2017/2/22 Meeting Preparation

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1. Answer for comments

   1) Consistent?
      - Even if using different Γ, estimated $\hat{\beta}$ would converge to real value through iteration.

   2) How to making new convex object function?

      - First estimating equation from (Tsiatis(1990))

$$W_n(\beta) = \sum_{i=1}^{n} w_i(\beta)\Delta_i[X_i - \frac{\sum_{j=1}^{n} X_j I[e_j(\beta) \geq e_i(\beta)]}{\sum_{j=1}^{n} I[e_j(\beta) \geq e_i(\beta)]}] = 0$$

      - Typically, solution for $W_n(\beta) = 0$ is not exist. Therefore we have to find generalized solution which is the βthat satisfies $\min \|W_n(\beta)\|$.

      - Use Gehan weight function($\sum_{i=1}^{n} I[e_j(\beta) \geq e_i(\beta)]$) as a $w_i(\beta)$ to make $W_n(\beta)$ monotone function

$$W_n(\beta) = \sum_{i=1}^{n} \sum_{j=1}^{n} \Delta_i (X_i - X_j) I[e_j(\beta) - e_i(\beta) \geq 0]$$

      - $\because e_j(\beta) - e_i(\beta) = (\log(T_j) - X_j\beta) - (\log(T_i) - X_i\beta)$, new objective function has same generalized solution with $W_i(\beta)$

$$O_n(\beta) = \sum_{i=1}^{n} \sum_{j=1}^{n} \Delta_i (e_j - e_i) I[e_j(\beta) - e_i(\beta) \geq 0]$$

      - Find β minimizing $O_n(\beta)$

2. Applying induced smoothing method to Quantile residual life model fitting estimating equation (Kim et al.(2012))
   Estimaitng Equation

$$S(\beta) = \textstyle\sum_{i=1}^{n} w_i I[Z_i > t_0] X_i \psi_\tau\{\log(Z_i - t_0) - X_i^T \beta\} = 0$$

   Using induced smoothing

$$\begin{aligned}
\bar{S}(\beta) &= E_z[S(\beta + \Gamma^{\frac{1}{2}} Z)] \\
&= E_Z[\sum_{i=1}^{n} w_i I[Z_i > t_0] X_i \psi_\tau\{\log(Z_i - t_0) - X_i^T(\beta + \Gamma^{\frac{1}{2}} Z)\}] \\
&= E_Z[\sum_{i=1}^{n} w_i I[Z_i > t_0] X_i(\tau - I\{\log(Z_i - t_0) - X_i^T(\beta + \Gamma^{\frac{1}{2}} Z) \leq 0\})] \\
&= \sum_{i=1}^{n} w_i I[Z_i > t_0] X_i(\tau - E_z[I\{\log(Z_i - t_0) - X_i^T(\beta + \Gamma^{\frac{1}{2}} Z) \leq 0\}]) \\
&= \sum_{i=1}^{n} w_i I[Z_i > t_0] X_i(\tau - P[X_i^T \Gamma^{\frac{1}{2}} Z \geq \log(Z_i - t_0) - X_i^T \beta]) \\
&= \sum_{i=1}^{n} w_i I[Z_i > t_0] X_i(\tau - P[Z \geq \frac{\log(Z_i - t_0) - X_i^T \beta}{\sqrt{X_i^T \Gamma X_i}}]) \\
&= \sum_{i=1}^{n} w_i I[Z_i > t_0] X_i(\tau - 1 + \Phi(\frac{\log(Z_i - t_0) - X_i^T \beta}{\sqrt{X_i^T \Gamma X_i}}))
\end{aligned}$$