Appendix A CCD Details

Algorithm 1 Cyclic Causal Discovery (CCD)

Input: A conditional independent oracle for a distribution $\mathcal{P}$, satisfying global directed Markov property and faithfulness conditions with respect to a directed graph $\mathcal{G}$ with vertex set $\mathcal{V}$.

Output: A PAG Ψ for the Markov equivalence class of DCGs, $Equiv(\mathcal{G})$.

Step 1. Form a complete graph (Ψ) with the edge $\circ\!\!-\!\!\circ$ between every pair of vertices in $\mathcal{V}$.
$n = 0$
repeat
  repeat
    Select an ordered pair of variables X and Y that are adjacent in Ψ such that the number of vertices in $\textbf{Adjacent}(\Psi, X)\backslash\{Y\} \geq n$, and select a subset $\mathcal{S}$ of $\textbf{Adjacent}(\Psi, X)\backslash\{Y\}$ with n vertices.
    If $X \perp\!\!\!\perp Y \mid \mathcal{S}$, then delete the edge $X\circ\!\!-\!\!\circ Y$ and record $\mathcal{S}$ in $\textbf{Sepset}\langle X, Y\rangle$ and $\textbf{Sepset}\langle X, Y\rangle$.
  until all pairs of adjacent variables X and Y such that the number of vertices in $\textbf{Adjacent}(\Psi, X)\backslash\{Y\} \geq n$ and all sets $\mathcal{S}$ such that the number of vertices in $\mathcal{S} = n$ have been tested.
    $n = n + 1$;
until for all ordered pairs of adjacent vertices X and Y, $\textbf{Adjacent}(\Psi, X)\backslash\{Y\} < n$.
Step 2. For each triple of vertices A, B, C such that each of the pair of A, B and the pair B, C are adjacent in Ψ but the pair A, C are not adjacent in Ψ, then:
(i) orient $A{*\!\!-\!\!*}B{*\!\!-\!\!*}C$ as $A{\longrightarrow}B{\longleftarrow}C$ *iff* $B \notin \textbf{Sepset}\langle A, C\rangle$.
(ii) orient $A{*\!\!-\!\!*}B{*\!\!-\!\!*}C$ as $A{*\!\!-\!\!*}\underline{B}{*\!\!-\!\!*}C$ *iff* $B \in \textbf{Sepset}\langle A, C\rangle$.
Step 3. For each triple of vertices A, X, Y in Ψ such that (i) A is not adjacent to X or Y, (ii) X and Y are adjacent, (iii) $X \notin \textbf{Sepset}\langle A, Y\rangle$, then orient $X{*\!\!-\!\!*}Y$ as $X{\longleftarrow}Y$ if $A \not\perp\!\!\!\perp X \mid \textbf{Sepset}\langle A, Y\rangle$.
Step 4. For each vertex V in Ψ form the following set: $X \in \textbf{Local}(\Psi, V)$ *iff* X is adjacent to V in Ψ, or there is a vertex Y such that $X{\longrightarrow}Y{\longleftarrow}V$ in Ψ.
$m = 0$
repeat
  repeat
    Select an ordered triple $\langle A, B, C\rangle$ such that $A{\longrightarrow}B{\longleftarrow}C$, A and C are not adjacent, and $\textbf{Local}(\Psi, A)\backslash\{B, C\}$ has $\geq m$ vertices.
    Select a set $T \subseteq \textbf{Local}(\Psi, A)\backslash\{B, C\}$ with m vertices. If $A \perp\!\!\!\perp C \mid T \cup \{B\}$, then orient $A{\longrightarrow}B{\longleftarrow}C$ as $A{\longrightarrow}\underset{\cdots}{B}{\longleftarrow}C$ and record $T \cup \{B\}$ in $\textbf{Supset}\langle A, B, V\rangle$.