1. $m(a+bx) = a + b \cdot m(X)$

$$\frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$$

$$\frac{1}{N}\sum_{i=1}^{N} a + \frac{1}{N}\sum_{i=1}^{N} bx_i$$

$$a + b\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$$

summation of any constant is constant

$\leftrightarrow m(X) = \frac{1}{N}\sum_{i=1}^{N} x_i$

$a + bm(X)$ ✓

2. $cov(X, a+bY) = b\,cov(X,Y)$

$$\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - m(a+bY))$$

$$\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - (a+bm(Y)))$$

$$\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))\,b(y_i - m(Y))$$

$$b\left(\frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))\right)$$

$\leftrightarrow cov(X,Y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y))$

$b\,cov(X,Y)$ ✓

3. $\text{cov}(a+bX, a+bX) = b^2\,\text{cov}(X,X) = \sigma^2$

$$\frac{1}{N}\sum_{i=1}^{N}((a+bx_i)-m(a+bx_i))((a+bx_i)-m(a+bx_i))$$

$$\frac{1}{N}\sum_{i=1}^{N}((a+bx_i)-m(a+bx_i))^2$$

$$\frac{1}{N}\sum_{i=1}^{N}((a+bx_i)-(a+bm(x)))^2$$

$$\frac{1}{N}\sum_{i=1}^{N}(bx_i - bm(x))^2$$

$$\frac{1}{N}\sum_{i=1}^{N}(b(x_i - m(x)))^2$$

$$b^2\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2$$

↑ $\text{cov}(X,X) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(x_i - m(x)) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2 = \sigma^2$

$b^2\,\text{cov}(X,X)$ ✓