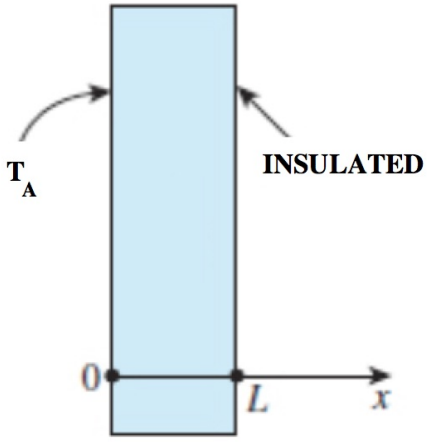
Homework number 1

We have a large plate of thickness L with a constant thermal conductivity k and uniform heat generation q. The face A is at T_A while at the face B is insulated. We can assume that the dimensions in the y- and z-directions are so large that the temperature gradients are significant just in the x-direction only. The governing equation is

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+q=0 \tag{1}$$



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1. Calculate the analytical solution.
2. Discretise equation 1 with dividing the domain in 4 control volumes. You have to arrive to a system of algebraic equations. Please obtain this system. Until here please use just symbols since we did not define yet the numerical values.
3. Use the TDMA to solve the system of algebraic equations. First write the coefficients β_j, D_j, α_j and C_j. Afterwards find the coefficients A_j and C'_j. Use as values: $T_A = 120^o C$, $k = 0.6 \frac{W}{mK}$, $q = 500 \frac{W}{m^3}$, $L = 16cm$. Compare numerical and analytical results. What is the point-by-point percentage error?
4. Now write a Matlab (or Python) Code to solve point 3. Compare the solution with TDMA to the solution obtained using the command "\".
5. Calculate the percentage error between numerical and analytical solution at $x = 14cm$ with dividing the domain in 12, 20 and 36 control volumes instead. Plot the error vs grid spacing. Make your comments.