Comparison of Constraint-Based Cyclic Causal Discovery Algorithms

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1. Introduction

Background

- In many dynamical systems, the presence of cycles of feedback loops is common.
- Cyclic causal discovery methods are not as well studied as the acyclic counterparts.

Goal

- To assess the performance of three different cyclic causal discovery algorithms by means of a simulation study and identify the most effective one.

2. Methods

Considered Algorithms

- Cyclic Causal Discovery (CCD)[1]
- Fast Causal Inference (FCI)[2]
- Cyclic Causal Inference (CCI)[3]

Data

- Data are simulated from different cyclic models with linear relationships and independent Gaussian error terms, as illustrated in *Figure 1*.

Evaluation Metric

structural Hamming distance (SHD) = *A (edge addition)* + *D (edge deletion)* + *C (edge-endpoint change)*[4]

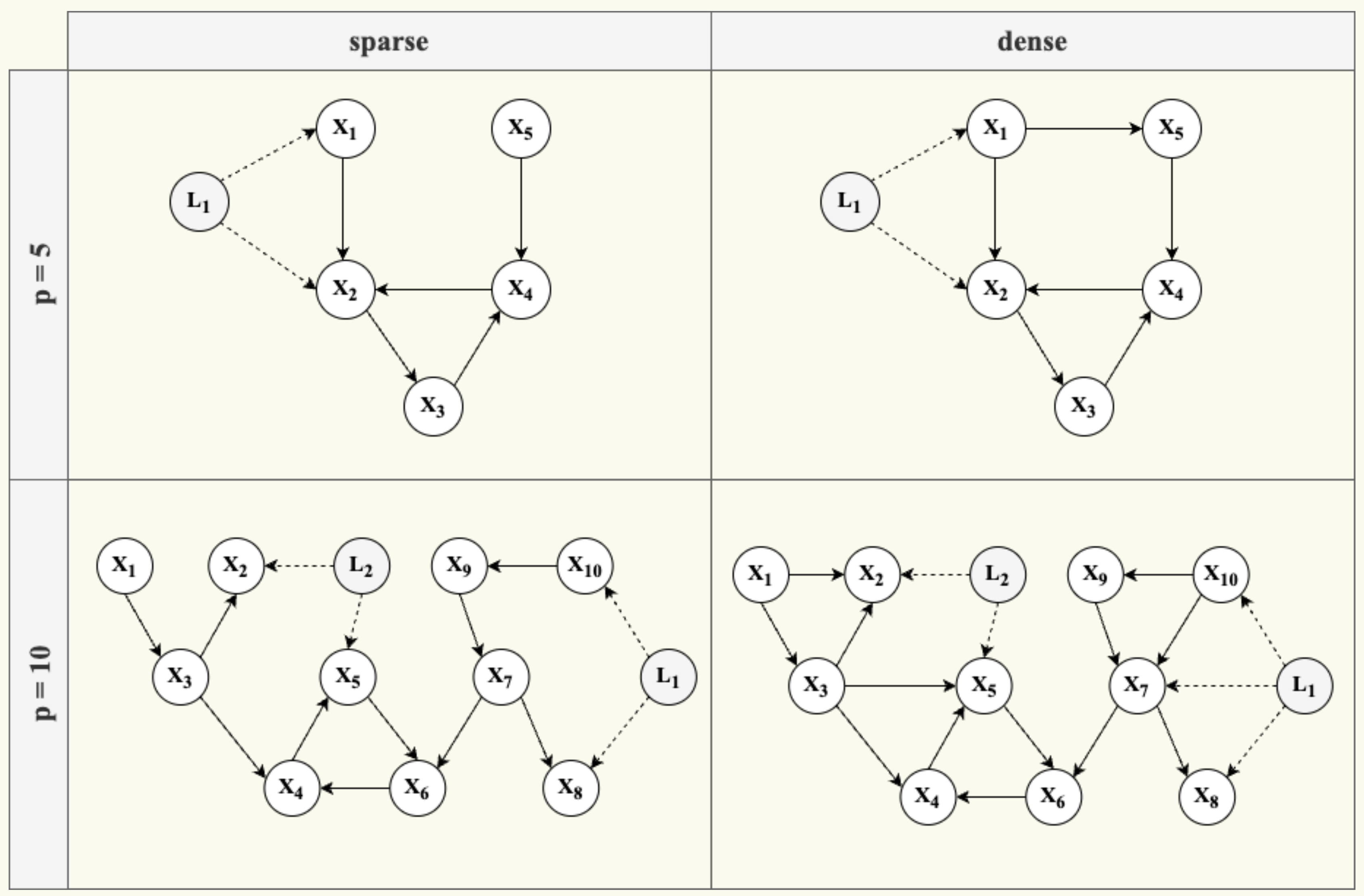

Figure 1. Simulation Settings

We vary the number of variables (rows in Figure 1), density (columns in Figure 1), influence of latent confounders (L_1 and L_2 in Figure 1), and sample size ranging from 50 to 10000. p = number of variables.

3. Results

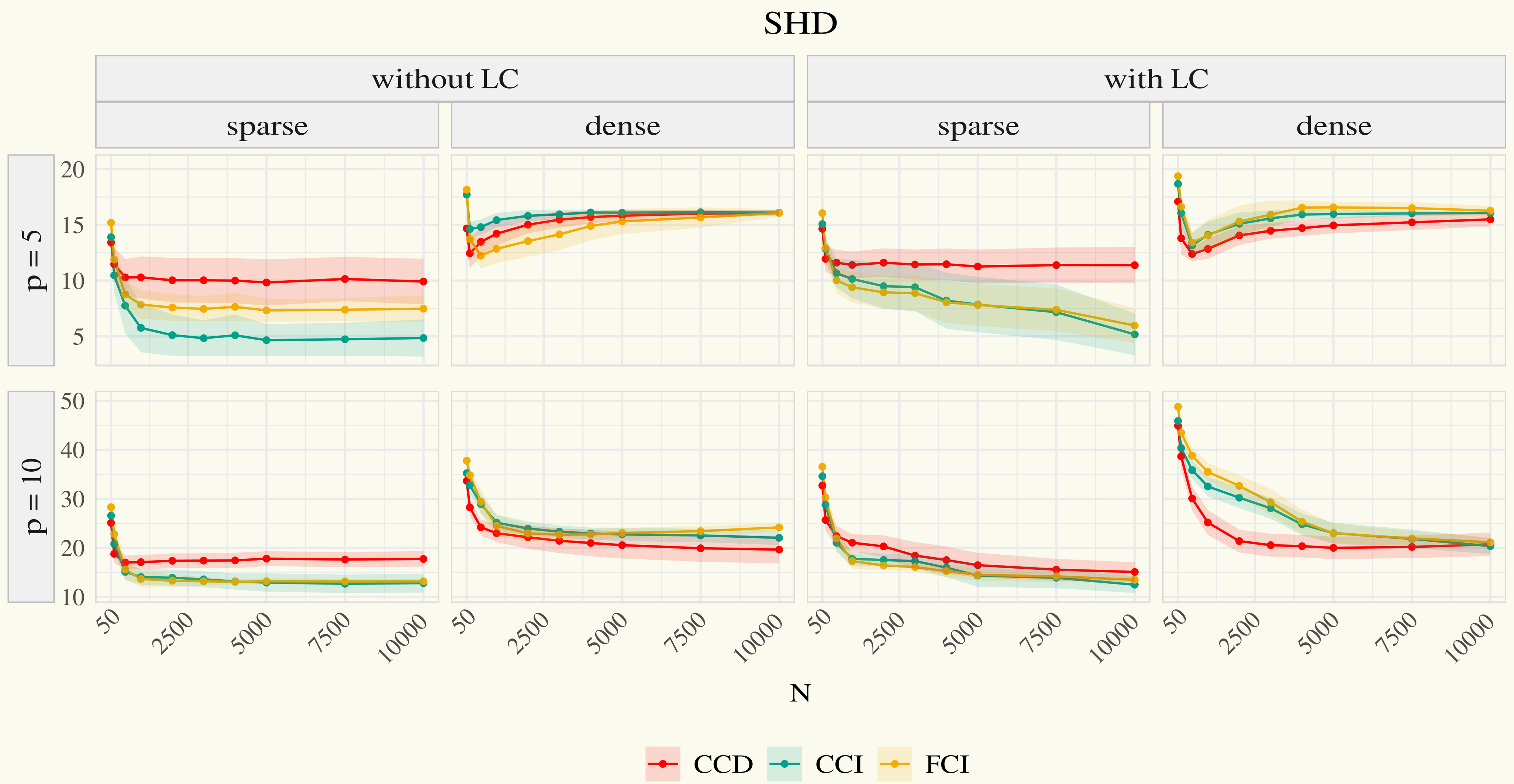

Figure 2. Structural Hamming Distance (SHD)

The sample size (N) is shown on the x-axis and the SHD values are shown on the y-axis. Each point in graph represents the average SHD values across 500 iterations, while the shaded area represents the interquartile range (IQR). p = number of variables; LC = latent confounder.

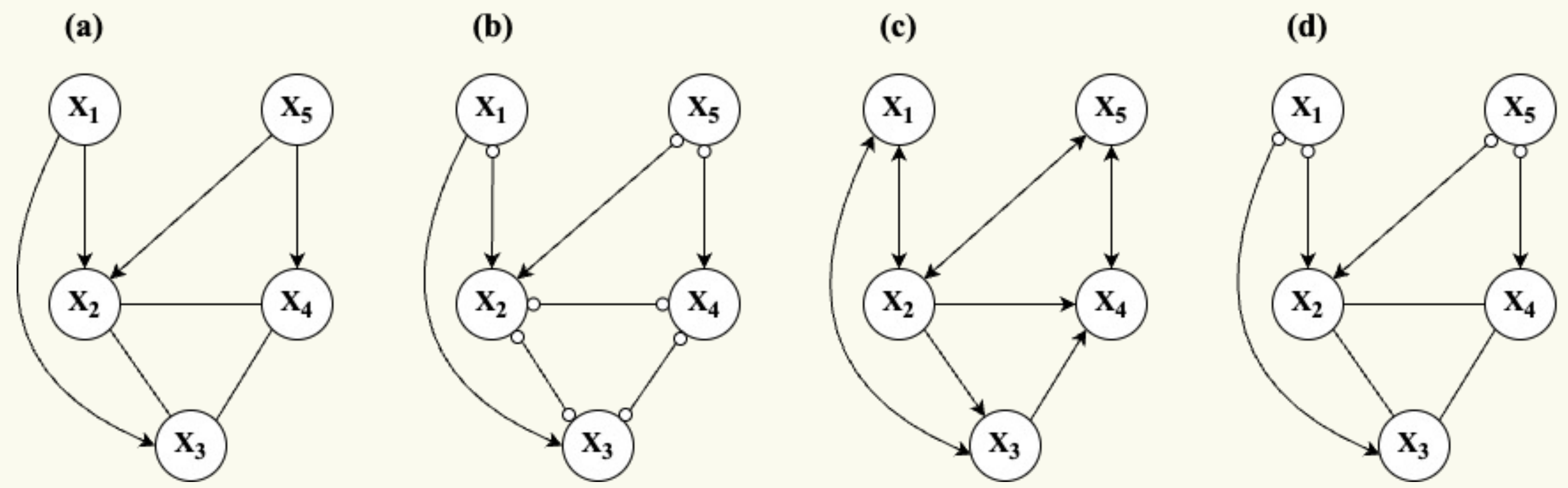

Figure 3. Frequently Estimated Partial Ancestral Graphs (PAGs) in the 5-variable Sparse Condition Without a Latent Confounder Condition

Panel (a) shows the true ancestral graph of the 5-variable sparse condition without a latent confounder. Panels (b), (c), and (d) present the most frequently occurring PAGs in the corresponding condition resulted from CCD, FCI, and CCI algorithms, respectively. They are obtained by selecting the most frequent type of edge-endpoints produced by each algorithm from 500 simulations with the sample size of 1000.

- *Figure 2* shows that the FCI and CCI algorithms overall perform better in sparse conditions, while the CCD algorithm outperforms the others in dense conditions.
- We do not observe any significant contrasting patterns between the conditions with and without latent confounders (LCs).
- *Figure 3* shows the typical behavior of algorithms. CCD tends to produce more circle endpoints (○), whereas FCI tends to guess directions for all edges. CCI appears to be outperform the others in identifying mutual ancestry in cycles with undirected edges (–).

For more detailed results, check the **supplementary material** here.

4. Conclusions

- No single algorithm is suitable for all cases, and the choice of algorithm should be based on the characteristics of the causal system of interest.
- Researchers need to consider their priorities when selecting an algorithm (e.g., if avoiding incorrect edge orientation is a priority, then the more conservative CCD algorithm may be preferred).

Limitations

- We restricted ourselves to a set of fixed causal structures.
- Our simulation was limited in scope as we only considered Gaussian linear cases.

Future Research

- Future research can improve the generalizability of our findings by randomly sampling the structures and exploring more general scenarios beyond linear Gaussian cases.

References

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[2] Mooij, J. M. & Claassen, T. (2020). Constraint-based causal discovery using partial ancestral graphs in the presence of cycles, In *Proceedings of the 36th Conference on Uncertainty in Artificial Intelligence* (pp. 1159–1168). Proceedings of Machine Learning Research.

[3] Strobl, E. V. (2019). A constraint-based algorithm for causal discovery with cycles, latent variables and selection bias. *International Journal of Data Science and Analytics*, *8*(1), 33–56.

[4] de Jongh, M. & Druzdzel, M. J. (2009). A comparison of structural distance measures for causal Bayesian network models. *Recent Advances in Intelligent Information Systems*.