42-302/42-782 A: Biomedical Engineering Systems Modeling and Analysis (Fall 2021)

Quiz 2: Thu 28 Oct 2021

Instructions

1. This test is open book, open notes.
2. You may not receive assistance from others in completing this Quiz.
3. You may not submit other's work as your own.
4. You may clarify the Quiz questions with the instructor and TAs via Zoom during normal lecture time or email. **Do NOT use Piazza.**
5. Please start each problem on a new page.
6. You have **80 minutes to complete the Quiz + 20-minute grace period** to download the problem set and to upload the answers (i.e. total of 100 minutes).
7. The Quiz is set to be **available from 8 am to 6 pm EST**. You may start the Quiz at any time between the stated time period, but you must submit your answers within the given time limit of 100 minutes (counting from the start time), and before 6 pm EST.
8. Both typed up answers and scanned handwritten documents are acceptable. Please make sure that your scans have high enough resolution and your handwriting is legible.
9. For any problem that requires sketching, you may sketch directly onto the provided figure. If you cannot sketch directly on the PDF file, you may replicate the figure by hand and sketch your answer on top of it. Make sure that you label the sketch clearly.
10. Report your answer with the appropriate unit (when applicable).
11. Submit your answers as a **single PDF file on Gradescope**. **Late submission will be penalized** (see late submission policy on the syllabus).

There are 5 questions.

42-302/42-782 Quiz 2

1. **[Required, 2 pts]** Sign or type your full name below to acknowledge the following statement.

   Note: *If you do not have access to a printer, scanner or cannot sign/type directly on a PDF file, you may write your full name, Andrew ID and sign your signature on a piece of paper, AND indicate that they are for Q.1 in Quiz 2.*

   I pledge to uphold the highest academic standards and integrity. I affirm that I have not used any unauthorized materials in completing this exam, and have neither given assistance to others nor received assistance from others. Further, I affirm that 1) I have not observed any other students in this class acting to gain and unfair advantage, 2) if so, I have reported to my instructor any activity I have observed that is not in accordance with CMU academic integrity. I do so to sustain CMU culture of integrity, responsibility and community. I understand that there are significant consequences for violating academic integrity and that suspected violations will be reported to the School and the University. (https://www.cmu.edu/policies/student-and-student-life/academic-integrity.html).

   Full name (print): Nakya Ni

   Andrew ID: qinyu

   Electronic signature: Nakya Ni

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2. **[Total: 15 pts (5 pts each)]** A manufacturer runs a quality control of ventilator to ensure the equipment safety. The plots below show the measured airflow, $Flow(t)$, from the ventilator.

- Match the airflow plot (i, ii, and iii) to each of the given scenarios (a, b and c).
  a) The ventilator produces a constant airflow.

  Airflow plot: ___iii___

  b) The ventilator produces airflow that oscillates at two frequencies.

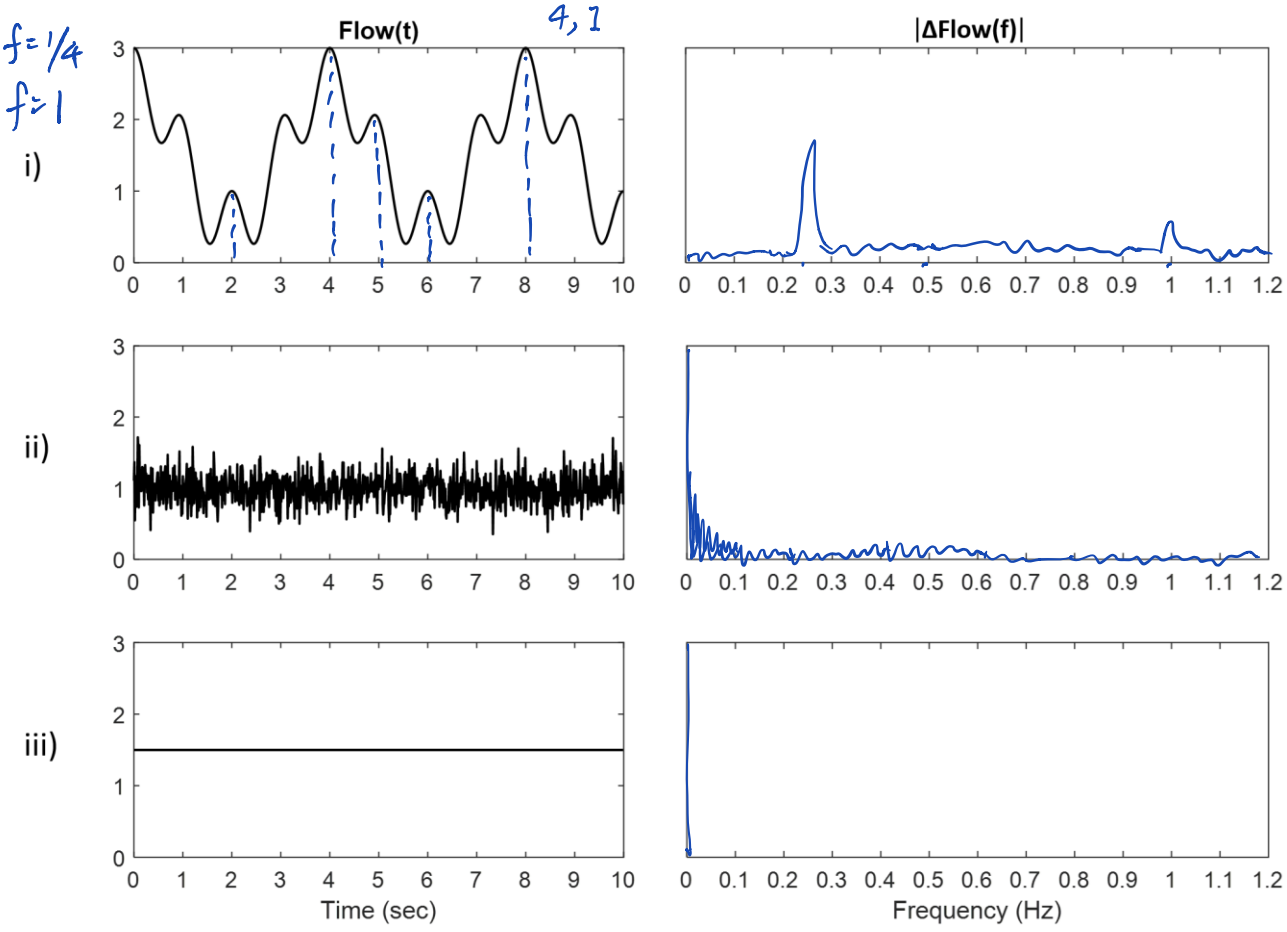
  Airflow plot: ___i___

  c) An intern accidentally detached the ventilator from the scope (the equipment used to display the measured airflow). So now the scope only shows random noise. This random noise contains a broad range of frequencies of roughly equal contribution from each frequency.

  Airflow plot: ___ii___

- Sketch the magnitude of the frequency spectrum of **the fluctuations of airflow around its mean value**, $|\Delta Flow(f)|$. Your sketch should reflect the frequency and magnitude of each frequency content in the airflow. You do *not* need to determine the exact height of the spectral peak, but the peak height should reflect the relative contribution of that frequency in airflow.
- Briefly explain your answer – why did you choose such plot and why should the magnitude spectrum look like what you sketched. (Space provided on the next page.)

T = 4 f = 1/4
T = 1 f = 1



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Explanation:

a) The ventilator produces a constant airflow.

Because it produces a constant flow, it has all kinds of periodicities, the period or frequency can bye any number, and it also be highest at zero.

b) The ventilator produces airflow that oscillates at two frequencies.

Because this flow has two frequencies, it has two peak magnitude

c) An intern accidentally detached the ventilator from the scope (the equipment you use to display the measured airflow). So now the scope only shows random noise. This random noise contains a broad range of frequencies of roughly equal contribution from each frequency.

Because this flow is random noise, it has all kinds of frequencies, and has highest frequency at zero.

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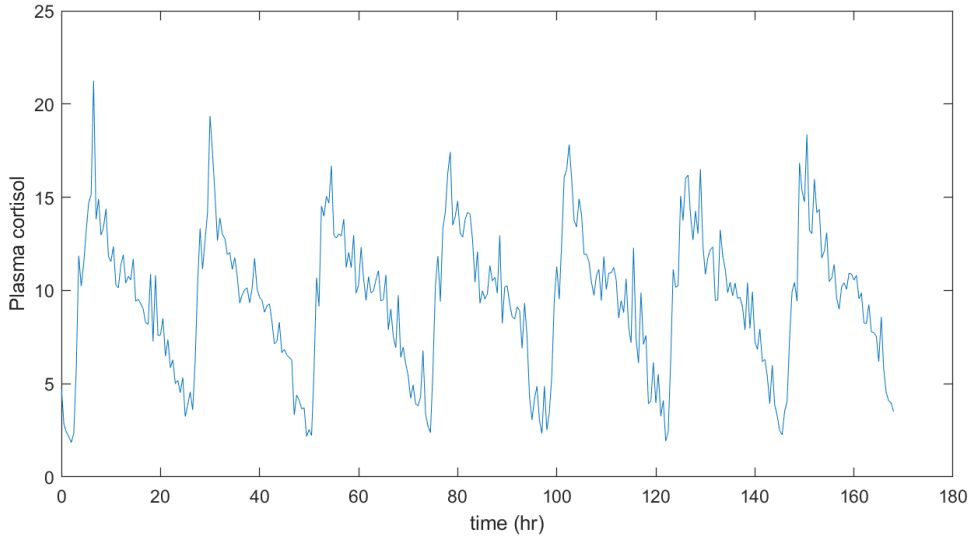
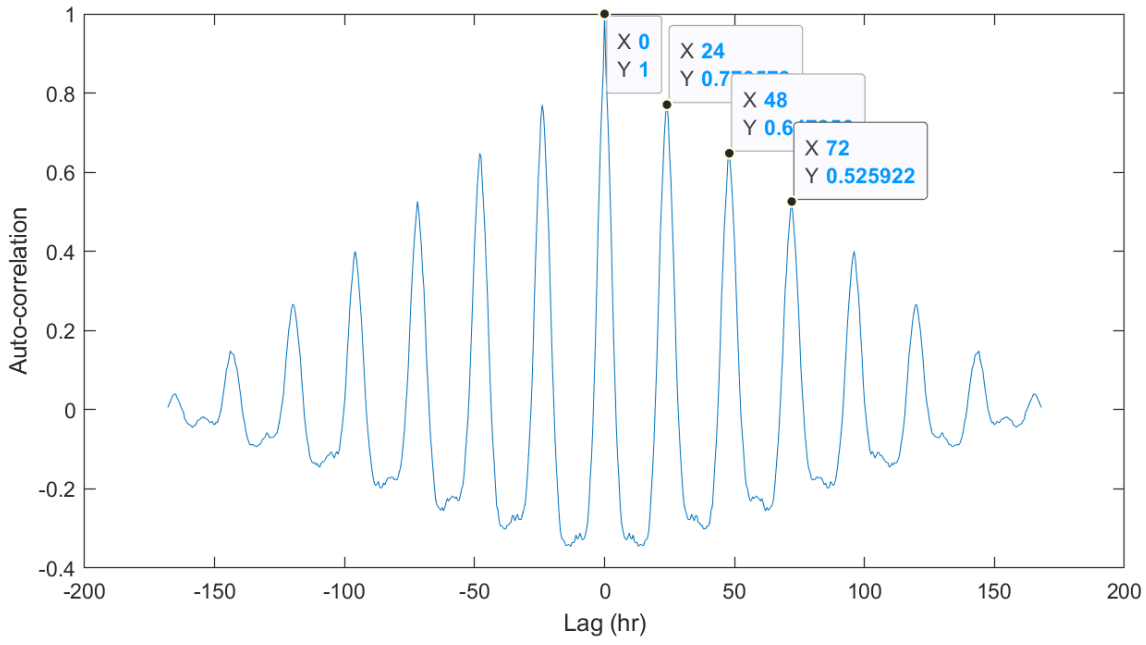
3. **[Total: 20 pts]** Cortisol, the primary stress hormone, plays an important role in controlling blood sugar levels, regulate metabolism, assist with memory formulation, and other essential functions in the body. The MATLAB file, cortisol.mat, contains plasma cortisol (µg/dL) data **measured every 0.5 hours over seven days**. The data file can be downloaded from:
   - https://drive.google.com/file/d/13-tQeCsY-FuvNHY2QqUbqban55FZKzpo/view?usp=sharing or
   - The Quiz 2 announcement (posted at 6:30 AM today).

   a) **[5 pts]** Use MATLAB to plot the plasma cortisol vs. time **in hours**. Label both axes. Copy and paste the plot into the solution file.



   b) **[5 pts]** Use MATLAB to calculate the **auto-correlation of the fluctuations of the plasma cortisol around its mean value**. Normalize the auto-correlation using the 'coeff' option. Plot the auto-correlation vs. time **lag in hours**. Label both axes. Copy and paste the plot into the solution file.



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c) **[4 pts]** From part b), determine **the most prominent period** at which the cortisol level oscillates over time from the auto-correlation of the cortisol level. Explain how you obtained your answer.

From the graph, the pattern repeats every 24 hours, thus the most prominent period is 24hours.

d) **[3 pts]** If you were to plot the magnitude of the frequency spectrum of **the fluctuations of the plasma cortisol around its mean value**, at which frequency would the spectral peak appear, based on your answer in c)? Please also specify the unit of the reported frequency.
You do NOT need to calculate or plot the spectrum.

The period of 24 hours corresponds to frequency of 1/24 = 0.041666 hr, as the unit of Hz is second, then the frequency should be 1/(24*60*60) = 0.000011574 Hz, thus the highest peak appear in the spectral plot at 0.000012 Hz.

e) **[3 pts]** Copy and paste the MATLAB code you used in this question. You may take a picture of your MATLAB script, but please make sure that the code is clearly readable from the image. **You will receive NO CREDIT if the code is NOT provided, or the code is NOT readable.**

```
% Load data
load('cortisol.mat');

T = 0.5;
fs = 1/T;
% time = linspace(0, length(cortisol)/fs, length(cortisol));
time = (0:length(cortisol)-1)'/fs;
plot(time,cortisol);
xlabel('time (hr)');
ylabel('Plasma cortisol');
```

```
%% Correlation
[c,lag] = xcorr(cortisol - mean(cortisol), 'coeff');
figure; plot(lag/fs, c);
xlabel('Lag (hr)'); ylabel('Auto-correlation');
```

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4. **[Total: 12 pts]** A study investigated the relationship between brain activity and heart rate variability in healthy young men during sleep. The figure below shows sleep stages (hypnogram, *Panel a*), brain activity and cardiac vagal activity of one subject. The brain activity is represented by the power of delta wave in the EEG signal (Delta P, thick line in *Panel b*). The cardiac vagal activity is represented by the normalized high-frequency power of R-R intervals (HFnu, thick line in *Panel c*).

Read the figure caption for further information that may be useful in answering the questions below.

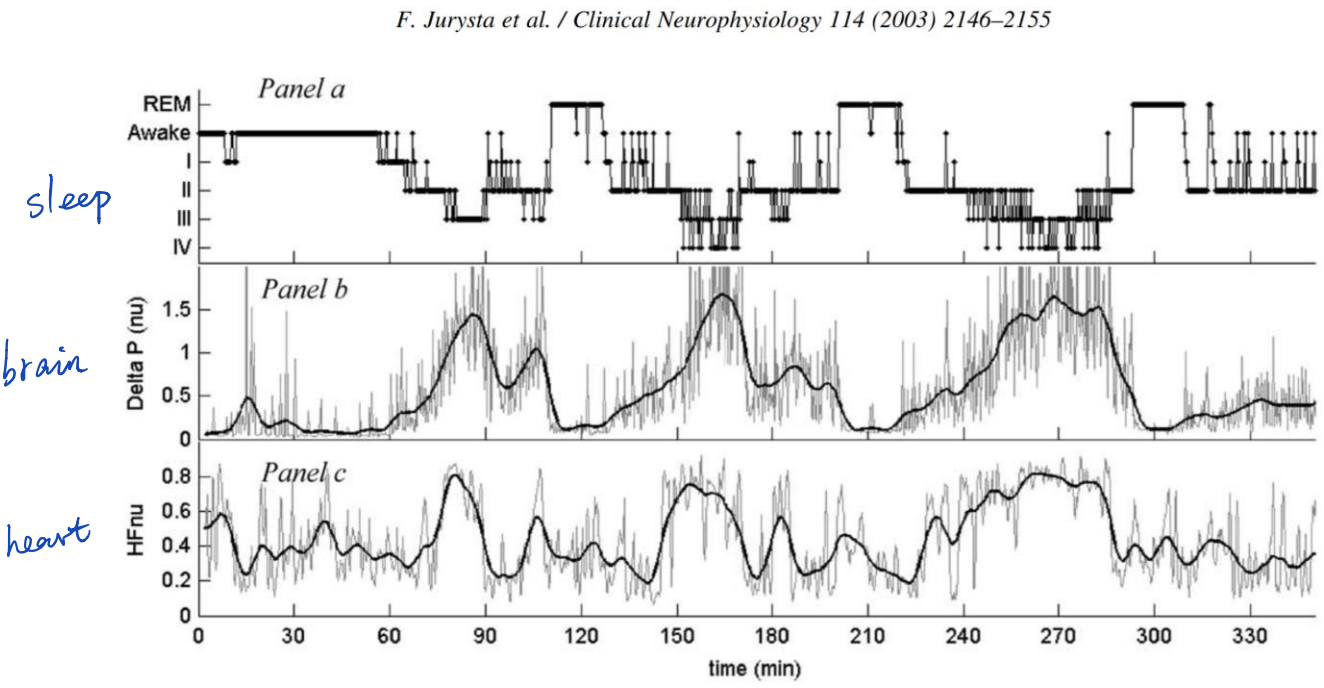

Fig. 5 Delta EEG power (Delta P) in normalized units (nu) (b) and normalized high frequency (HFnu) of the RRI (c) across sleep stages (a). Visual inspection of the readings reveals that HFnu fluctuations precede the delta frequency oscillations. For this subject, the phase advance between cardiac vagal activity and the occurrence of delta waves in the EEG is of 23° corresponding to a lead in shift of 7 min. Sleep stages are visualized on the hypnogram. REM, rapid-eye movement sleep; I, stage I; II, stage II; III, stage III; IV, stage IV of sleep.

For the following questions related to cross-correlation, use this definition of cross-correlation (the same definition we have used in class and MATLAB):

$$\hat{R}_{xy}[m] = \begin{cases} \sum_{n=0}^{N-m-1} x[n+m]y[n], & m \geq 0 \\ \hat{R}_{yx}[-m], & m < 0 \end{cases}$$

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a) **[6 pts]** If you were to compute the cross-correlation between sleep stages (*Panel a*, treat this signal as "x") and Delta P (*Panel b*, treat this signal as "y"),
   i) Would the strongest peak of the cross-correlation function be positive or negative?
   ii) At which time lag (give an estimate) would the peak in i) occur?
   iii) Explain your answers in i) and ii).

The cross-correlation between sleep and Delta P will be negative, as the two signals change in the opposite direction.
The peak will occur around 0 second, because the pattern of two matches but in the opposite direction.

b) **[6 pts]** If you were to compute the cross-correlation between Delta P (*Panel b*, treat this signal as "x") and HFnu (*Panel c*, treat this signal as "y"),
   i) Would the peak of the cross-correlation function be positive or negative?
   ii) At which time lag (give an estimate) would the peak in i) occur?
   iii) Explain your answers in i) and ii).

This peak will be positive because the two signals change in the same direction.
The peak will occur around like +8~+10 seconds, because the pulse in HFnu appears before Delta P, thus HFnu leads the Delta P.

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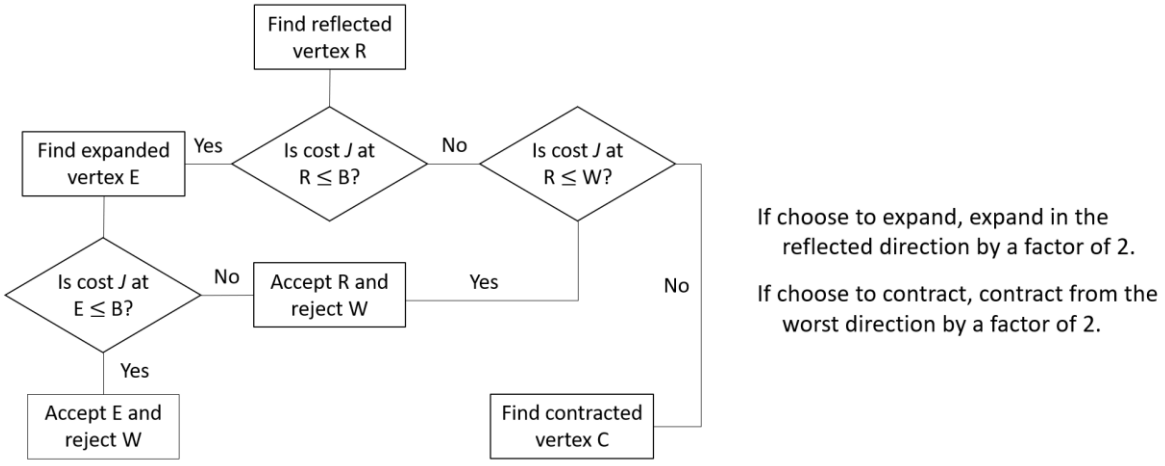
5. **[Total: 22 pts]** A researcher wants to estimate the drug concentration in the plasma $c(t)$ [mg/L] following a bolus injection of the drug amount D [mg]. The drug is eliminated from the plasma of volume V [L] at the rate of k [hr^{-1}]. The drug concentration can be modeled as an exponential decaying function as follows

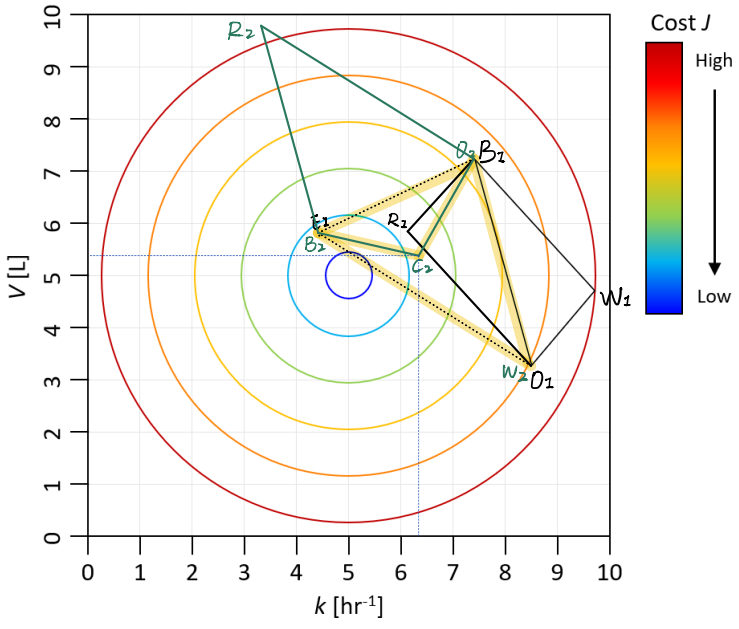
$$\hat{c}(t) = \frac{D}{V} e^{-kt} \quad \text{where } \hat{c}(t) \text{ is the predicted drug concentration.}$$

The drug concentration was measured from the plasma at every hour for 12 hours, and the injected amount of drug is known. The unknown parameters in the above equation are k and V. Use **the simplex algorithm** to estimate these unknown parameters. The optimization cost function J is

$$J(k,V) = \sum_{i=1} \left(prediction\ error(t_i)\right)^2 = \sum_{i=1} \left(c(t_i) - \hat{c}(t_i)\right)^2$$

The flow chart of the simplex algorithm and the contour plot of the cost function J are given below. The **color bar** next to the contour plot indicates **the height of the contour**.





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a) **[10 pts]** Let the initial simplex be the black triangle on the contour plot. Draw as accurately as possible **the next two iterations** of the simplex optimization. **Show all your work** e.g. label each vertex and show how you transform the initial simplex. **Clearly indicate the accepted shape at each iteration. If possible, use different colors for each iteration in your sketch.**

You may insert your sketch here if you did not sketch directly onto the provided figure.

Iteration 1: Find R1, R1 < B1, expend E1, E1 < B1, accept E1, reject W1.

Iteration 2: Find R2, R2 > B2, R2>W2, find contract C.

b) **[4 pts]** If we were to stop the algorithm after two iterations, determine the values of k and V based on your answer in a). Explain how you decided that k and V should be the reported values.

k is around 6.3, and V is around 5.4
I follow the coordinations to decide the values. As to report the optimal value of k and v, it depends when the value and cost function converge, it means the cost function's value will not be lower anymore.

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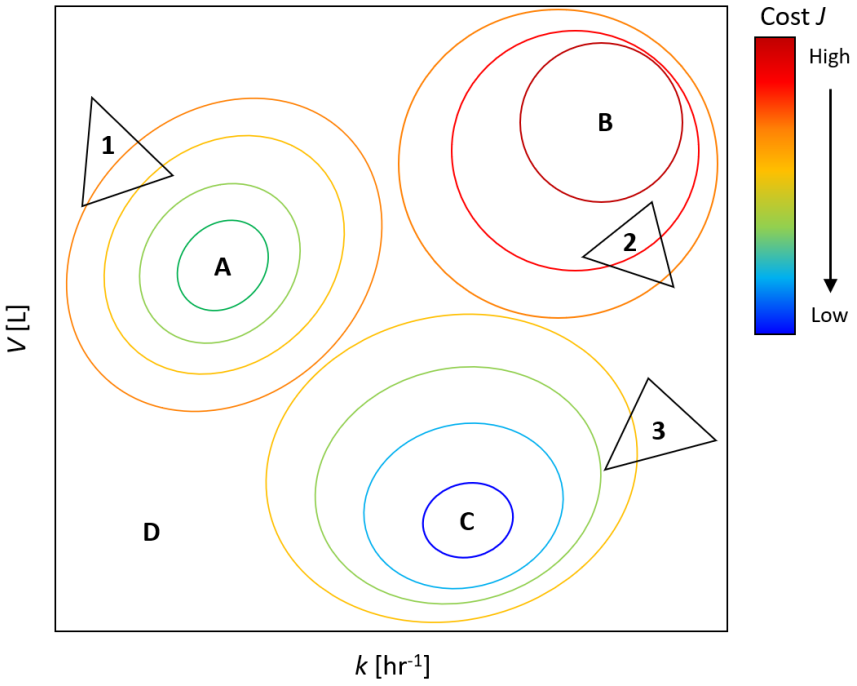
5. (Cont.) Let's say the cost function is redefined and its contour plot is shown below in colors.

Contour plot for Q. 5c) and 5d)



c) **[3 pts]** Which initial simplex (1, 2 or 3) will likely take **the least number of iterations to converge** to the optimal (best) values of k and V? Which point (A, B, C, or D) will the selected simplex converge to? Explain your answers.

The simplex 3 will take least steps to the optimal. It will converge to C. Because C close most to the optimal values, and has no risk to converge to the local optimal site as simplex A dose.

d) **[3 pts]** Which initial simplex (1, 2 or 3) is the **worst** starting point? Explain your answer.

Simplex 2, because it stays at a site with very high cost, and the cost function for it is convex, it has difficulty to find the optimal values.

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e) **[2 pts]** State **one disadvantage** of the iterative optimization algorithm, such as the simplex optimization.

It has local optimal sites, which add difficulty for the simplex to find the global optimal values. Also, the algorithm is not convex, if the a bad simplex initial site is selected, the function can not converge.

---------- **End of Quiz 2** ----------