Multilevel analysis

Longitudinal data and contextual effects

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Course outline

Week 1 & 2
- Monday: when/why multilevel analysis
- Monday: the multilevel regression model
- Friday: The three-level MLM
- Friday: MLM assumptions

Week 3 & 4
- Monday: Longitudinal model
- Friday: Contextual effects

Week 5 & 6
- Analyzing dichotomous and ordinal data

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Today

1) Multilevel analysis for longitudinal data

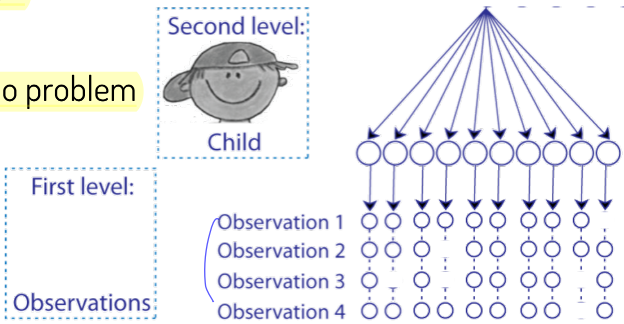
2) Two examples

Longitudinal data

Measurements nested within subjects

Longitudinal data

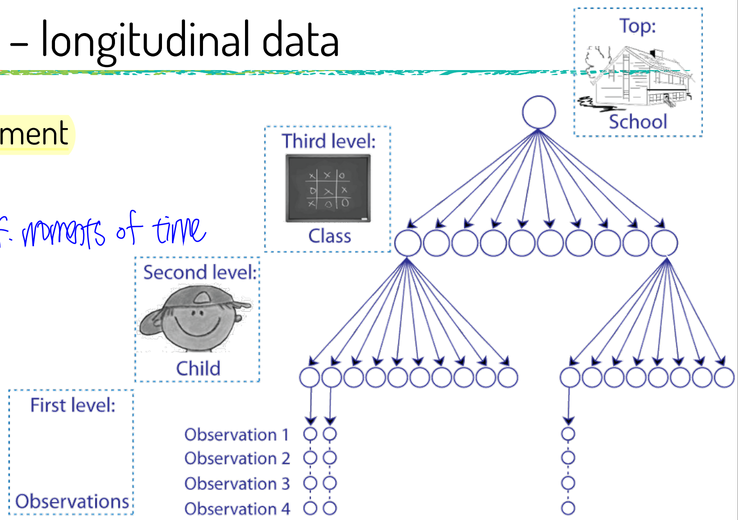
- Collected data on ten children measured on four occasions
- First level: measurement occasion : each children are measured 4 times
- Second level: Child
- Note: missing observations are no problem



Why multilevel analysis? – longitudinal data

4. Balanced data is not a requirement
5. Varying occasions possible !!

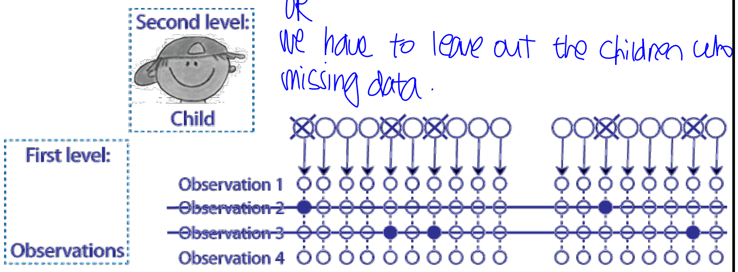
   every child is measured at diff. moments of time



Why multilevel analysis? – longitudinal data

4. Balanced data is not a requirement
5. Varying occasions possible

If we'd choose repeated ANOVA, then we need to leave out either obs 2 & obs 3 cuz they contain missing data,
OR
we have to leave out the children who has missing data.



→ BUT, w/ MLM we can use all info. that're present, w/o adjusting any missingness in any ways.

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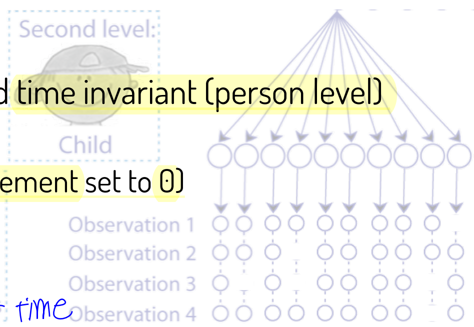
Longitudinal data

- Collected data on ten children measured on four occasions
- First level: measurement occasion
- Second level: Child
- We can mix time variant (occasion level) and time invariant (person level) predictors
- Time included as predictor (with first measurement set to 0)
  - linear: 0,1,2,3, ...
  - quadratic: 0, 1, 4, 9, ... ~ if you think that the progression over time doesn't follow a straight (linear) line but it levels off at some point, becuz we have a ceiling effect / floor effect, then we can easily incorporate that our mlm analysis!

∵ The reason that we don't need a balanced data



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Here the measurement location is equal for each person

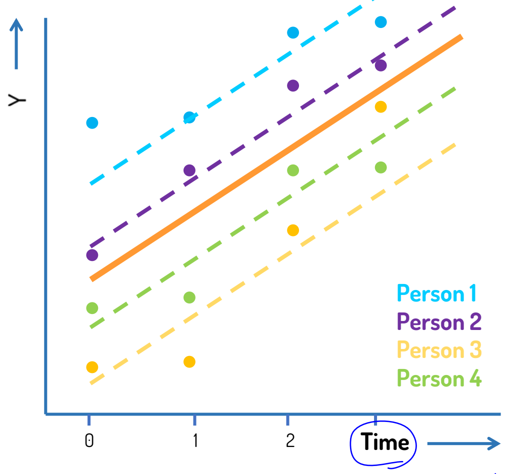
Occasion level model – person specific starting points

Multilevel regression with one explanatory variable time:

fixed slope

$$y_{ti} = \pi_{0i} + \pi_1 T_{ti} + e_{ti}$$

- subscript t is a time indicator
- y_{tj} outcome at time point t for person i
- T_{ti} time variable
- π_{0j} person dependent intercept
- π_1 regression slope for time
- e_{ti} residual error term, are assumed to have mean zero and variance σ_e^2
- We use π such that at person level we can still use β



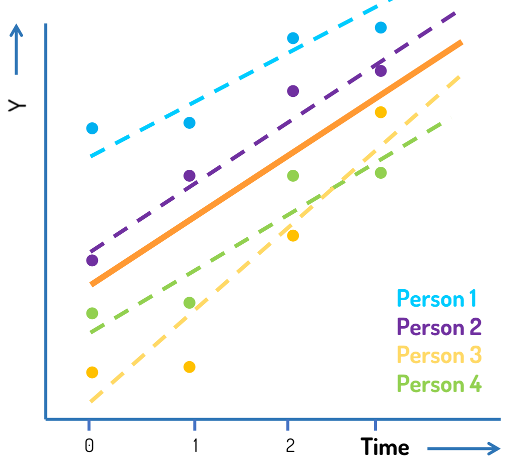
just a predictor in our model

Occasion level model – person specific rates of change

Multilevel regression with one explanatory variable time:

$$y_{ti} = \pi_{0i} + \pi_{1j} T_{ti} + e_{ti}$$

- subscript t is a time indicator
- y_{tj} outcome at time point t for person i
- T_{ti} time variable
- π_{0j} person dependent intercept
- π_{1j} person dependent regression slope for time
- e_{ti} residual error term, are assumed to have mean zero and variance σ_e^2



not only that a person can have a diff. starting point (π_{0j}: person-specific intercept) but also progress differently over time (π_{1j}: person-specific slope)

Person Level Model

- *Occasion level*: $y_{ti} = \pi_{0i} + \pi_{1i}T_{ti} + e_{ti}$
- *Person level*: predict starting points and rates of change with person level regression model

person-specific int → $\pi_{0i} = \overset{\text{overall int.}}{\beta_{00}} + \overset{\text{person-specific deviation}}{u_{0i}}$ from the overall int.

person-specific rate of change (slope) → $\pi_{1i} = \overset{\text{overall slope}}{\beta_{10}} + \overset{\text{person-specific deviation}}{u_{1i}}$ from the overall slope

  - β_{00} is the overall intercept, u_{0i} is the person specific deviation from the overall intercept β_{00} (i.e., the residual error term in the equation for π_{0i})
  - β_{10} is the overall slope, u_{1i} is the person specific deviation from the overall slope β_{10} (i.e., the residual error term in the equation for π_{1i})

Person Level Model

- *Occasion level*: $y_{ti} = \pi_{0i} + \pi_{1i}T_{ti} + e_{ti}$
- *Person level*: predict starting points and rates of change with person level regression model

Z_i: time-invariant predictor

- Add predictor at person level (e.g., gender):
- $\pi_{0i} = \beta_{00} + \beta_{01}Z_i + u_{0i}$
- $\pi_{1i} = \beta_{10} + \beta_{11}Z_i + u_{1i}$
  - β_{00} and β_{01} are the intercept and slope to predict π_{0i} from Z_i
  - u_{0i} is the residual error term in the equation for π_{0i}
  - β_{10} and β_{11} are the intercept and slope to predict π_{1i} from Z_i
  - u_{1i} is the residual error term in the equation for π_{1i}

person-specific deviation from the overall intercept taking into account of Z_i (person-level predictor)

what variation in the slope can we not explain using Z_i (person level covariate)

predict the starting point
Ex) "Do boys start off higher/lower compared to girls?"
OR
they can partly explain why the rate of change differ
Ex) "If we have sig. slope variance, then maybe this is becuz girls change faster compared to boys for ex."

So this explains within a person "occasion-level" variance

this time-varying covariate is an explaining why the observation over time within the same person varies

Extended Model

- Occasion level: *time varying covariates*
  - $y_{ti} = \pi_{0i} + \pi_{1i}T_{ti} + \pi_{2i}X_{ti} + e_{ti}$

outcome at time t of person i = $\underbrace{\pi_{0i}}_{\text{PS int.}} + \underbrace{\pi_{1i}T_{ti}}_{\text{PS slope for the rate of change over time}} + \underbrace{\pi_{2i}X_{ti}}_{\text{another time-varying covariate}} + \underbrace{e_{ti}}_{\text{residual error term}}$

- Person level: *time invariant covariates*
  - $\pi_{0i} = \beta_{00} + \beta_{01}Z_i + u_{0i}$
  - $\pi_{1i} = \beta_{10} + \beta_{11}Z_i + u_{1i}$
  - $\pi_{2i} = \beta_{20} + \beta_{21}Z_i + u_{2i}$

explains the starting point diff. or the rate of change differences

This terms explain why the means of persons differ: why the averages of each person vary! (time-invariant covariates)

*정리하자면,

$\beta_{01}Z_i$: explains why the means of a person differ

$\beta_{11}Z_i$, $\beta_{21}Z_i$: cross-level interaction → explaining why the slopes differ

ICC in longitudinal data

- ICC:
  1. Expected correlation between two randomly sampled individuals in *same* group
  2. Percentage of variance at the cluster level

- DIY: what does ICC represent in longitudinal multilevel analysis?

ICC is now telling us how similar the observations within the same person are, so the expected corr. between observations that are originated from the same person

But also if we look at all the diff. in all the observations that we collected, then what percentage is due to the diff. between persons that we have within our dataset.

An example

Growth in GPA over time

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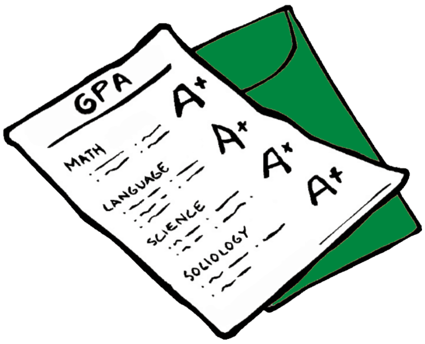
Data set

- 200 Students
- GPA measured on 6 occasions (range: 1-4)
- Time varying covariate:
  - *Job:* number of hours worked in off-campus job
- Time invariant covariates:
  - *HighGPA:* high school GPA
  - *Sex* (0=male; 1=female)

Artificial data, SPSS file *GPA2* and *GPA2long*



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Fixed measurement occasions

Here: All individuals provide data at the same time points

Typically in longitudinal data:

- Measurements often regularly spaced
- Observations may be incomplete
    - Dropout, intermittently missing observations
- Analysis methods:
    - Anova and Manova approaches
    - Multilevel analysis
    - Latent Growth Curve modelling (SEM)

almost identical

Longitudinal Data File: wide format

→ you need to restructure this to a LONG format.

	student	sex	highgpa	gpa1	gpa2	gpa3	gpa4	gpa5	gpa6
1	1	1	2.8	2.3	2.1	3.0	3.0	3.0	3.3
2	2	0	2.5	2.2	2.5	2.6	2.6	3.0	2.8
3	3	1	2.5	2.4	2.9	3.0	2.8	3.3	3.4
4	4	0	3.8	2.5	2.7	2.4	2.7	2.9	2.7
5	5	0	3.1	2.8	2.8	2.8	3.0	2.9	3.1
6	6	1	2.9	2.5	2.4	2.4	2.3	2.7	2.8
7	7	0	2.3	2.4	2.4	2.8	2.6	3.0	3.0
8	8	1	3.9	2.8	2.8	3.1	3.3	3.3	3.4
9	9	0	2.0	2.8	2.7	2.7	3.1	3.1	3.5
10	10	0	2.8	2.8	2.8	3.0	2.7	3.0	3.0
11	11	1	3.9	2.6	2.9	3.2	3.6	3.6	3.8
12	12	1	2.9	2.6	3.0	2.3	2.9	3.1	3.3
13	13	0	3.7	2.8	3.1	3.5	3.6	3.9	3.9
14	14	1	3.5	2.4	3.0	2.9	3.0	3.3	3.4
15	15	0	3.6	2.2	2.7	2.6	2.6	2.8	2.8

Longitudinal Data File: "long format"

you need a

each student now takes the same amount of rows as the number of measurements

	student	occas	gpa	job	sex	highgpa
1	1	0	2.3	2	1	2.8
2	1	1	2.1	2	1	2.8
3	1	2	3.0	2	1	2.8
4	1	3	3.0	2	1	2.8
5	1	4	3.0	2	1	2.8
6	1	5	3.3	2	1	2.8
7	2	0	2.2	2	0	2.5
8	2	1	2.5	3	0	2.5
9	2	2	2.6	2	0	2.5
10	2	3	2.6	2	0	2.5
11	2	4	3.0	2	0	2.5
12	2	5	2.8	2	0	2.5
13	3	0	2.4	2	1	2.5
14	3	1	2.9	2	1	2.5
15	3	2	3.0	2	1	2.5

Restructure data

```
##   student sex highgpa gpa1 gpa2 gpa3 gpa4 gpa5 gpa6 job1 job2 job3 job4
## 1       1   2     2.8  2.3  2.1  3.0  3.0  3.0  3.3    2    2    2    2
## 2       2   1     2.5  2.2  2.5  2.6  2.6  3.0  2.8    2    3    2    2
## 3       3   2     2.5  2.4  2.9  3.0  2.8  3.3  3.4    2    2    2    3
## 4       4   1     3.8  2.5  2.7  2.4  2.7  2.9  2.7    3    2    2    2
```

- Restructure (reshape) data from wide format to long format

```
library(tidyr)
GPA_long <- pivot_longer(data = GPA,
        cols = c(4:15),
        names_to = c(".value", "time"),
        names_pattern = "(gpa|job)(.)")
```

```
## # A tibble: 6 x 6
##   student     sex highgpa  time   gpa   job
##   <fct>     <dbl>   <dbl> <dbl> <dbl> <dbl>
## 1 "    1 "      1     2.8     0   2.3     2
## 2 "    1 "      1     2.8     1   2.1     2
## 3 "    1 "      1     2.8     2   3       2
## 4 "    1 "      1     2.8     3   3       2
## 5 "    1 "      1     2.8     4   3       2
## 6 "    1 "      1     2.8     5   3.3     2
```

Simple Analysis of GPA data: Means

MANOVA

- occasion effect significant
- high school GPA and sex also significant
- no interactions
- job status not tested
  (time varying covariate which Manova cannot handle)

Occasion	1	2	3	4	5	6	Total
male	2.6	2.7	2.7	2.8	2.9	3.0	2.8
female	2.6	2.8	2.9	3.0	3.1	3.2	2.9
all	2.6	2.7	2.8	2.9	3.0	3.1	2.9

female scores a bit higher in average

if you look over time, we see that GPA increases over time

→ Both main effects of sex & Time are significant!

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- we cannot include Job cuz it's time-variant covariate (MANOVA cannot)
- W/ MANOVA, we don't get sig. interaction between SEX & TIME. So according to MANOVA, the progression over time is the same between males & females.

Let's see what happens if we use MLA!

Multilevel analysis of GPA data

- Model for calculating ICC, significance of σ_{u0}^2 (int. var.): do the means of diff. children sig. vary?
  - Time not included: $GPA_{ti} = \beta_{00} + u_{0i} + e_{ti}$ → This model has overall int. (β_{00}) + person-specific deviation (u_{0i}) + residual (e_{ti})
    - used to calculate if the int. var is sig? → Do we need to do ML analysis or not?
    - used to be a bench mark model for R^2. but here w/ longitudinal data, the benchmark model for R^2 = zero model + Time!!
- Baseline model for calculating R^2 values
  - Time fixed: $GPA_{ti} = \beta_{00} + \beta_{10}T_{ti} + u_{0i} + e_{ti}$
- Allow for random time-effect:
  - Time random: $GPA_{ti} = \beta_{00} + \beta_{10}T_{ti} + u_{0i} + u_{1i}T_{ti} + e_{ti}$: trajectory over time is diff. over students.
- With covariates to predict intercept and slope variability (full model)
  - Adding sex and job: $GPA_{ti} = \beta_{00} + \beta_{10}T_{ti} + \beta_{20}Job_{ti} + \beta_{01}Sex_i + \beta_{21}Sex_iJob_{ti} + u_{0i} + u_{1i}T_{ti} + e_{ti}$

you need to include predictor Time in order to get the correct model for baseline model to compute R^2!

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M1) random int.
Int = 2.87 : average GPA over all students.
residual var. @ occ = 0.098
intercept var. = 0.057 : variance of diff. in mean between persons.
SE is much smaller compared to the par. → we know they're sig!

M2) Add level 1 predictor "Time"
This model w/ Time is used as a benchmark model to calculate R^2!
- Int : (assuming 1st measurement is coded as 0), our predicted GPA = 2.60 for all students.
- Time: w/ every time point, their GPA increases about 0.11

M3) next, we add all other time-variant covariates into the model.
- Job = -0.17 ; neg. effect. so more hours you work, the lower your GPA is.

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M4) Add level 2 predictors
- High GPA = 0.08 : The higher HighGPA on average, you'll have higher GPA
- Sex (female is coded as 1) = 0.15 : If you're female, you're expected to get a higher GPA

Multilevel Analysis of GPA Data

2nd step of adding level 1 predictors are splitted up in 2 steps (in longitudinal data)!

Model	M1: random intercept		2-1) M2: + time		2-2) M3: + job status		M4: +high sch GPA & sex	
Predictor	par est	SE	par est	SE	par est	SE	par est	SE
Intercept	2.87	0.02	2.60	0.02	2.61	0.02	2.53	0.03
Time			0.11	0.00	0.10	0.00	0.10	0.00
Job					-0.17	0.04	-0.17	0.02
High GPA							0.08	0.03
sex							0.15	0.03
Var_occ	0.098	0.004	0.058	0.003	0.055	0.002	0.055	0.002
Var_sub	0.057	0.007	0.063	0.007	0.052	0.006	0.045	0.006
Deviance	913.5		393.7		308.4		282.8	
AIC	919.5		401.7		318.4		296.8	

these two values are the ones you need to compute R^2

Var_sub increases when time added???

normally if we add predictors to the model, variances decrease. What's going on here!?

level 1 var. remains the same, which is as it should be & we further explain some of the differences in the mean between persons

level 1 predictor Job partly explains the occ. level var. but also explains large part of why GPA differs over persons. It's not strange, as the hours work off-campus actually varies more between persons than within-person. So, the amount of hours work is allowed to vary over the measurement occasions, it usually remains the same within the same person. But it differs between-person. So it makes sense that Job explains much of Var_sub.

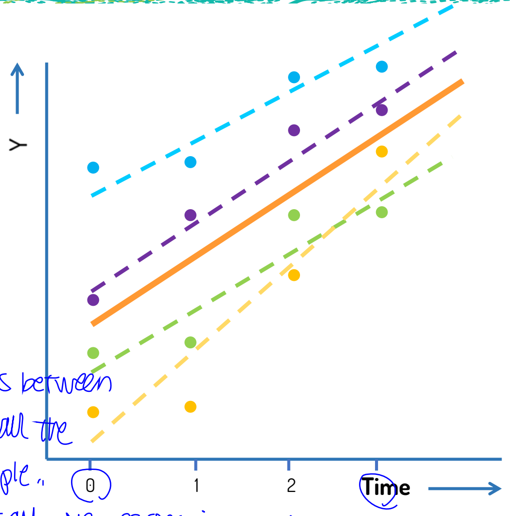
*Mert Q: MLM assumes random sampling at both levels. What are two levels?
level 1) occasion level
level 2) subject level

Why model including time is used as R^2 benchmark model

Remember: multilevel model assumes random sampling at both levels

DIY: is this the case with longitudinal data? Why?

No! only the level 2 observations are randomly sampled.
Population to use the same terms of the time ranges between zero to three, then it's not that we're sampling all the points between 0 & 3, we don't really even sample. we just use 0-3, then there's no variation at all, over persons in occasions. They're fixed, they're not randomly sampled from all the possible time points. They're sort of fixed over persons.



Why model including time is used as R^2 benchmark model:

Mathematical interpretation

avg. time point for person i   diff. between time we look at now & person avg.

- $time_{ti} = \overline{time}_{.i} + (time_{ti} - \overline{time}_{.i}) = time_{pers} + time_{occ}$

int. var. for M2

- $\sigma^2_{u0:M2} = \sigma^2_{u0:M1} - \beta^2_{10}\left(var\left(time_{pers}\right) - \frac{1}{n_{clus}-1} var(time_{occ})\right)$
- $\sigma^2_{e:M2} = \sigma^2_{e:M1} - \beta^2_{10}\left(\frac{n_{clus}}{n_{clus}-1} var(time_{occ})\right)$

Which of the two 'var' components is larger? Why?

→ becuz all the measurement locations are the same for each person, there's no variation between the means of the person time. They'll be all exactly the same. So the var. will be zero.

Becuz of that,
- the var. at the person-level is under-estimated in M1.
- Subsequently, var. at the occ. level is over-estimated. (There is total amount of var. to be redistributed over the 2 levels)

⇓

To correct for that, we need include time into our model.
& It straightens out the variance components, and that's why we're using the model including time as our benchmark model for computing R^2.

Multilevel Analysis of GPA Data

Model	M5: time random		M6: + cross level interac	
Predictor	par est	SE	par est	SE
Intercept	2.55	0.02	2.57	0.03
Time	0.10	0.01	0.09	0.02
Job	-0.13	0.02	-0.13	0.02
High GPA	0.09	0.03	0.09	0.03
sex	0.12	0.03	0.08	0.03
Time*sex			0.03	0.01
Var_occ	0.042	0.002	0.042	0.002
Var_sub	0.038	0.006	0.038	0.006
Var_time	0.004	0.001 sig	0.004	0.001
Covar(sub*time)	-0.002	0.002	-0.002	0.001
Deviance	170.1		163.0	
AIC	188.1		183.0	

: GPA of female increase faster over time compared to males.

M5) random time: time to be diff. for each person
this is sig: meaning that persons significantly vary in their rate of change in GPA over time

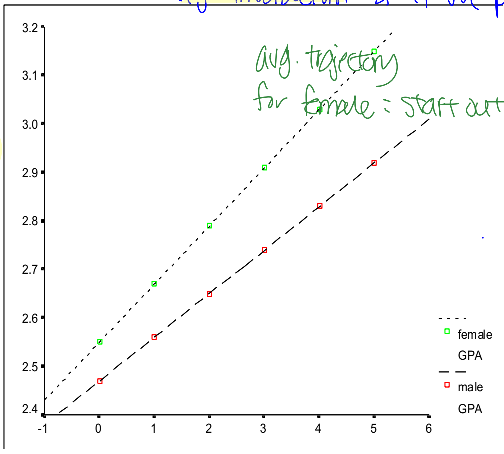
M6) add cross-level interaction, tryna explain that

⇒ So, in this dataset: not only that females start out w/ higher GPAs (0.08) at the 1st measurement occasion, but also they increase faster (0.03) over time. & This partly explains why we had diff. rate of change between students.

NOTE that we didn't find this sig. interaction w/ MANOVA, becuz we have more statistical power when we do ML analysis becuz we treat Time as a continuous variable instead of these categories, so we can actually discover this sig. interaction & if we plot this

Multilevel Analysis of GPA Data

- Note in the multilevel analysis, a significant interaction of time with *sex*
- Difference with Manova
  - Including *job status* in multi-level (> power)



avg. trajectory for female: start at higher & increase faster over time

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① 1st occasion = 0 : conventional way.

27 You can play around w/ scale of Time

"what I expect you to do: rescale the time such that 0 is included becuz then intercept has a meaningful interpretation it also helps interpreting cross-level interaction"

Intercept = avg. outcome at 1st occasion, in this case for boys. becuz boys are coded as 0, assuming that they're centered.

② last occasion = 0 → Intercept = predicted outcome at last measurement occasion (e.g. measurement 6 for boys)

DIY: what changes in these models?

Model	① 1st occasion = 0		② last occasion = 0		③ time centered	
Predictor	par est	SE	par est	SE	par est	SE
Intercept	2.55	0.02	2.96	0.03	2.70	0.02
Time	0.10	0.01	0.10	0.01	0.10	0.01
Job	-0.13	0.02	-0.13	0.02	-0.13	0.02
High GPA	0.09	0.03	0.09	0.03	0.09	0.03
sex	0.12	0.03	0.12	0.03	0.12	0.03
Var_occ	0.042	0.002	0.042	0.002	0.042	0.002
Var_sub	0.038	0.006	0.109	0.013	0.050	0.006
Var_time	0.004	0.001	0.004	0.001	0.004	0.001
Covar(sub*time)	-0.002	0.002	0.017	0.003	0.007	0.001
Corr(sub*time)	-0.21		0.82		0.51	
Deviance	170.1		170.1		170.1	
AIC	188.1		188.1		188.1	

→ Intercept = avg. outcome at right in the middle of our study (@ mean time). for boys.

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28 ⇒ These are all interesting options to use, conventional one is ① 1st occasion = 0

If you do ML analysis on longitudinal data, remember always to re-scale the time such that at least 0 is included.

※ Another reason that it's impo. is cuz the var. @ subject level also changes, on top of the intercept value. as a function of how we scale time

This happens becuz we have slope variance, which means that trajectory over time differs for each person. Then, how much variance there is between ppl in intercept depends on where the intercept is located

If look at the "var-sub", then the farther along we're in time, the more GPA scores diverge between persons.
→ This is also why it's impo. To think about how to rescale Time becuz it doesn't only connects to the intercept, but also to the int. variance. So it'd be nice if you can relate a mngful interpretation to this.

21/02/2022

Multilevel Analysis of GPA Data

Conclusions thus far:

- Initial GPA status differs across students
- Average increase in GPA of about 0.10 per semester (Time par: 0.09)
- Rate of change differs across students (∵ sig. slope variance for Time)
- Initial status of GPA predicted by *sex* and *high school GPA* (i.e. female has higher GPA & also higher Highschool GPA)
- *Job status* has a negative effect (par: −0.13)
- Rate of increase faster for females than for males (cross level int)

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slope var (var_Time) ~ interesting: students don't progress the same over time.
let's say sth. more about this using predictive interval

Multilevel Analysis of GPA Data

Calculation of predictive interval

- Average increase about 0.10 per semester
- *Rate of change* differs across students
  - Standard deviation of the slope variance of 0.06 (= $\sqrt{.004}$) implies that about 68% of students have a rate of change between 0.04 and 0.16 (estimate ± standard deviation)
    - slope variance
    - 1SD
    - → If we wanted 95%, then multiply use 1.96 x sd, then about [−0.02, 0.22]
  - The correlation between the initial status and the rate of change is −0.21 (if 1st occasion = 0)
    - So if you start out really high, you increase not as fast as when you start out quite low.

for small percentage of students actually decrease in their GPA over time, instead of increase.
→ This would be super interesting that actually not everyone increases over time but there're also ppl who are predicted to decrease over time.

Emmeke Aarts | Multilevel analysis - lecture 2

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Coffee break

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Example II

Children's Reading Skills

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Data set

- Reading Skill measured at four occasions
- 405 children, a few missings & some dropout
- Fixed occasions if we consider measurement occasion
- Varying occasions if we consider the children's age as the relevant *metric of time*! would be the right operationalization of time
  - Age varies from 6-8 at the first measurement occasion
  - Measured in months: 25 different values

From theoretical perspective, it makes more sense to predict reading skills based on how old kids are, rather than when we decide to measure.

Data set made by Patrick Curran for a workshop on comparative analysis

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Varying Measurement Occasions

Individuals measured at different time points

If we use ML analysis this doesn't matter at all!

- Often not regularly spaced
- Dropout occurs
- Intermittently missing observations possible
- Examples:
  - growth studies,
  - diary studies (experience sampling, intensive longitudinal data)
  - accelerated longitudinal design
- Analysis methods: multilevel regression most flexible method !!

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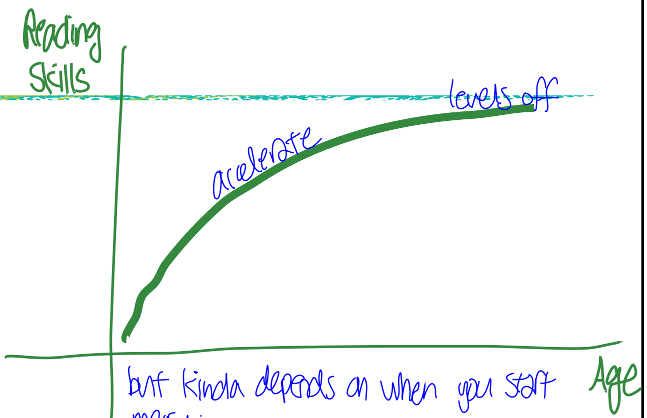
DIY: Overall Trend

- Do you think the relationship between age and reading ability will be linear? Why? No, not linear.
- If not, how can you accommodate this in the model?



but kinda depends on when you start measuring them.

⇒ we don't expect linear relationship becuz at some point, it will level off & not continuously grow.

add a quadratic term:

"non-linear"

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Description of variables

- Variables – occasion level:
  - Age (centered on first age in data: KidAge-6) : our (Time) predictor
  - Age squared : to take care of this non-linear relationship → "If we expect a quadratic term, it's impo. to include *both the normal version of time (AGE, in this case) & the squared version of time.
    So that, we know what the rate of change is, over time in general. Then, we add this curvature to the line as well.
- Variables – child level
  - kid gender
  - home cognitive stimulation
  - home emotional support
  - mother's age

fixed predictors "time-invariant"

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M1) Only include Time
Reading Score = 1.74 @ 1st measurement occasion
Time = 0.93 : As kids get older, RS increases!
$Age^2 = -.05$: negative quadratic term → As time increases, the growth of RS becomes less & less! Indeed we level off.

M2) Add level 2 predictors
σ_e^2 : diff. in RS over time within the same kids remain the same → we don't explain any extra.
σ_{u0}^2 : we do explain some of the diff. in average RS

21/02/2022

M3) Add slope var. for AGE — σ_{u1}^2 which is significant meaning that the rate of change in RS is diff. over children.

M4) Add cross-level interaction trying to explain this.

Results

Model	M1	M2	M3	M4
Fixed part				
Predictor	coeff. (s.e.)	coeff. (s.e.)	coeff. (s.e.)	coeff. (s.e.)
Intercept	1.74 (.06)	0.71 (.48)	0.71 (.48)	1.06 (.49)
Child age	0.93 (.03)	0.92 (.03)	0.92 (.03)	0.49 (.14)
Child age sq	-.05 (.003)	-.05 (.003)	-.05 (.003)	-.05 (.003)
Mother age		0.05 (.02)	0.03 (.02)^{ns}	0.02 (.02)^{ns}
Cogn. Stim.		0.05 (.02)	0.04 (.01)	0.04 (.01)
Emot. support		0.04 (.02)	0.003 (.02)^{ns}	-.01 (.02)^{ns}
Age*Momage				0.01 (.005)
Age*Emot				0.01 (.004)
Random part				
σ_e^2	0.39 (.02)	0.39 (.02)	0.27 (.02)	0.28 (.02)
σ_{u0}^2	0.66 (.06)	0.60 (.04)	0.21 (.04)	0.21 (.04)
σ_{u1}^2			0.02 (.003)	0.01 (.003)
σ_{u01}			0.04 (.001)	0.04 (.001)
r_{u01}				0.64
Deviance	3245.0	3216.2	3015.4	2995.3
AIC	3255.0	3232.2	3035.4	3019.3

remove these two insig. predictors

Becuz we have these cross-level interaction between AGE * Momage, AGE * Emot, we still need include these into the model, Despite their main effects are not sig!

This is the same as OLS; If we include an interaction, then both the main effects of the interaction term need to be present in the model.
That's why these 2 are still included.

100%. If we have a slope variance, and then we proceed to try to explain the slope variance.
Then you shouldn't forget to see if the slope variance can be explained by level 2 predictors that you kicked out of the model becuz they were insignificant!
Even tho they may not explain differences in average, they might be able to explain why we have a diff. rate of growth between the children!!

Summary

- Longitudinal data can be seen as measurements nested in persons
- Multilevel analysis provides a very flexible method to analyze longitudinal data. Here:
  - We use π at the lowest level instead of γ in the notation
  - The baseline model + time should be used as benchmark model for calculating R^2