Homework 4

The fluid in a very thin box with porous eastern/western porous walls is initially at rest with a constant temperature T_i. We change very abruptly the temperature of the eastern wall of the box to T_∞, while the western one is considered insulated. The box has length L and we settle the origin of the axis at the western wall. Then the eastern side is located at $x^* = L$. We can assume that the problem is 1D and then the governing equation is (where x^* is the dimensional east/west direction)

$$\frac{1}{\alpha}\frac{\partial T^*}{\partial t^*} = \frac{\partial^2 T^*}{\partial x^{*2}}$$

where $\alpha = \frac{k}{\rho c}$ and k, c and ρ are the thermal conductivity, the heat specific and the density of the fluid, respectively. At the eastern boundary we have a heat transfer rate given by $\dot{Q} = hA_{wall}(T^*_{wall} - T_\infty)$ where h is the convective heat transfer coefficient, A_{wall} is the eastern wall surface and T^*_{wall} is the fluid temperature at the wall.

1. Define the initial and the boundary conditions. Explain carefully what are you doing!

2. We can define non-dimensional parameters, such as the temperature $T = \frac{T^* - T_\infty}{T_i - T_\infty}$, $Nu = \frac{hL}{k}$ (Nu is the Nusselt number), $x = \frac{x^*}{L}$ and $t = \frac{\alpha t^*}{L^2}$. Show that the non-dimensional equation is

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$$

with boundary conditions: $\frac{\partial T}{\partial x} = 0$ at $x = 0$ and $\frac{\partial T}{\partial x} = -Nu T_{wall}$ at $x = 1$ with $T = 1$ at $t = 0$.

3. Discretise the governing equations with the FVM and obtain the standard general equation for both internal and boundary nodes (node 1 and node N) with using the central differencing scheme for the diffusion terms and the implicit scheme for the time advancement.

4. Settle $Nu = 10$, $L = 1$, and use a grid with 400 nodes. Write a Matlab code to obtain the transient solution and compare at $t = 4$ with the analytical solution for $t > 0.2$ given by

$$T(x,t) = A_1 \exp\left(-\lambda_1^2 t\right) \cos\left(\lambda_1 x\right)$$

where $A_1 = 1.2620$ and $\lambda_1 = 1.4289$. Settle $\Delta t = 0.001$.

5. At $t = 4$ a flux through the porous eastern and western walls is introduced. Now the fluid moves with a constant non-dimensional velocity. In particular we study two cases $u = \frac{u^* L}{\alpha} = 200$ and $u = \frac{u^* L}{\alpha} = -200$. The non-dimensional governing equation is now

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2}$$

What will be the steady temperature of the system for the two cases?

6. Discretise the convective term with upwind and write a Matlab code to describe the transient temperature profile of the system. Use the same grid and the same time step of the previous questions. How will you define the boundary conditions (node 1 and node *N*) for the convective term? Be very careful at it! For which velocity the solution is more accurate (say tolerance 10^{-9})? (Bonus: explain why).

7. And if the velocity is $u = \pm 20$? Which techniques would you use to discretise the convection terms and the time advancement? Why?