CS1555 Recitation 10 - Solution

Objective: to practice normalization, canonical forms, decomposing relations into BCNF and checking for lossless decompositions.

Part 1: For each of the following relations R and sets of functional dependencies F, do the following:

1) Find the canonical cover (minimal cover) of F.
2) Using the canonical cover, find the keys of the R.

1. Consider the following set of functional dependencies F on a relation R (A, B, C, D, E):

A → BC
A → D
B → C
C → D
DE → C
BC → D

Finding the canonical form:

- Transform all FDs to canonical form (i.e., one attributes on the right):
  A → B
  A → C
  A → D
  B → C
  C → D
  DE → C
  BC → D
- Drop extraneous attributes:
  B in BC → D is extraneous, since we already have C → D. The set of FDs becomes:
  A → B
  A → C
  A → D
  B → C
  C → D
  DE → C
- Drop redundant FDs:
  A → B and B → C implies A → C, so we drop A → C.
  A → B, B → C and C → D implies A → D, so we drop A → D.
  The set of FDs becomes:

$A \rightarrow B$
$B \rightarrow C$
$C \rightarrow D$
$DE \rightarrow C$

which is the canonical cover of F.

<u>Finding the keys of R:</u>

Observations:

- A and E do not appear in the right hand side of any FDs, so they have to appear in all keys of R.
- AE+ : AE → AEB (since A → B) → AEBC (since B → C) → AEBCD (since C → D). So AE is a key of R.
  In this case, we do not need to consider any other combination, because any other combination containing AE (e.g., AEB) is a super key and not minimal.

2. Consider the following set of functional dependencies F on relation R (A, B, C, D, E, H):

$A \rightarrow C$
$AC \rightarrow D$
$E \rightarrow AD$
$E \rightarrow H$
$A \rightarrow CD$
$E \rightarrow AH$

<u>Finding the canonical form:</u>

- Transform all FDs to canonical form (i.e., one attribute on the right):
  - $A \rightarrow C$
  - $AC \rightarrow D$
  - $E \rightarrow AD$ becomes $E \rightarrow A$ and $E \rightarrow D$
  - $E \rightarrow H$
  - $A \rightarrow CD$ becomes $A \rightarrow C$ and $A \rightarrow D$
  - $E \rightarrow AH$ becomes $E \rightarrow A$ and $E \rightarrow H$
- Remove redundant dependencies:
  - $A \rightarrow C$
  - $AC \rightarrow D$
  - $E \rightarrow A$
  - $E \rightarrow D$
  - $E \rightarrow H$
  - $A \rightarrow D$
- Drop extraneous attributes:

$AC \rightarrow D$ can be removed because we have $A \rightarrow D$ so C is redundant:

$A \rightarrow C$
$E \rightarrow A$
$E \rightarrow D$
$E \rightarrow H$
$A \rightarrow D$

- Drop redundant FDs: Try removing some dependencies in F and still have a set of dependencies equivalent to F.
  $E \rightarrow D$ can be deduced from $E \rightarrow A$ and $A \rightarrow D$ so we can remove $E \rightarrow D$.
  The set of FDs becomes:

  $A \rightarrow C$
  $E \rightarrow A$
  $E \rightarrow H$
  $A \rightarrow D$

  which is the canonical cover of F.

Finding the keys of R:

Observations:

- B and E do not appear in the right hand side of any FDs, so they have to appear in all keys of R.
- BE+: BE $\rightarrow$ AEB (because $E \rightarrow A$) $\rightarrow$ AEBC (because $A \rightarrow C$) $\rightarrow$ AEBCD (because $A \rightarrow D$) $\rightarrow$ AEBCDH (because $E \rightarrow H$). So BE is a key of R.
  In this case, we do not need to consider any other combination, because any other combination containing BE (e.g., AEB) is a super key and not minimal.

Part 2: Consider the following set of functional dependencies F on relation R (A, B, C, D, E, H):

$A \rightarrow C$
$AC \rightarrow D$
$E \rightarrow AD$
$E \rightarrow H$
$A \rightarrow CD$
$E \rightarrow AH$

The key for R is *EB* and the following set of functional dependencies constitutes the canonical cover:

$A \rightarrow C$, $E \rightarrow A$, $E \rightarrow H$, $A \rightarrow D$

1) Using Synthesis Method, construct a set of 3NF relations.

We are starting from a canonical cover. → We can skip steps 1, 2 and 3.

Step 4: Group FDs with same determinant:

$A \rightarrow CD$

E → AH

Step 5: Construct a relation for each group:

R1(A,C,D)

R2(E,A,H)

Step 6: If none of the relations contain the key for the original relation, add a relation with the key.

R3(E,B)

R1, R2 and R3 are in 3NF and in BCNF.

2) Using Universal Method, decompose R into a set of BCNF relations.

Using A → C to decompose R, we get:

R1(A,B,D,E,H) in 1NF

R2(A,C) is already in BCNF form

Using A → D to decompose R1, we get:

R11(A,B,E,H) in 1NF

R12(A,D) is already in BCNF form

Using E → A to decompose R11, we get:

R111(B,E,H) in 1NF

R112(E,A) is already in BCNF form

Using E → H to decompose R111, we get:

R1111(B,E) in BCNF form

R1112(E,H) is already in BCNF form

Group the relations with the same key:

R1(A,C,D)

R2(E,A,H)

R3(E,B)

R1, R2 and R3 are in BCNF form.

Part 3: Assume that R is decomposed into:

R1 (A, B), F1 = {A → B}, key (A)
R2 (B, C), F2 = {B → C}, key (B)
R3 (C, D, E), F3 = {C → D, DE → C), key (DE), (CE)
Is this decomposition a lossless-join decomposition? Use the table method.

Checking for lossless-join:

Initially the Table looks like this:

	A	B	C	D	E
R1(A,B)	a1	a2	U13	U14	U15
R2(B,C)	U21	a2	a3	U24	U25
R2(C,D,E)	U31	U32	a3	a4	a5

Using B → C: we can replace U13 by a3

	A	B	C	D	E
R1(A,B)	a1	a2	**a3**	U14	U15
R2(B,C)	U21	a2	a3	U24	U25
R2(C,D,E)	U31	U32	a3	a4	a5

Using C → D: we can replace U14 and U24 by a4

	A	B	C	D	E
R1(A,B)	a1	a2	**a3**	**a4**	U15
R2(B,C)	U21	a2	a3	**a4**	U25
R2(C,D,E)	U31	U32	a3	a4	a5

We cannot proceed and there is no row of all known values → the decomposition is lossy.