Homework number 2

Consider the general form of a 1D dimensionless convection-diffusion equation with an additional source $S = \rho u$:

$$\rho u \frac{d\phi}{dx} - \Gamma \frac{d^2\phi}{dx^2} = \frac{\rho u}{L} \tag{1}$$

with $0 \leq x \leq L$ and $\phi(0) = 0$ and $\phi(L) = 0$. Note that we are considering that the continuity equation is satisfied. The analytical solution is

$$\phi(x) = \frac{x}{L} - \frac{1 - \exp\left(\frac{\rho u x}{\Gamma}\right)}{1 - \exp\left(\frac{\rho u L}{\Gamma}\right)} \tag{2}$$

We want to discretise eq. 1 but we can use only a grid with 10 nodal points at the most.

Homework number 2

1. If we have a fluid with $\rho = 2$, $\Gamma = 0.5$ in a domain with $L = 2$, which is the limiting velocity u in order to ensure boundedness with using the Central Differencing (CD) scheme?
2. If the velocity is $u = 4$, find the the profile of ϕ in function of x with using the best method in your opinion between CD and First-order upwind (UW). Find the percentage error in function of the location. Use Matlab/Python.
3. And if we would be able to use a grid with N=100. Which method would you choose? CD or UW? Why?
4. Sorry, I'm not convinced of your choice (whatever it is). Can you convince me with some data to compare the two methods? Use Matlab/Python.
5. But unfortunately we cannot use such a fine grid! We have just our coarse one with $N = 10$. And the velocity now is not constant along the domain! But it varies linearly from between $-2 \leq u \leq 4$. Which method would you use? Pick one that you learnt in the course. Why?