FMAN95 Computer Vision

Assignment 1

Exercise 1:

To find the 2D coordinates of the points with known homogeneous coordinates, we simply divide them by the third coordinate.

X1 = (2,-1)

X2 = (-3,2)

X3 = (2,-1)

The interpretation of points with 0 as the third coordinate:

It can be interpreted as a point infinitely far away in the direction (2,-1). It is a vanishing point or a point at infinity. It is an intersection point of two parallel lines.

Computer Exercise 1:

Plot:

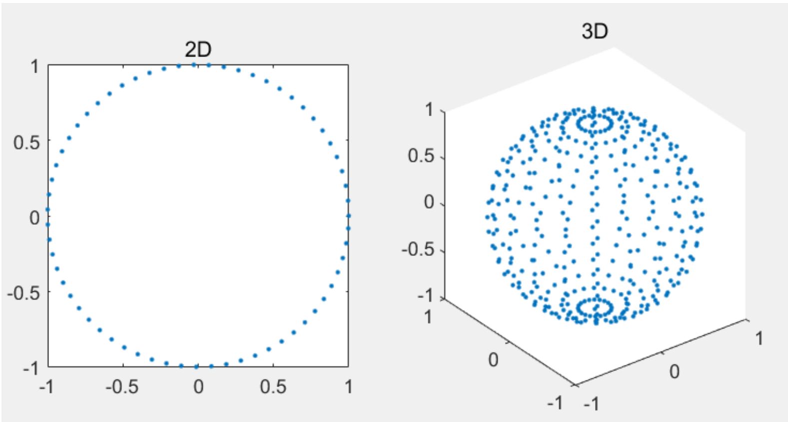

Figure 1: Plots of CE1

Exercise 2:

If x is on both the lines l_1 and l_2, we have

$$\begin{cases} l_1^T x = 0 \\ l_2^T x = 0 \end{cases} \iff \begin{cases} x+y+z=0 \\ 3x+2y+z=0 \end{cases} \iff \begin{cases} x = s \\ y = -2s \\ z = s \end{cases}, \quad s \in \mathbb{R}$$

The intersection point is $x \sim (1, -2, 1)$

The corresponding point in $\mathbb{R}^2$ is $(1, -2)$

If x is on both the lines l_3 and l_4, we have

$$\begin{cases} l_3^T x = 0 \\ l_4^T x = 0 \end{cases} \iff \begin{cases} x+2y+3z=0 \\ x+2y+z=0 \end{cases} \iff \begin{cases} x = -2s \\ y = s \\ z = 0 \end{cases}, \quad s \in \mathbb{R}$$

The intersection point is $x \sim (-2, 1, 0)$

The homogeneous coordinates of x_1, x_2

$$x_1 \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad x \sim \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

If line l pass through both points we have

$$\begin{cases} l^T x_1 = 0 \\ l^T x_2 = 0 \end{cases} \iff \begin{cases} a+b+c=0 \\ 3a+2b+c=0 \end{cases} \iff \begin{cases} a = s \\ b = -2s \\ c = s \end{cases}, \quad s \in \mathbb{R}$$

Therefore the line is $l \sim (1, -2, 1)$

It can be interpreted as a point infinitely far away in the direction (-2,1). It is a vanishing point or a point at infinity. It is an intersection point of two parallel lines.

Exercise 3:

The intersection of two lines $l_1 = \begin{pmatrix} a_1 \\ b_1 \\ c_1 \end{pmatrix}$, $l_2 = \begin{pmatrix} a_2 \\ b_2 \\ c_2 \end{pmatrix}$

$x \sim (x, y, z)$

we have

$$\begin{cases} l_1^T x = 0 \\ l_2^T x = 0 \end{cases} \iff \begin{cases} a_1 x + b_1 y + c_1 z = 0 \\ a_2 x + b_2 y + c_2 z = 0 \end{cases} \iff \underbrace{\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix}}_{M} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{x} = 0$$

All the points x satisfy that $Mx = 0$

so the intersection point x is in the null space of the matrix $M = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix}$

All the non-zero vectors in the null space of M are the intersection point. So there is no other non-zero points besides the intersection point.

Computer Exercise 2:

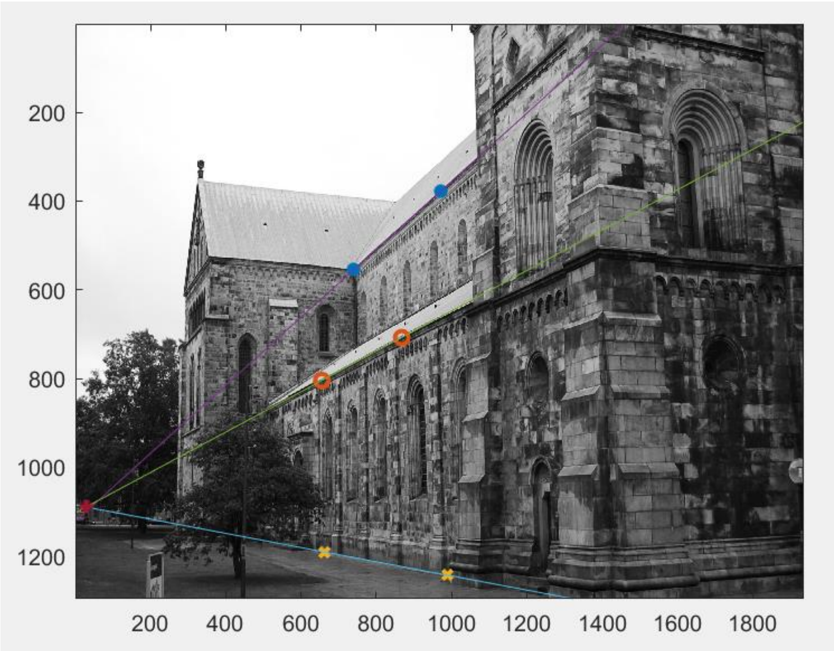

Figure 2: Plot of CE2

These lines appear to be parallel in 3D because them are basically on the same height level.

The intersection point of the second and third lines are plotted in the image.

The distance between the first line and the intersection point is 8.195. Compared to the image size of 1296*1936. This value is fairly close to zero.

Because these three lines are basically parallel in real world, the perspective projections of parallel lines in three-dimensional space appear to converge at a vanishing point. One of the reason why the distance is not 0 is, maybe the points collected are not absolutely accurate.

Exercise 4:

$$y_1 \sim Hx_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$y_2 \sim Hx_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$l_1 \sim (a, b, c)$ contains $x_1 x_2$

$$\begin{cases} l_1^T x_1 = 0 \\ l_1^T x_2 = 0 \end{cases} \Longleftrightarrow \begin{cases} a + \phantom{b+{}} c = 0 \\ \phantom{a+{}} b + c = 0 \end{cases} \Longleftrightarrow \begin{cases} a = -s \\ b = -s \\ c = s \end{cases} \Longleftrightarrow l_1 \sim (-1, -1, 1)$$

$l_2 \sim (a, b, c)$ contains $y_1 \, y_2$

$$\begin{cases} l_2^T y_1 = 0 \\ l_2^T y_2 = 0 \end{cases} \Longleftrightarrow \begin{cases} a = 0 \\ a+b+c = 0 \end{cases} \Longleftrightarrow \begin{cases} a = 0 \\ b = -s \\ c = s \end{cases} \Longleftrightarrow l_2 \sim (0, -1, 1)$$

$$(H^{-1})^T l_1 = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = l_2$$

proof: we have $\quad 0 = l_1^T x = l_1^T H^{-1} H x = \left((H^{-1})^T l_1\right) H x \sim$

$$(H^{-1})^T l_1 \, y = 0$$

which means y lies on the line $(H^{-1})^T l_1$

So we proved that transformation $y \sim Hx$ belongs to l_2.

Computer Exercise 3:

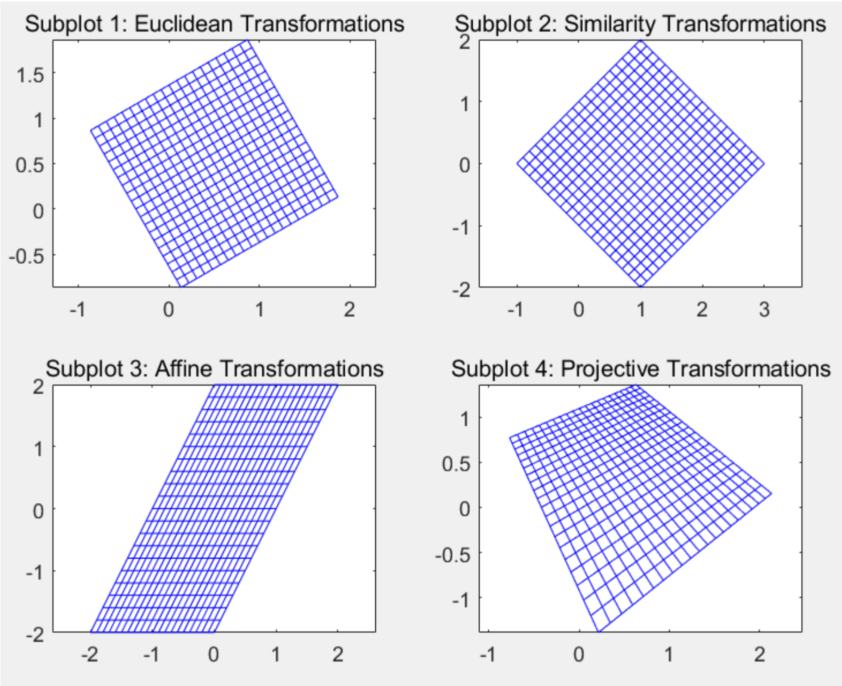

Figure 3: Plots of CE3

H1 is Euclidean transformation which preserve lengths between points.
H2 is similarity transformation which preserve angles between lines.
H3 is affine transformation where parallel lines are mapped to parallel lines.
H4 is projective transformation which is not a special case.

Exercise 5:

Ex 5.

$$x_1' = PX_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1/4 \\ 1/2 \\ 1 \end{pmatrix}$$

$$x_2' = PX_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \\ 1 \end{pmatrix}$$

$$x_3' = PX_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

camera center : $C = -R^T t = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

principal axis : R_3^T (the third row of R) $= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

It can be interpreted as a point infinitely far away in the direction (1,1). It is a vanishing point or a point at infinity. It is an intersection point of two parallel lines.

Computer Exercise 4:

The camera centers:

P1: (0,   0,   0)

P2: (6.6352, 14.8460, -15.0691)

The principal axes normalized to length one:

P1: (0.3129, 0.9461, 0.0837)

P2: (0.0319, 0.3402, 0.9398)

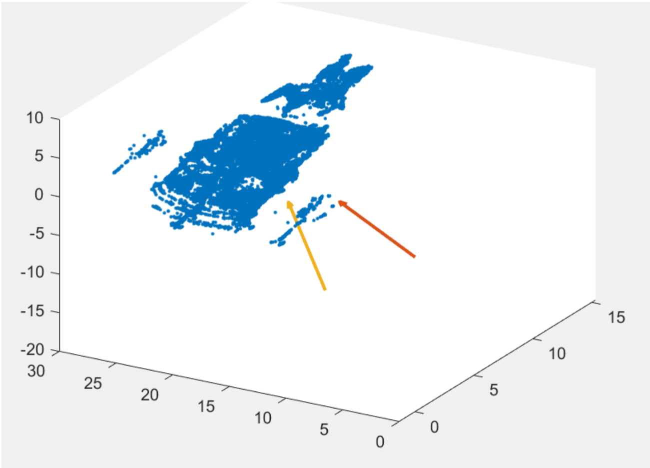

Figure 3: 3D points and vector of principal axes

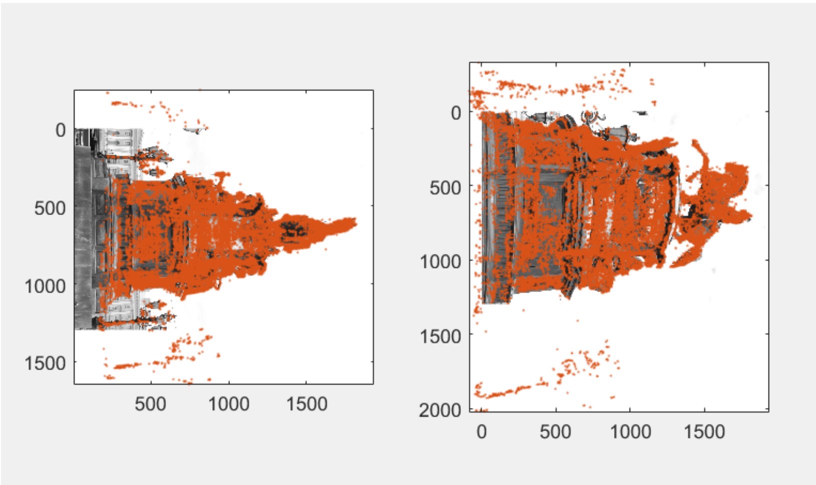

Figure 4: Projection into camera P1 and P2

The results look reasonable. Most of the points lie on the statue and match the shape.

Exercise 6:

Ex 6

$P_1 = [I\ 0] \qquad U \sim \begin{pmatrix} X \\ s \end{pmatrix} \sim \begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ s \end{pmatrix}$

$$x \sim P_1 U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ s \end{bmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

we can see s is eliminated during the calculation

so we can not determine s only using information from P_1

U belong to the plane $\Pi = \begin{pmatrix} \pi \\ 1 \end{pmatrix} \qquad \Pi^T U = 0$

$$(\pi\ \ 1)\begin{pmatrix} X \\ s \end{pmatrix} = \pi X + s = 0$$

$$s = -\pi^T X$$

$$x \sim P_1 U = X$$

$$y \sim P_2 U = [R\ t]\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ s \end{bmatrix} = [Rx + ts]$$

$$= [Rx - t\pi^T x]$$

$$= [R - t\pi^T]X$$

so $H = (R - t\pi^T)$ maps x to y

Computer Exercise 5:

Originally, the origin of the image coordinate system is located in the upper left corner in the image.

After normalization, the origin is located in the middle part of the image, shown in figure 5.

Figure 7 shows the plots of 3D corner points, camera centers and camera principal axes for camera P1 and the new cameraP2. The new camera matrix is

$$P_2 = \begin{bmatrix} 0.8660 & 0 & 0.5000 & -1.7321 \\ 0 & 1.0000 & 0 & 0 \\ -0.5000 & 0 & 0.8660 & 1.0000 \end{bmatrix}$$

Figure 8 shows the plots of final transformed image and the corner points.

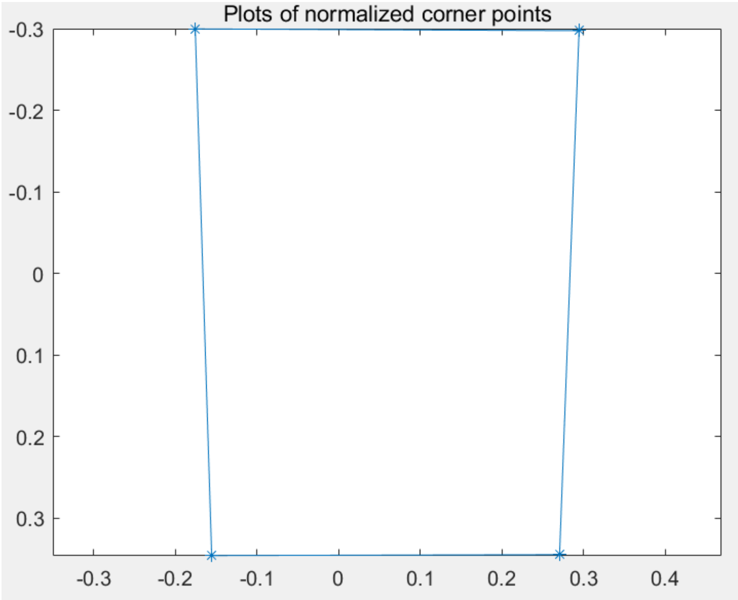

Figure 5: The plot of normalized corners

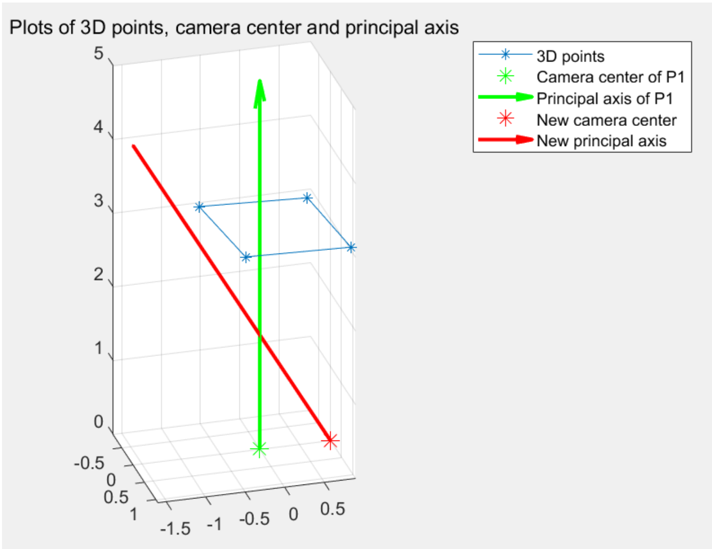

Figure 6: Plots of camera centers and principal axes

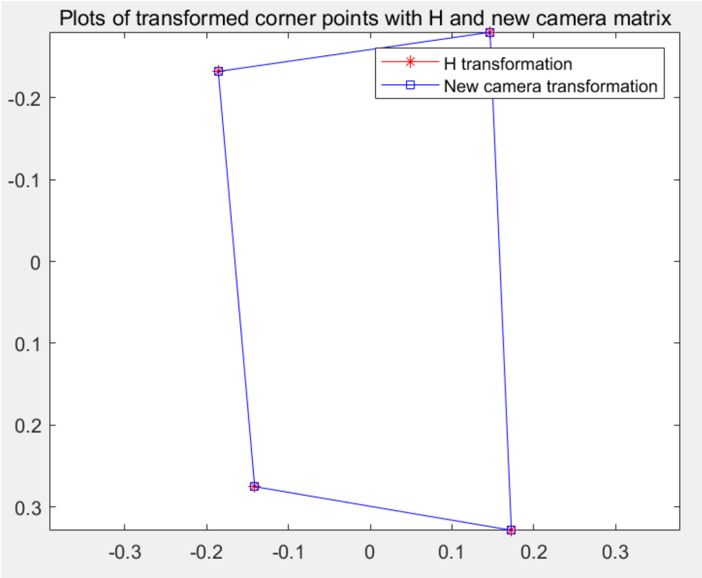

Figure 7: The transformed corner points

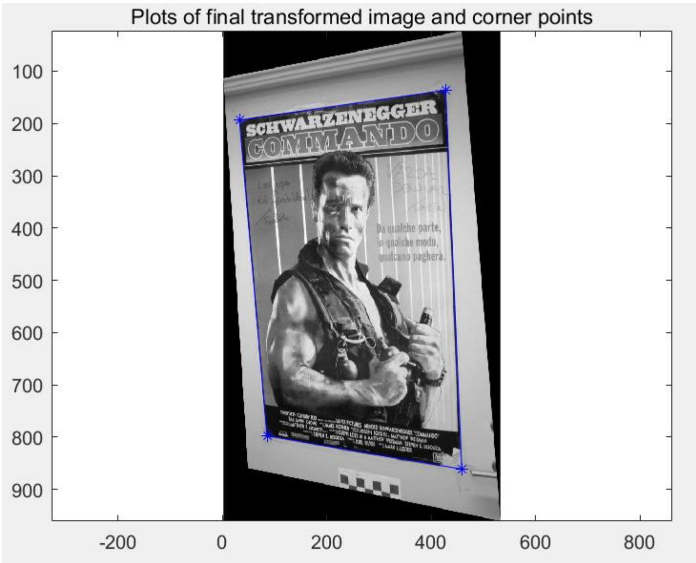

Figure 8: The transformed image