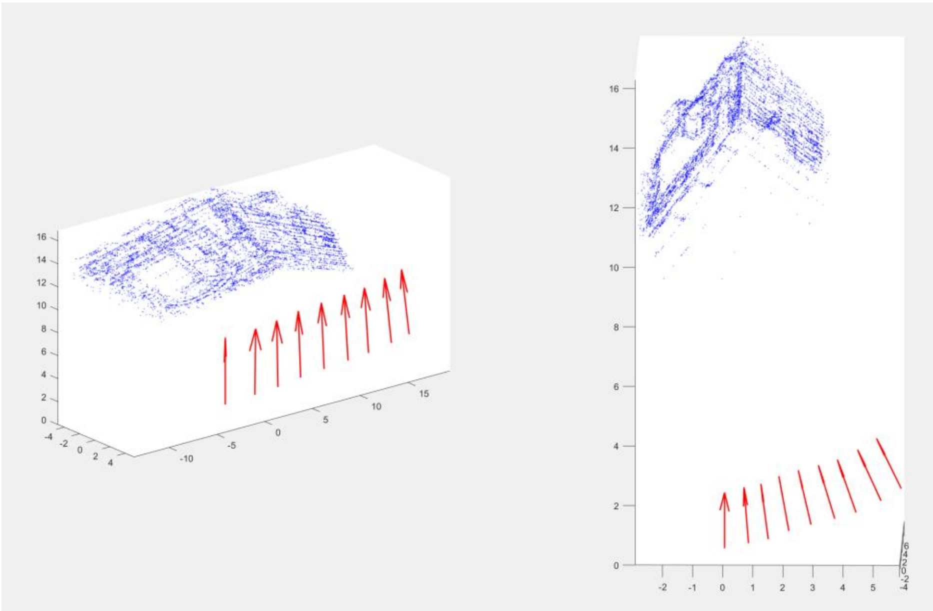

Figure: Plots of 3D construction using T1 and T2 transformation

Among these two reconstructions, the second one seems reasonable, very close to the structure in real life.

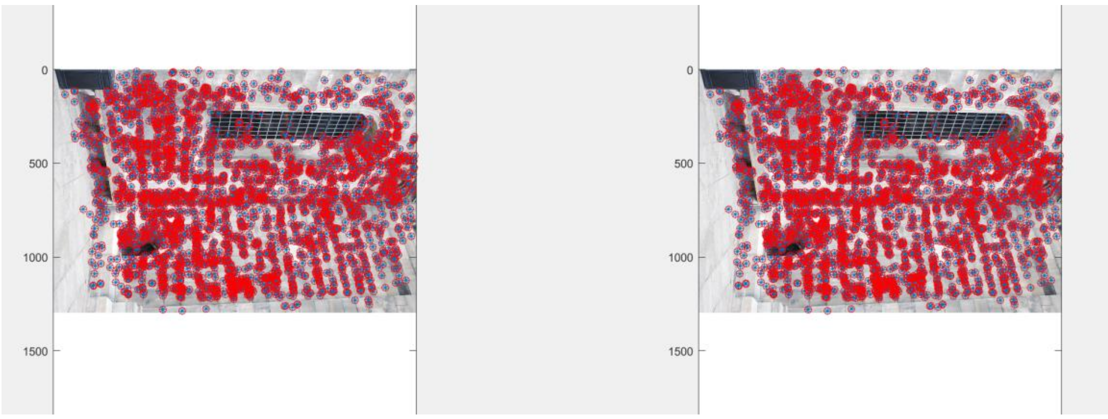

Figure: Plots of the projections of the 3D points and the corresponding image points

The projections appear to be not changed. Since we know a new solution can always be obtained from TX as in Exercise 1, so the new projections should be the same.

Exercise 2

When we use calibrated cameras,

$$\tilde{x} = K^{-1}x \rightarrow \tilde{x} \sim K^{-1}K[R \ \ t]X = [R \ \ t]X$$

The statement is $\lambda\tilde{x} = [R \ \ t]X$ where $\tilde{x} = K^{-1}x$

To achieve a valid solution, H need to be a similarity transformation and it preserves angles and paralled lines, so there is no distortion. However, it can still be re-scaled.

Exercise 4

$$f = 1000 \qquad (x_0 \ y_0) = (500, 500)$$

$$K = \begin{pmatrix} 1000 & 0 & 500 \\ 0 & 1000 & 500 \\ 0 & 0 & 1 \end{pmatrix} \quad K^{-1} = \begin{pmatrix} 1/1000 & 0 & -1/2 \\ 0 & 1/1000 & -1/2 \\ 0 & 0 & 1 \end{pmatrix} \quad P = \begin{pmatrix} 1000 & -250 & 250\sqrt{3} & 500 \\ 0 & 500(\sqrt{3}-\frac{1}{2}) & 500(1+\frac{\sqrt{3}}{2}) & 500 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \end{pmatrix}$$

$$K^{-1} \cdot P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & 0.5 & 0 \\ 0 & -0.5 & \sqrt{3}/2 & 1 \end{pmatrix}$$

$$(0,0) \rightarrow K^{-1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix}$$

$$(0,1000) \rightarrow K^{-1}\begin{pmatrix} 0 \\ 1000 \\ 1 \end{pmatrix} = (-\tfrac{1}{2}, \ \tfrac{1}{2}, \ 1)^T$$

$$(1000, 0) \rightarrow K^{-1}\begin{pmatrix} 1000 \\ 0 \\ 1 \end{pmatrix} = (\tfrac{1}{2}, \ -\tfrac{1}{2}, \ 1)^T$$

$$(1000, 1000) \rightarrow K^{-1}\begin{pmatrix} 1000 \\ 1000 \\ 1 \end{pmatrix} = (\tfrac{1}{2}, \ \tfrac{1}{2}, \ 1)^T$$

$$(500, 500) \rightarrow K^{-1}\begin{pmatrix} 500 \\ 500 \\ 1 \end{pmatrix} = (0 \quad 0 \quad 1)^T$$

Exercise 3

$$K \cdot K^{-1} = \begin{pmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{pmatrix}$$

geometric interpretation : A is to rescale the image by $1/f$.
B is to translate the coordinates by $(-x_0, -y_0)$

K^{-1} is first translate by $(-x_0, -y_0)$ then rescale by $1/f$.
it's a transformation from the real image in pixels back to the image plane.

The principal point $(x_0\ y_0)$ end up at $(0, 0)$
The points with distance f to the principal point end up at a circle with distance 1 to $(0, 0)$

$$K^{-1} = \begin{pmatrix} 1/320 & 0 & -1 \\ 0 & 1/320 & -3/4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$K^{-1} \cdot \begin{pmatrix} 0 \\ 240 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$K^{-1} \cdot \begin{pmatrix} 640 \\ 240 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\tilde{x}_1 \cdot \tilde{x}_2 = |\tilde{x}_1| |\tilde{x}_2| \cdot \cos\theta = -1 + 1 = 0$$

$$\theta = \frac{\pi}{2}$$

$K[R\ t]$ and $[R\ t]$ have the same nullspace. so they have the some camera center.

The principal axis is the last row of $[R\ t]$. when multiply with K, the last row is $(0, 0, 1)$
the last row keep the same. so the principal axis keeps the same.

Exercise 5

Ex 5

$$K \cdot R = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix} = \begin{pmatrix} aR_1^T + bR_2^T + cR_3^T \\ dR_2^T + eR_3^T \\ fR_3^T \end{pmatrix} = \begin{bmatrix} A_1^T \\ A_2^T \\ A_3^T \end{bmatrix}$$

A is the first 3×3 matrix of P $\qquad$ R_1 R_2 R_3 have the length 1.

$$A_3 = fR_3 = \left(-\tfrac{1}{\sqrt{2}}, 0, \tfrac{1}{\sqrt{2}}\right)^T \qquad f = \|A_3\| = 1 \qquad R_3 = \frac{1}{\|A_3\|} A_3 = \left(-\tfrac{1}{\sqrt{2}}, 0, \tfrac{1}{\sqrt{2}}\right)^T$$

$$A_2 = dR_2 + eR_3 = \left(\frac{-700}{\sqrt{2}}, 1400, \frac{700}{\sqrt{2}}\right)^T \qquad e = A_2^T R_3 = \frac{700}{\sqrt{2}\cdot\sqrt{2}} + 0 + \frac{700}{2} = 700$$

$$dR_2 = A_2 - eR_3 = (0, 1400, 0)^T$$

$$d = 1400 \qquad R_2 = (0, 1, 0)^T$$

$$A_1 = aR_1 + bR_2 + cR_3 \qquad b = A_1^T R_2 = 0$$

$$= \left(\frac{800}{\sqrt{2}}, 0, \frac{2400}{\sqrt{2}}\right)^T \qquad c = A_1^T R_3 = (-400 + 0 + 1200) = 800$$

$$aR_1 = A_1 - bR_2 - cR_3 = \left(\frac{800}{\sqrt{2}}, 0, \frac{2400}{\sqrt{2}}\right)^T - 0 - \left(-\frac{800}{\sqrt{2}}, 0, \frac{800}{\sqrt{2}}\right)^T$$

$$= \left(\frac{1600}{\sqrt{2}}, 0, \frac{1600}{\sqrt{2}}\right)^T$$

$$a = 1600 \qquad R_1 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)^T$$

$$K = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} = \begin{pmatrix} 1600 & 0 & 800 \\ 0 & 1400 & 700 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R = \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

focal length : $f = 1400$

skew : $s = 0$

aspect ratio : $\gamma = \frac{8}{7}$

principal point : $(800, 700)$

Computer Exercise 2

```
K_1 =

   1.0e+03 *

    2.3915    0.2257    0.9427
         0    0.6239    0.8136
         0         0    0.0010


K_2 =

   1.0e+03 *

    2.3940    0.0000    0.9324
         0    2.3981    0.6283
         0         0    0.0010
```

They are not the same transformation.

Exercise 7

$$p = N^{-1}\tilde{p}$$

Computer Exercise 3

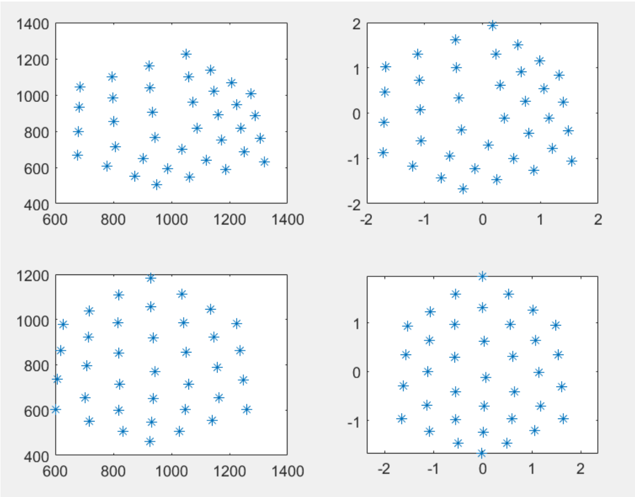

Figure: Original and normalized points

The points are centered around (0,0) with mean distance 1 to (0,0), after normalization.

After performing DLT equations and solve the least squares system using SVD.

We can find an eigenvector corresponding to the smallest eigenvalue by selecting the last column of V. The smallest eigenvalue can be find as the last element of S(T)S.

The smallest eigenvalues are 2.27e – 04 and 1.48e – 04 respectively, which are close to 0.

The $\|Mv\|$ are 0.015 and 0.012 respectively.

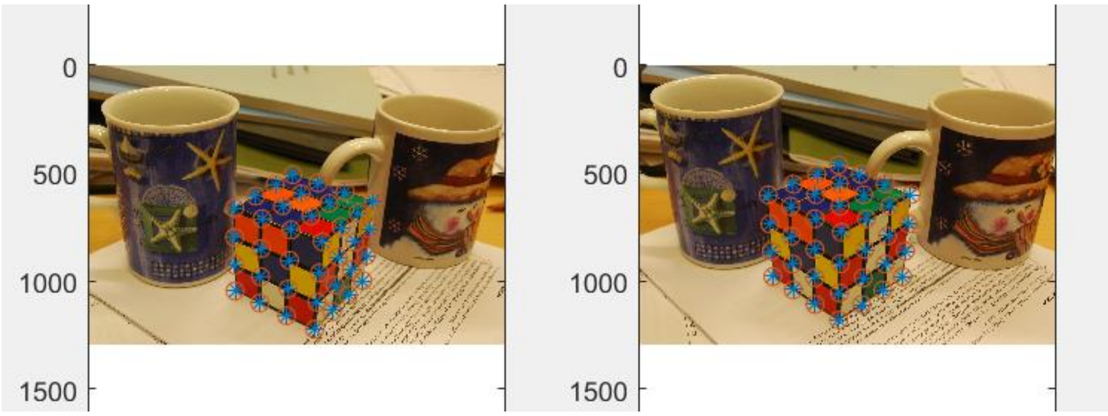

Figure: Plot of the projection points and the measured points in the same image.

The projection points and the measured points are quite closed to each other.

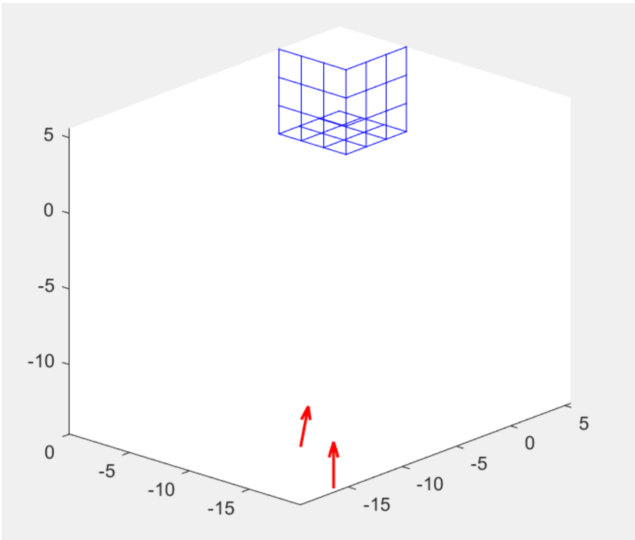

Figure: Plot of the cube model and two camera centers and viewing directions

The result looks reasonable.

```
K1 =

   1.0e+03 *

    2.4486   -0.0181    0.9598
         0    2.4468    0.6759
         0         0    0.0010


K2 =

   1.0e+03 *

    2.3885   -0.0245    0.8142
         0    2.4009    0.7905
         0         0    0.0010
```

The inner parameters of the first camera:

Focal length: 2448.6 principal point: (959.8,675.9) aspect ratio: 1.0007 skew: −0.0074

Computer Exercise 5

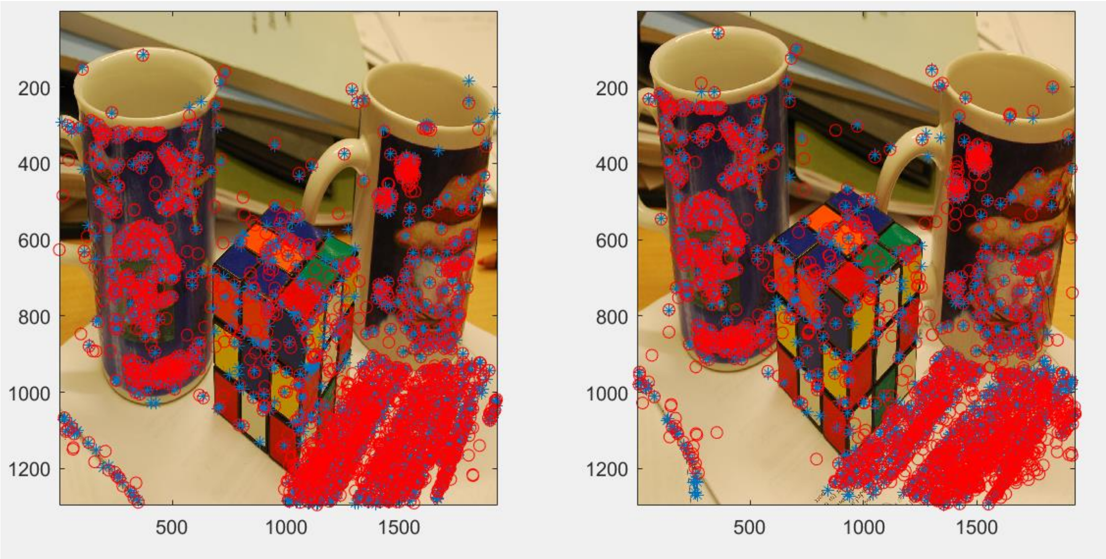

Figure: Plot of the projection points and SIFT points with normalization

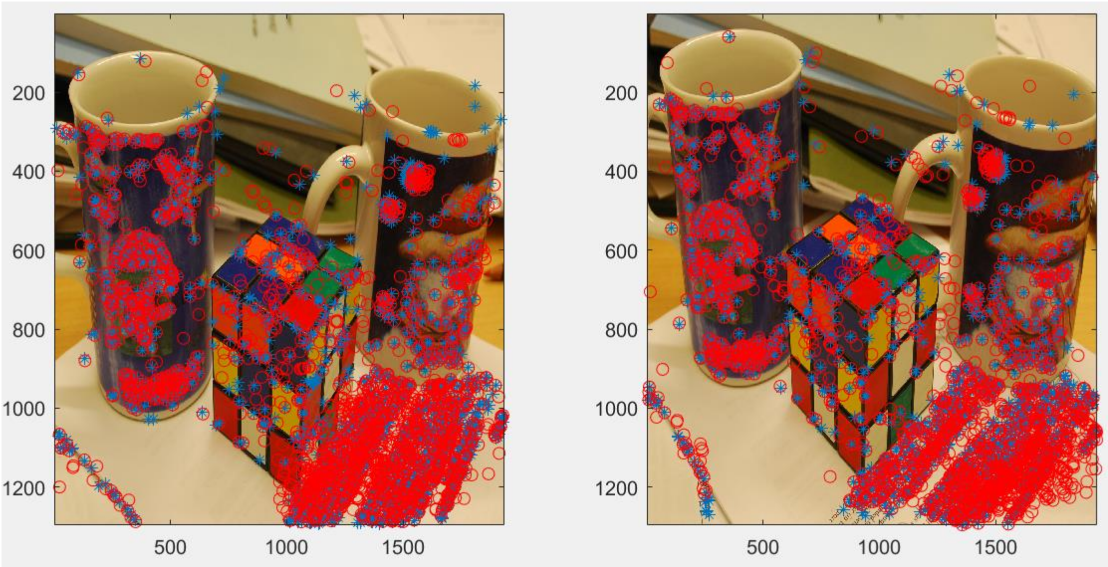

Figure: Plot of the projection points and SIFT points without normalization

Comparing the plots of points between normalized ones and non-normalized ones, we can see obvious improvements after normalization.

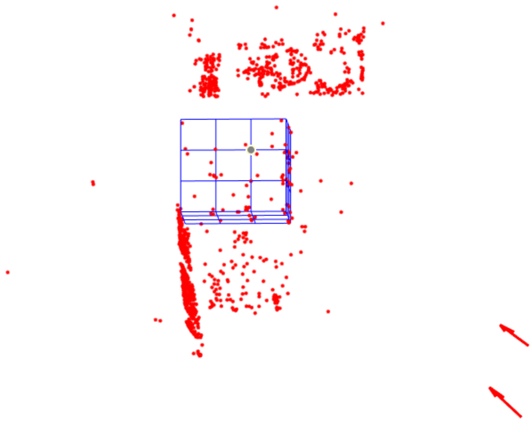

Figure: Plot of Plot of the 3D points and camera centers and viewing directions

We can distinguish the dominant the contour of cups and a flat plane which is the paper.