Further Graphics Exercise Set II

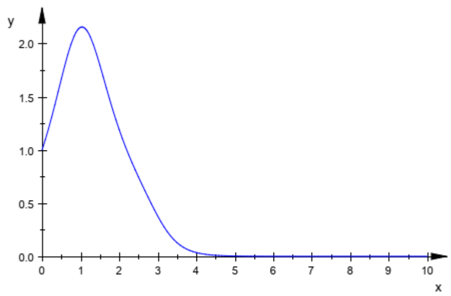
1. Simple 3D engines often approximate lighting as the superposition/combination of diffuse ambient light and direct illumination from point sources. This is a rather crude (yet efficient) approximation to the full rendering equation. What kind of effects are not captured by this simple approximation, i.e. what kind of visual effects cannot be rendered?

2. If there is only diffuse ambient light, we can easily determine the radiosity of a point on the surface through ambient occlusion, i.e. by measuring how much of the 'sky' is visible from that specific point.

   Briefly describe how information about the curvature of a surface can help in this situation.

3. Consider the function $f(x)$ with the graph shown below. In the range 0 to 10, it covers an area of $I = \int_0^{10} f(x)\mathrm{d}x \approx 4.329268516$.

$$f(x) = \frac{40}{(1+e^x)\cdot\big((x-2)^4+4\big)}$$



   (a) Approximate the integral I through uniform sampling at the five points $x_k = 1, 3, 5, 7, 9$, i.e.:

$$\langle I \rangle = \sum_k \frac{f(x_k)}{p(x_k)}$$

   Since there are five uniformly distributed points, the density for all of them is $p = \frac{5}{10} = \frac{1}{2}$. What is the relative error of the number you get?

   (b) According to the idea of importance sampling, you concentrate the evaluation of function values on the region where the values are largest, i.e. between 0 and 4 in our case.

   Sample the function at the points $x = 0.5, 1.5, 2.5, 3.5$ as well as $x = 7$ and compute again the estimator $\langle I \rangle$ for the integral I. The density for the first four points (each covering an interval of length 1) is $p = \frac{1}{1} = 1$ whereas the density for $x = 7$ (which covers an interval of length 6) is $p = \frac{1}{6}$.

4. You can calculate the surface integral over a hemisphere $\mathcal{H}^2$ with the following formula:

$$\int_{\mathcal{H}^2} f(\vec{\omega})\mathrm{d}\vec{\omega} = \int_0^{\pi/2} \int_0^{2\pi} f(\varphi, \theta) \sin(\varphi)\mathrm{d}\theta\mathrm{d}\varphi$$

   Now consider the 'top' part of the hemisphere ranging from $\varphi = 0$ to some arbitrary $\varphi = \Phi$, which we will denote as $\mathbf{H}_\Phi f$:

$$\mathbf{H}_\Phi f = \int_0^{\Phi} \int_0^{2\pi} f(\varphi, \theta) \sin(\varphi)\mathrm{d}\theta\mathrm{d}\varphi$$

   (a) Show that $\mathbf{H}_{\pi/3}$ covers exactly half of the entire surface area $\mathbf{H}_{\pi/2}$, i.e. if you have φ range from 0 to $\frac{\pi}{3}$ (instead of 0 to $\frac{\pi}{2}$), you cover $50\,\%$.

   (b) Let $f(\varphi, \theta) = \cos(\varphi)$ and compute $I = \mathbf{H}_{\pi/2} f$ as well as $I' = \mathbf{H}_{\pi/3} f$. What fraction of I is 'covered' by I'? That is, if we were to approximate I by I', how good would that approximation (relatively) be?

   (c) Briefly explain the significance of (b) with respect to incident light at a point on a surface. In particular, why is *importance sampling Monte Carlo integration* in the context of the rendering equation a good idea?

   (d) Using the cosine as weight for importance sampling MC integration of the rendering equation is based on some basic assumptions. What are these assumptions and under what circumstances might they fail? That is, under what circumstances would cosine-weighted importance sampling *not* deliver better results?

5. *(Past exam question)* Write down the directional form of the rendering equation. Briefly explain each of the terms and the integration domain.

$$L_0(\mathbf{x}, \vec{\omega}) = \ldots$$

6. *(Past exam question)* Assume a scene containing only diffuse surfaces with diffuse reflectance (i.e. albedo) $\rho(\mathbf{x})$. The scene is surrounded by a single *distant* light source with constant (i.e. directionally-invariant) emission $\bar{L}$. Let $V(\mathbf{x}, \vec{\omega})$ be a function returning the visibility of the light source from point $\mathbf{x}$ along direction $\vec{\omega}$.

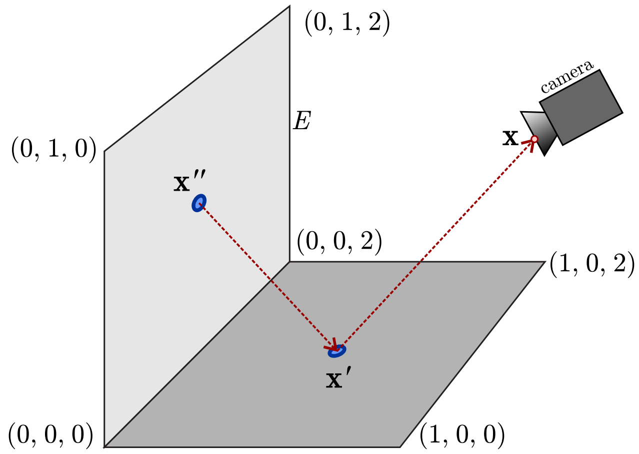
   Change/simplify the rendering equation you provided in (a) as much as possible to estimate *only direct illumination* due to the light source.

7. *(Past exam question)* Recall the surface area form of the rendering equation:

$$L(\mathbf{x},\mathbf{z}) = L_e(\mathbf{x},\mathbf{z}) + \int_A f_r(\mathbf{x},\mathbf{y},\mathbf{z}) \cdot L(\mathbf{x},\mathbf{y}) \cdot G(\mathbf{x},\mathbf{y})\, \mathrm{d}A(\mathbf{y})$$

   (a) Provide the mathematical formulation of $G(\mathbf{x},\mathbf{y})$ and explain its terms.

   (b) Consider the following scene configuration. The rectangle E on the left is an area emitter with $L_e = 1$ and a black BSDF, i.e. $f_r = 0$. The coordinates of its corners are labeled. The ground plane is a non-emissive surface with $f_r = \frac{1}{\pi}$. Provide pseudo-code for a Monte-Carlo estimator of L, given as input a point on the ground plane $\mathbf{x}'$, a point on the camera $\mathbf{x}$, and the desired amount of samples N. You can obtain random numbers uniformly in $[0, 1)$ by calling `RAND()`.



8. *(Past exam question)* Consider the following *single-sample* Monte Carlo (MC) estimator $\langle F \rangle$ of an integral F of an arbitrary non-negative integrand $f(x)$ over an arbitrary domain D with volume 1, driven by a random variable $X \sim p(x)$.

$$F = \int_D f(x)\mathrm{d}x \approx \frac{f(X)}{p(X)} = \langle F \rangle, \qquad p(x) = \frac{1}{2}\big(p_{\text{good}}(x) + p_{\text{bad}}(x)\big)$$

   The "good" probability density function (PDF) $p_{\text{good}}(x)$ is proportional to $f(x)$ (i.e. $p_{\text{good}}(x) = cf(x)$ for some constant c) and the "bad" PDF $p_{\text{bad}}(x)$ is zero wherever $f(x)$ is *non*-zero (i.e. $\forall x. f(x) \neq 0 \Rightarrow p_{\text{bad}}(x) = 0$).

   Derive *step-by-step* the variance of the estimator $\langle F \rangle$ as a function of F.

   Hint: Recall that the variance is defined as:

$$\mathrm{Var}[Y] = E\left[(Y - E[Y])^2\right] = E\left[Y^2\right] - E[Y]^2$$

9. *(Past exam question)* Consider a spotlight emitting radiant intensity $I = 20\,\frac{\mathrm{W}}{\mathrm{sr}}$ confined in a cone of directions with solid angle $\omega = 3\,\mathrm{sr}$ (recall that sr stands for steradian). Compute the total emitted radiant flux from this light.