induced smoothing method to Quantile residual life model fitting estimating equation (Kim et al.(2012))
Estimaitng Equation

$$S(\beta) = \textstyle\sum_{i=1}^{n} w_i I[Z_i > t_0] X_i \psi_\tau\{\log(Z_i - t_0) - X_i^T \beta\} = 0$$

Using induced smoothing

$$\begin{aligned}
\bar{S}(\beta) &= E_z[S(\beta + \Gamma^{\frac{1}{2}} D)] \\
&= E_Z[\sum_{i=1}^{n} w_i I[Z_i > t_0] X_i \psi_\tau\{\log(Z_i - t_0) - X_i^T(\beta + \Gamma^{\frac{1}{2}} D)\}] \\
&= E_Z[\sum_{i=1}^{n} w_i I[Z_i > t_0] X_i(\tau - I\{\log(Z_i - t_0) - X_i^T(\beta + \Gamma^{\frac{1}{2}} D) \le 0\})] \\
&= \sum_{i=1}^{n} w_i I[Z_i > t_0] X_i(\tau - E_z[I\{\log(Z_i - t_0) - X_i^T(\beta + \Gamma^{\frac{1}{2}} D) \le 0\}]) \\
&= \sum_{i=1}^{n} w_i I[Z_i > t_0] X_i(\tau - P[X_i^T \Gamma^{\frac{1}{2}} D \ge \log(Z_i - t_0) - X_i^T \beta]) \\
&= \sum_{i=1}^{n} w_i I[Z_i > t_0] X_i(\tau - P[D \ge \frac{\log(Z_i - t_0) - X_i^T \beta}{\sqrt{X_i^T \Gamma X_i}}]) \\
&= \sum_{i=1}^{n} w_i I[Z_i > t_0] X_i(\tau - 1 + \Phi(\frac{\log(Z_i - t_0) - X_i^T \beta}{\sqrt{X_i^T \Gamma X_i}}))
\end{aligned}$$

where $D \sim N_P(0, I_P)$, $Z_i = min(T_i, C_i)$ and X_i is covariate.