CS1555 Recitation 4 - Solution

Objective: to practice normalization, canonical forms, decomposing relations into BCNF and checking for lossless decompositions.

Part 1: For each of the following relations R and sets of functional dependencies F, do the following:

1) Find the canonical cover (minimal cover) of F.
2) Using the canonical cover, find the keys of the R.

1. Consider the following set of functional dependencies F on a relation R (A, B, C, D, E):

$A \rightarrow BC$
$A \rightarrow D$
$B \rightarrow C$
$C \rightarrow D$
$DE \rightarrow C$
$BC \rightarrow D$

Finding the canonical form:

- Transform all FDs to canonical form (i.e., one attributes on the left):
  $A \rightarrow B$
  $A \rightarrow C$
  $A \rightarrow D$
  $B \rightarrow C$
  $C \rightarrow D$
  $DE \rightarrow C$
  $BC \rightarrow D$
- Drop extraneous attributes:
  B in $BC \rightarrow D$ is extraneous, since we already have $C \rightarrow D$. The set of FDs becomes:
  $A \rightarrow B$
  $A \rightarrow C$
  $A \rightarrow D$
  $B \rightarrow C$
  $C \rightarrow D$
  $DE \rightarrow C$
- Drop redundant FDs:
  $A \rightarrow B$ and $B \rightarrow C$ implies $A \rightarrow C$, so we drop $A \rightarrow C$.
  $A \rightarrow B$, $B \rightarrow C$ and $C \rightarrow D$ implies $A \rightarrow D$, so we drop $A \rightarrow D$.
  The set of FDs becomes:

$A \to B$
$B \to C$
$C \to D$
$DE \to C$

which is the canonical cover of F.

Finding the keys of R:

Observations:

- A and E do not appear in the right hand side of any FDs, so they have to appear in all keys of R.
- AE+ : $AE \to AEB$ (since $A \to B$) $\to$ AEBC (since $B \to C$) $\to$ AEBCD (since $C \to D$). So AE is a key of R.
  In this case, we do not need to consider any other combination, because any other combination containing AE (e.g., AEB) is a super key and not minimal.

2. Consider the following set of functional dependencies F on relation R (A, B, C, D, E, H):

$A \to C$
$AC \to D$
$E \to AD$
$E \to H$
$A \to CD$
$E \to AH$

Finding the canonical form:

- Transform all FDs to canonical form (i.e., one attribute on the right):
  $A \to C$
  $AC \to D$
  $E \to AD$ becomes $E \to A$ and $E \to D$
  $E \to H$
  $A \to CD$ becomes $A \to C$ and $A \to D$
  $E \to AH$ becomes $E \to A$ and $E \to H$
- Remove redundant dependencies:
  $A \to C$
  $AC \to D$
  $E \to A$
  $E \to D$
  $E \to H$
  $A \to D$
- Drop extraneous attributes:

AC→D can be removed because we have A→D so C is redundant:

$A \rightarrow C$
$E \rightarrow A$
$E \rightarrow D$
$E \rightarrow H$
$A \rightarrow D$

- Drop redundant FDs: Try removing some dependencies in F and still have a set of dependencies equivalent to F.
  E→D can be deduced from E→A and A→D so we can remove E→D.
  The set of FDs becomes:

  $A \rightarrow C$
  $E \rightarrow A$
  $E \rightarrow H$
  $A \rightarrow D$

  which is the canonical cover of F.

<u>Finding the keys of R:</u>

Observations:

- B and E do not appear in the right hand side of any FDs, so they have to appear in all keys of R.
- BE+: BE → AEB (because E → A) → AEBC (because A → C) → AEBCD (because A → D) → AEBCDH (because E → H). So BE is a key of R.
  In this case, we do not need to consider any other combination, because any other combination containing BE (e.g., AEB) is a super key and not minimal.

<u>Part 2:</u> Consider the following set of functional dependencies F on relation R (A, B, C, D, E, H):

$A \rightarrow C$
$AC \rightarrow D$
$E \rightarrow AD$
$E \rightarrow H$
$A \rightarrow CD$
$E \rightarrow AH$

The key for R is *EB* and the following set of functional dependencies constitutes the canonical cover:

$A \rightarrow C,\ E \rightarrow A,\ E \rightarrow H,\ A \rightarrow D$

1) Using Synthesis, construct a set of 3NF relations.

We are starting from a canonical cover. → We can skip steps 1, 2 and 3.
Step 4: Group FDs with same determinant:

$A \rightarrow CD$

$E \rightarrow AH$

Step 5: Construct a relation for each group:

R1($\underline{A}$,C,D)

R2($\underline{E}$,A,H)

Step 6: If none of the relations contain the key for the original relation, add a relation with the key.

R3($\underline{E,B}$)

R1, R2 and R3 are in 3NF and in BCNF.

2) Apply the decomposition method on R to end up with BCNF relations.

Using $A \rightarrow C$ to decompose R, we get:

R1(A,$\underline{B}$,D,$\underline{E}$,H) in 1NF

R2($\underline{A}$,C) is already in BCNF form

Using $A \rightarrow D$ to decompose R1, we get:

R11(A,$\underline{B}$,$\underline{E}$,H) in 1NF

R12($\underline{A}$,D) is already in BCNF form

Using $E \rightarrow A$ to decompose R11, we get:

R111($\underline{B,E}$,H) in 1NF

R112($\underline{E}$,A) is already in BCNF form

Using $E \rightarrow H$ to decompose R111, we get:

R1111($\underline{B,E}$) in BCNF form

R1112($\underline{E}$,H) is already in BCNF form

Group the relations with the same key:

R1($\underline{A}$,C,D)

R2($\underline{E}$,A,H)

R3($\underline{E,B}$)

R1, R2 and R3 are in BCNF form.

Part 3: Assume that R is decomposed into:

R1 (A, B), F1 = {A → B}, key (A)
R2 (B, C), F2 = {B → C}, key (B)
R3 (C, D, E), F3 = {C → D, DE → C), key (DE), (CE)
Is this decomposition a lossless-join decomposition? Use the table method.

Checking for lossless-join:

Initially the Table looks like this:

	A	B	C	D	E
R1(A,B)	a1	a2	U13	U14	U15
R2(B,C)	U21	a2	a3	U24	U25
R2(C,D,E)	U31	U32	a3	a4	a5

Using B → C: we can replace U13 by a3

	A	B	C	D	E
R1(A,B)	a1	a2	**a3**	U14	U15
R2(B,C)	U21	a2	a3	U24	U25
R2(C,D,E)	U31	U32	a3	a4	a5

Using C → D: we can replace U14 and U24 by a4

	A	B	C	D	E
R1(A,B)	a1	a2	**a3**	**a4**	U15
R2(B,C)	U21	a2	a3	**a4**	U25
R2(C,D,E)	U31	U32	a3	a4	a5

We cannot proceed and there is no row of all known values → the decomposition is lossy.