

1. induced smoothing method to Quantile residual life model fitting estimating equation (Kim et al.(2012))
Estimating Equation

$$S(\beta) = \sum_{i=1}^n w_i I[Z_i > t_0] X_i \psi_\tau \{\log(Z_i - t_0) - X_i^T \beta\} = 0$$

Using induced smoothing

$$\begin{aligned} \bar{S}(\beta) &= E_z[S(\beta + \Gamma^{\frac{1}{2}} D)] \\ &= E_z\left[\sum_{i=1}^n w_i I[Z_i > t_0] X_i \psi_\tau \{\log(Z_i - t_0) - X_i^T (\beta + \Gamma^{\frac{1}{2}} D)\}\right] \\ &= E_z\left[\sum_{i=1}^n w_i I[Z_i > t_0] X_i (\tau - I\{\log(Z_i - t_0) - X_i^T (\beta + \Gamma^{\frac{1}{2}} D) \leq 0\})\right] \\ &= \sum_{i=1}^n w_i I[Z_i > t_0] X_i (\tau - E_z[I\{\log(Z_i - t_0) - X_i^T (\beta + \Gamma^{\frac{1}{2}} D) \leq 0\}]) \\ &= \sum_{i=1}^n w_i I[Z_i > t_0] X_i (\tau - P[X_i^T \Gamma^{\frac{1}{2}} D \geq \log(Z_i - t_0) - X_i^T \beta]) \\ &= \sum_{i=1}^n w_i I[Z_i > t_0] X_i (\tau - P[D \geq \frac{\log(Z_i - t_0) - X_i^T \beta}{\sqrt{X_i^T \Gamma X_i}}]) \\ &= \sum_{i=1}^n w_i I[Z_i > t_0] X_i (\tau - 1 + \Phi(\frac{\log(Z_i - t_0) - X_i^T \beta}{\sqrt{X_i^T \Gamma X_i}})) \end{aligned}$$

where $D \sim N_P(0, I_P)$, $Z_i = \min(T_i, C_i)$ and X_i is covariate.

2. Sandwich estimator $A(\beta)$

$$\begin{aligned}
A(\beta) &= \frac{\partial U(\beta)}{\partial \beta} \\
&= \frac{\partial \sum w_i I[Z_i > T_0] X_i \left(\tau - 1 + \Phi \left(\frac{\log(Z_i - t_0) - X_i^T \beta}{\sqrt{X_i^T \Gamma X_i}} \right) \right)}{\partial \beta} \\
&= \sum w_i I[Z_i > T_0] X_i \phi \left(\frac{\log(Z_i - t_0) - X_i^T \beta}{\sqrt{X_i^T \Gamma X_i}} \right) \left(\frac{-X_i^T}{\sqrt{X_i^T \Gamma X_i}} \right)
\end{aligned}$$