FMAN95 Computer Vision
Assignment 3
Epipolar Geometry

Exercise 1 and 3

Epipolar Geometry

Exercise 1

$$P_1 = [I \; 0] \qquad P_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} = [A \; t]$$

$$F = [t]_\times A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{pmatrix}$$

$$l = FX = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$$

$l^T x_1 = (2 \; 0 \; -4)\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 0$ could be a projection

$l^T x_2 = (2 \; 0 \; -4)\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0$ could be a projection

$l^T x_3 = (2 \; 0 \; -4)\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} = 4$ could not

Exercise 3

$$\tilde{x}_2^T \tilde{F} \tilde{x}_1 = (N_2 x_2)^T \tilde{F} (N_1 x_1) = x_2^T (N_2^T \tilde{F} N_1) x_1 = 0$$

so when $F = N_2^T \tilde{F} N_1$

it fulfills $x_2^T F x_1 = 0$

Exercise 2

Exercise 2

$$P_1 = [I \; 0] \qquad P_2 = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} = [A \; t]$$

$$C_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad C_2 = null(P_2) = \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \begin{bmatrix} -A^{-1}t \\ 1 \end{bmatrix}$$

$$e_1 \sim P_1 C_2 = [I \; 0]\begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = -A^{-1}t$$

$$e_2 \sim P_2 C_1 = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = t$$

$$F = [t]_x A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix} \qquad \det(F) = 0$$

$$e_2^T F = \begin{pmatrix} 2 & 2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

$$F e_1 = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix}\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

optional:

$e_1 = -A^{-1}t$

$e_2 = t$

$$e_2^T F = t^T \cdot [t]_x A = 0 \times A = 0$$

$$F e_1 = [t]_x A \cdot -A^{-1}t = -[t]_x t = 0$$

From linear algebra we know if $FX = 0$ has non zero solution $|F| = 0$ which is the fundamental matrix has to have determinant 0.

we have already had a non zero solution $e_1 = -A^{-1}t$

Exercise 4

Exercise 4

$$F = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad \text{let } e_2 = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$e_2^T F = 0 \Rightarrow (x\ y\ z)\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} = (y,\ x+z,\ x+z) = 0$$

$$\Rightarrow \begin{cases} y = 0 \\ x+z = 0 \\ x+z = 0 \end{cases} \Rightarrow \begin{cases} x = t \\ y = 0 \\ z = -t \end{cases} \Rightarrow e_2 \sim \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$[e_2]_\times F = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 1 & 0 & 0 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

$X_1 = (1, 2, 3)$ $\quad x_1 = P_1\begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\quad x_2^T F x_1 = (2\ -10\ 0)\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$x_2 = P_2\begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -10 \\ 0 \end{pmatrix}$ $\quad = 0$

$X_2 = (3, 2, 1)$ $\quad x_1 = P_1\begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ $\quad x_2^T F x_1 = (4\ -6\ 2)\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

$x_2 = P_2\begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$ $\quad = 0$

$X_3 = (1\ 0\ 1)$ $\quad x_1 = P_1\begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $\quad x_2^T F x_1 = (2\ -2\ 0)\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$x_2 = P_2\begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$ $\quad = 0$

$$\text{null}(P_2) = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

camera center

Exercise 5 and Exercise 6 (revised)

Exercise 5

$$[t]_x^T [t]_x = (USV^T)^T USV^T = VS^T \underbrace{U^T U}_{I} SV^T = VS^T S V^T$$

S is diagonal $\quad S = S^T \quad = VS^2V^T$

S^2 is the diagonalization of $[t]_x^T [t]_x$

so the eigenvalues are the squared singular values.

Exercise 6

$$UV^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \end{pmatrix}$$

$$\det(UV^T) = \frac{1}{2} + \frac{1}{2} = 1$$

$$E = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad x_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$x_2^T E x_1 = (1 \ 1 \ 1) \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

so x_1, x_2 is a plausible correspondence

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = P_1 X = [I \ 0] \begin{pmatrix} x \\ y \\ z \\ s \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{cases} x = 0 \\ y = 0 \\ z = 1 \end{cases} \Rightarrow X = \begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix}$$

X must be one of the points $\begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix}$ if x_1 is the projection of X in P_1

$$UWV^T = \begin{pmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & -1 & 0 \end{pmatrix} \qquad X = \begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix} \qquad \det(\quad) = 1$$

$$UW^TV^T = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \end{pmatrix} \qquad P_2X \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \det(\quad) = 1$$

① $P_2 = [UWV^T \ \ u_3]$

$$P_2X = \begin{pmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ s \end{pmatrix} \Rightarrow s = -1/\sqrt{2}$$

$\mathrm{depth}(P_1, X) = \frac{1}{-1/\sqrt{2}} \cdot 1 < 0$

$\mathrm{depth}(P_2, X) = \frac{1}{-1/\sqrt{2}} \cdot -1/\sqrt{2} > 0$

② $P_2 = [UWV^T \ -u_3]$

$$P_2X = \begin{pmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ +1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & -1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ -s \end{pmatrix} \Rightarrow s = 1/\sqrt{2}$$

$\mathrm{depth}(P, X) = \frac{1}{1/\sqrt{2}} \cdot 1 > 0$

$\mathrm{depth}(P_2X) = \frac{1}{1/\sqrt{2}} \cdot -1/\sqrt{2} < 0$

③ $P_2 = [UW^TV^T \ \ u_3]$

$$P_2X = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ s \end{pmatrix} \Rightarrow s = 1/\sqrt{2}$$

④ $P_2 = [UW^TV^T \ -u_3]$

$$P_2X = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & -1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ -s \end{pmatrix} \Rightarrow s = -1/\sqrt{2}$$

$$\mathrm{depth}(P, X) = \frac{\mathrm{sign}(\det(A))}{\|A_3\| \rho} \cdot [A_3^T \ a_3] X \qquad \text{for } X = \begin{bmatrix} X \\ \rho \end{bmatrix}$$

③: $\mathrm{depth}(P_1, X) = \frac{1}{1/\sqrt{2}} \cdot 1 = \sqrt{2} > 0$

$\mathrm{depth}(P_2, X) = \frac{1}{1/\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 2 > 0$

④ $\mathrm{depth}(P_1, X) = \frac{1}{-1/\sqrt{2}} \cdot 1 = -\sqrt{2} < 0$

$\mathrm{depth}(P_2, X) = \frac{1}{-1/\sqrt{2}} \cdot 1/\sqrt{2} < 0$

only ③ 3D point X is in front of both cameras.

Computer Exercise 1

$$F = \begin{bmatrix} -3.39e^{-8} & -3.72e^{-6} & 0.0058 \\ 4.67e^{-6} & 2.89e^{-7} & -0.0267 \\ -0.0072 & 0.0263 & 1 \end{bmatrix}$$

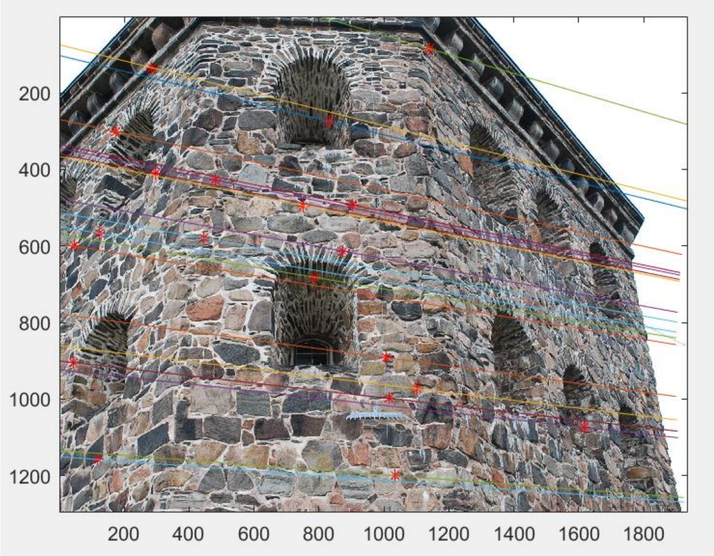

Figure 1: Plot of 20 randomly selected points with their epipolar lines.

From the figure we can see that the epipolar lines and points are quite close.

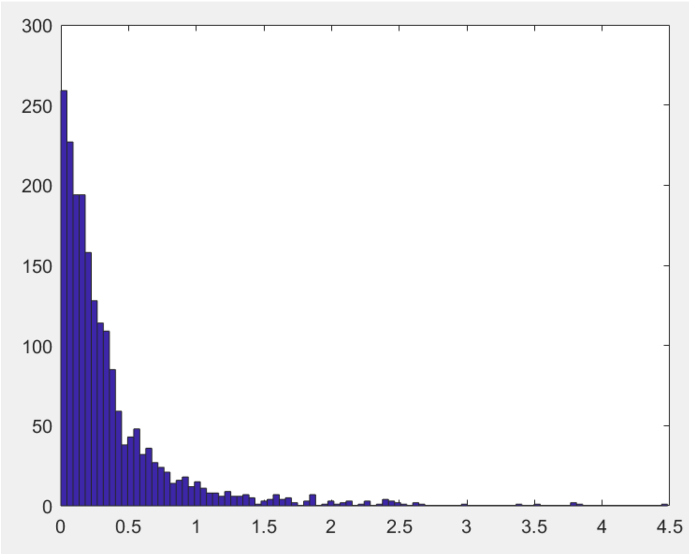

Figure 2: The histogram of the distances between all the points and epipolar lines.

The mean epipolar distances with normalization d = 0.3612

The mean epipolar distances without normalization d = 0.4878

Computer Exercise 2

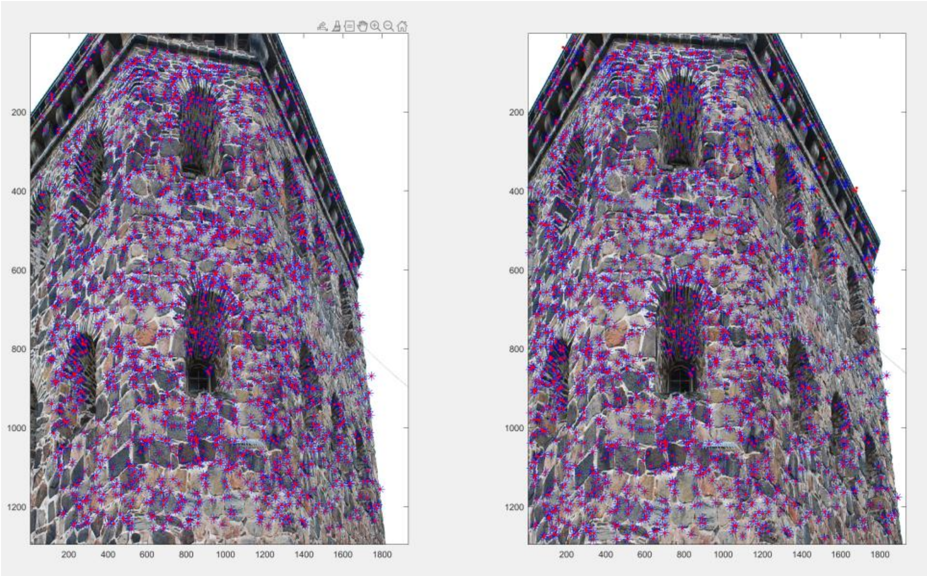

Figure 3: The image points and the projected 3D points in the same figure.

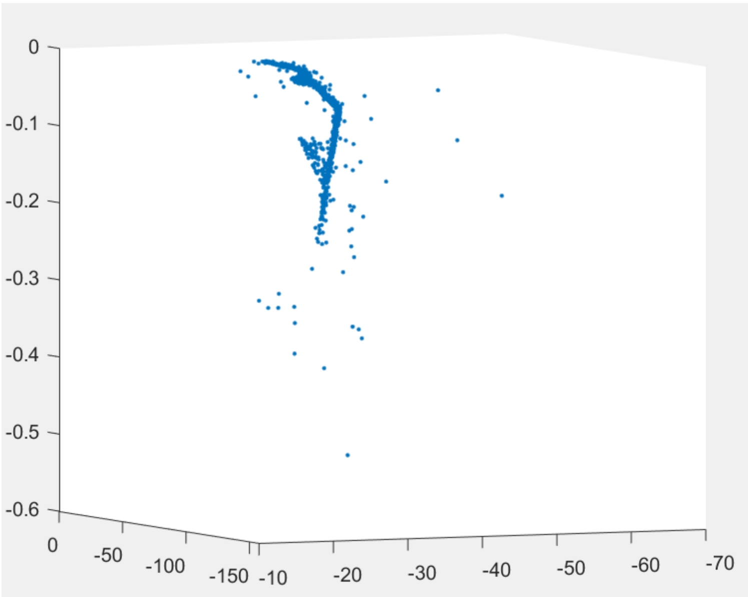

Figure 4: Plot of 3D points

It is not good enough because of distortion.

Computer Exercise 3

$$E = \begin{bmatrix} -8.89 & -1006 & 377 \\ 1252 & 78 & -2448 \\ -472.78 & 2550.2 & 1 \end{bmatrix}$$

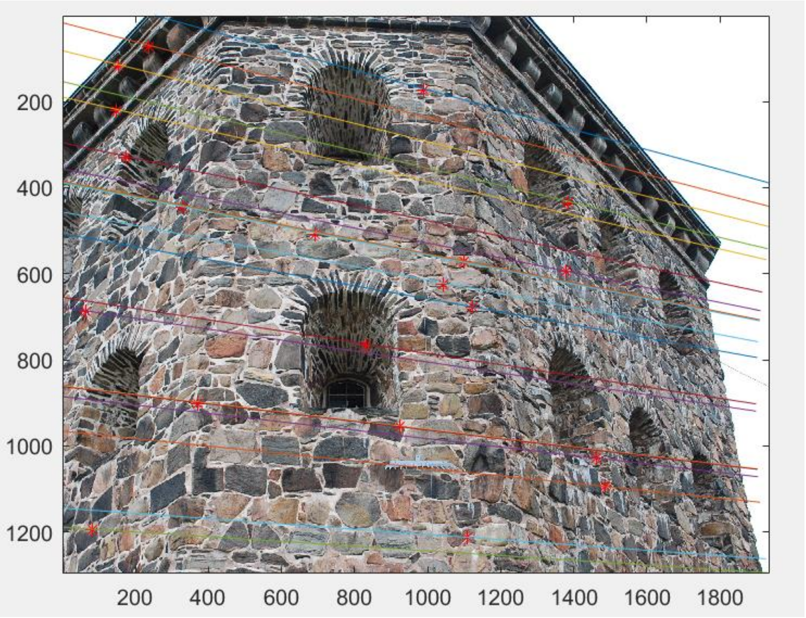

Figure 5: Plot of 20 randomly selected points with their epipolar lines.

From the figure we can see that there are distances between the epipolar lines and points. The results are worse compared to in computer exercise 1.

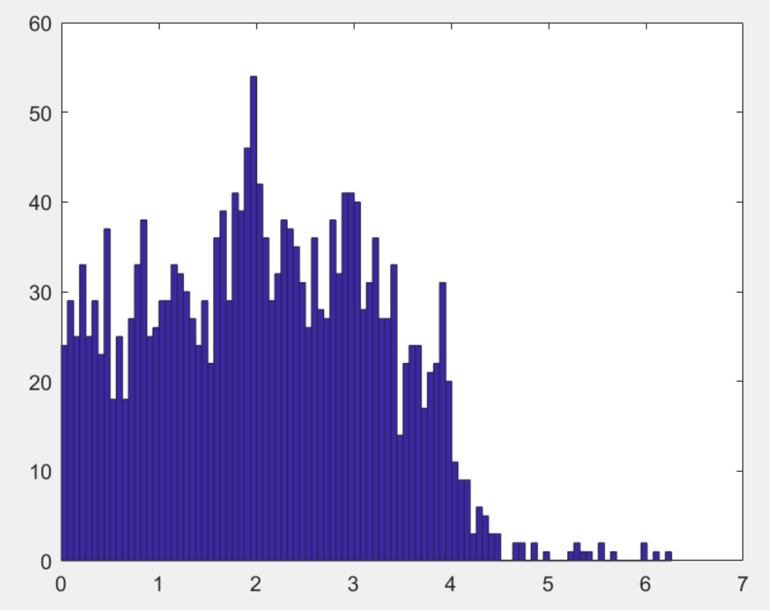

Figure 6: The histogram of the distances between all the points and epipolar lines.

Computer Exercise 4

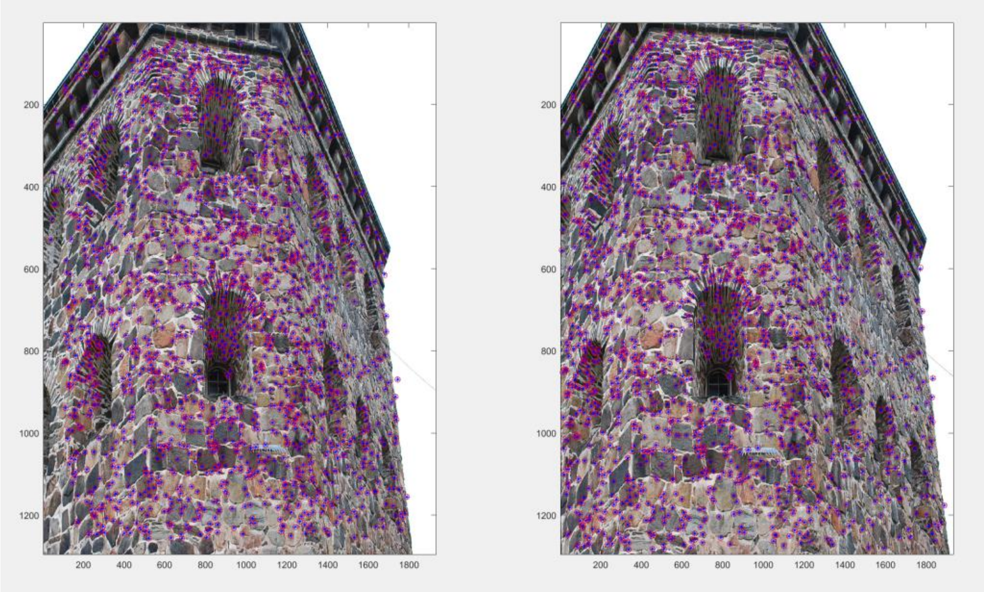

Figure 7: Plot of the image points and the projections of 3D points

They match well and the error look small.

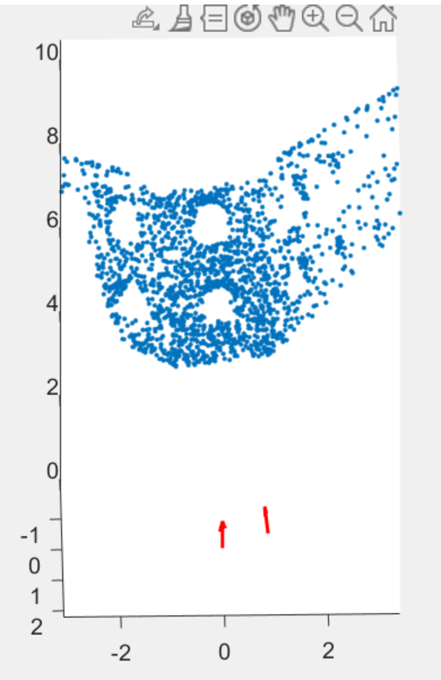

Figure 8: Plot of 3D points and camera centers and principal axes

They look reasonable and nice.