$$\hat{y}_i = b_0 + b_1 z_{i1} + b_2 z_{i2} \qquad z_{ij} = x_{ij} - m_j \qquad \frac{1}{N}\sum_{i=1}^{N} z_{ij} = 0$$

1) SSE model

$$e_i^2 = (y_i - \hat{y}_i)^2 \qquad SSE = \sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

$$SSE(b_0, b_1, b_2) = \sum_{i=1}^{N} (y_i - (b_0 + b_1 z_{i1} + b_2 z_2))^2$$

2) $$\frac{dSSE}{db_k} = \sum_{i=1}^{N} 2e_i \frac{de_i}{db_k}$$ ← chainrule

$$e_i = y_i - b_0 - b_1 z_{i1} - b_2 z_{i2}$$

$$\frac{de_i}{db_0} = -1 \qquad \frac{de_i}{db_1} = -z_{i1} \qquad \frac{de_i}{db_2} = -z_{i2}$$

$$\frac{dSSE}{db_0} = \sum_{i=1}^{N} 2e_i(-1) \qquad \frac{dSSE}{db_1} = \sum_{i=1}^{N} 2e_i(-z_{i1}) \qquad \frac{dSSE}{db_2} = \sum_{i=1}^{N} 2e_i(-z_{i2})$$

$$\frac{dSSE}{db_0} = \sum_{i=1}^{N} -2e_i = 0 \qquad \frac{dSSE}{db_1} = \sum_{i=1}^{N} -2z_{i1}e_i = 0 \qquad \frac{dSSE}{db_2} = \sum_{i=1}^{N} -2z_{i2}e_i = 0$$

3) $$\frac{dSSE}{db_0} = \sum_{i=1}^{N} -2e_i = 0$$ ← first order conditions

$\sum_{i=1}^{N} e_i = 0 \rightarrow \bar{e} = 0$ average error is zero

$$\frac{dSSE}{db_1} = \sum_{i=1}^{N} -2z_{i1}e_i = 0 \qquad \frac{dSSE}{db_2} = \sum_{i=1}^{N} -2z_{i2}e_i = 0$$

$\sum_{i=1}^{N} z_{i1}e_i = 0$ AND $\sum_{i=1}^{N} z_{i2}e_i = 0 \rightarrow$ e is orthogonal to each regressor column z_j

4) $$\sum_{i=1}^{N} e_i = 0$$

$$\sum_{i=1}^{N} y_i - b_0 - b_1 z_{i1} - b_2 z_{i2} = 0$$

$$\frac{1}{N}\sum_{i=1}^{N} z_{ij} = 0$$

$$\sum_{i=1}^{N} y_i - \sum_{i=1}^{N} b_0 - \underbrace{\cancel{\sum_{i=1}^{N} b_1 z_{i1}}}_{0} - \underbrace{\cancel{\sum_{i=1}^{N} b_2 z_{i2}}}_{0} = 0$$

$$\sum_{i=1}^{N} y_i - Nb_0 = 0$$

$$\sum_{i=1}^{N} y_i = Nb_0 \rightarrow b_0^* = \bar{y}$$

$$\hat{y}_i = b_0 + b_1 z_{i1} + b_2 z_{i2}$$
$$b_0^* = \bar{y}$$
$$e_i = y_i - \bar{y} - b_1 z_{i1} - b_2 z_{i2}$$
$$y_i^c = y_i - \bar{y}$$

$$\sum_{i=1}^{N} z_1 (y_i^c - b_1 z_{i1} - b_2 z_{i2}) = 0$$

$$\sum_{i=1}^{N} z_{i1} y_i^c - b_1 z_{i1}^2 - b_2 z_{i1} z_{i2} = 0$$

$$\sum_{i=1}^{N} z_2 (y_i^c - b_1 z_{i1} - b_2 z_{i2}) = 0$$

$$\sum_{i=1}^{N} z_{i2} y_i^c - b_1 z_{i1} z_{i2} - b_2 z_{i2}^2 = 0$$

5)
$$\sum_{i=1}^{N} z_{i1} y_i^c - b_1 z_{i1}^2 - b_2 z_{i1} z_{i2} = 0 \qquad \sum_{i=1}^{N} z_{i2} y_i^c - b_1 z_{i1} z_{i2} - b_2 z_{i2}^2 = 0$$

$$\sum_{i=1}^{N} b_1 z_{i1}^2 + \sum_{i=1}^{N} b_2 z_{i1} z_{i2} = \sum_{i=1}^{N} z_{i1} y_i^c \qquad \sum_{i=1}^{N} b_1 z_{i1} z_{i2} + \sum_{i=1}^{N} b_2 z_{i2}^2 = \sum_{i=1}^{N} z_{i2} y_i^c$$

$$\begin{bmatrix} \sum_{i=1}^{N} z_{i1}^2 & \sum_{i=1}^{N} z_{i1} z_{i2} \\ \sum_{i=1}^{N} z_{i1} z_{i2} & \sum_{i=1}^{N} z_{i2}^2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} z_{i1} y_i^c \\ \sum_{i=1}^{N} z_{i2} y_i^c \end{bmatrix}$$

6)
$$\begin{bmatrix} \overset{var(x_1)}{\frac{1}{N}\sum_{i=1}^{N} z_{i1}^2} & \overset{cov(x_1,x_2)}{\frac{1}{N}\sum_{i=1}^{N} z_{i1} z_{i2}} \\ \underset{cov(x_1,x_2)}{\frac{1}{N}\sum_{i=1}^{N} z_{i1} z_{i2}} & \underset{var(x_2)}{\frac{1}{N}\sum_{i=1}^{N} z_{i2}^2} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{N}\sum_{i=1}^{N} z_{i1} y_i^c & cov(x_1, y) \\ \frac{1}{N}\sum_{i=1}^{N} z_{i2} y_i^c & cov(x_2, y) \end{bmatrix}$$

$$z_{ij} = x_{ij} - m_{ij} \qquad y_i^c = y_i - \bar{y}$$

$$\begin{bmatrix} var(x_1) & cov(x_1, x_2) \\ cov(x_1, x_2) & var(x_2) \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} cov(x_1, y) \\ cov(x_2, y) \end{bmatrix}$$

A: covariance matrix – how the predictors vary / how redundant/collinear

C: covariance w/ respect to y – how predictors move w response

system apportions shared covariance w/ y across predictors, adjusting for their inter-correlation

→ x_1 and x_2 uncorrelated: $b_1 = \frac{cov(x_1, y)}{var(x_1)}$ and $b_2 = \frac{cov(x_2, y)}{var(x_2)}$

→ correlated: avoids double counting overlaps