Mathematica Module for ARAP Interpolation

MARAP User's Manual
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1 Introduction

This is a Mathematica Module for investigating an interpolation in Computer Graphics, called the ARAP (as rigid as possible) interpolation. Our module contains about 100 functions of lementary matrix operations, matrix and polygon interpolatins and drawing polygons.

To use this package "ARAPlibv024‘" and sample data "ARAPdatav021‘" users should set a directory where the modules is stored.

```
[Example]

SetDirectory[FileNameJoin[$HomeDirectory, "--- Some Folder ---"]];
<< ARAPlibv024‘;
<< ARAPdata021‘;
```

This module was used and introduced in the followings:

[1] S.Kaji, S.Hirose, S.Sakata, Y.Mizoguchi, Mathematical analysis on affine maps for 2D shape interpolagion (`http://dl.acm.org/citation.cfm?id=2422368`), SCA ’12 Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation, pp.71-76.

[2] Y. Mizoguchi, Mathematical Aspects of Interpolation Technique for Computer Graphics, Forum “Math-for-Industry” 2012, Information Recovery and Discovery, 22 October 2022. `http://fmi2012.imi.kyushu-u.ac.jp/`

[3] T.Hirano, A.hirakawa, N.Miyaki, C.Matsufuji and Y.Mizoguchi. Mathematica Module for ARAP Interpolation
`https://github.com/KyushuUniversityMathematics/MathematicaARAP`

2 Basic Functions

2.1 `MakePolygon`

`MakePolygon[`*V*`,`*tindex*`]`

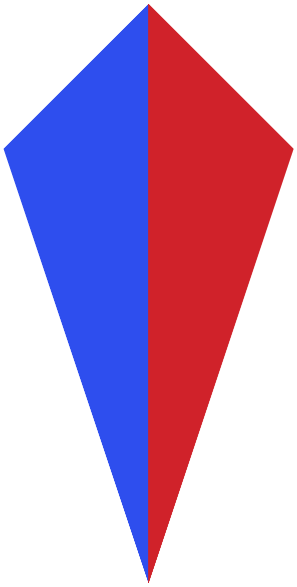
:: Make polygon

V vertex set

tindex constitution of triangle

return polygon

```
[Example]
Graphics[MakePolygon[dataS2,dataT2]]
```



2.2 `HorizontalSnake`

`HorizontalSnake[`*n*`]`

:: Make vertexes of horizontally long rectangle

n rectangle size

return vertex set of rectangle

```
[Example]
HorizontalSnake[5]
={{0, 0}, {0, 1}, {1, 0}, {1, 1}, {2, 0}, {2, 1}, {3, 0}, {3, 1}, {4, 0}, {4, 1}}
```

2.3 VerticalSnake

`VerticalSnake[n]`
:: Make vertexes of vertically long rectangle

n rectangle size

return vertex set

```
[Example]
VerticalSnake[5]
={{1, 0}, {0, 0}, {1, 1}, {0, 1}, {1, 2}, {0, 2}, {1, 3}, {0, 3}, {1, 4}, {0, 4}}
```

2.4 SnakeTriangle

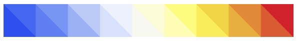
`SnakeTriangle[n]`
:: Triangulate Horizontal/Vertex Snake

n rectangle size

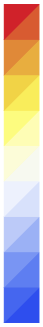
return constitution of triangle

```
[Example]

Graphics[MakePolygon[HorizontalSnake[10], SnakeTriangle[10]]]
```



```
Graphics[MakePolygon[VerticalSnake[10], SnakeTriangle[10]]]
```



2.5 AnimationRange

`AnimationRange[conf]`
:: size of Display conforming with polygon

conf { start vertex set , end vertex set}

return coordinates of display

```
[Example]

AnimationRange[Configuration2]
={{-2, 4}, {-3, 5}}
```

2.6 AnimationRange2

`AnimationRange2[`*range1*`,`*range2*`]`
:: size of Display

range1

range2

return coordinates of display

```
[Example]

AnimationRange[Configuration2]
={{-2, 4}, {-3, 5}}
```

2.7 Polar

`Polar[a]` :: Convert cartesian coordinates to polar coordinates

a cartesian coordinates

return polar coordinates

```
[Example]
Polar[{0,1}]
={1,2π}
```

2.8 Cartesian

`Cartesian[a]`
:: Convert polar coordinates to cartesian coordinates

a polar coordinates

return cartesian coordinate

```
[Example]
Cartesian[{1,2π}]
={0,1}
```

2.9 LinearInterpolate

`LinearInterpolate[a,b,t]`
:: Perform linear interpolation of point

a a start point

b an end point

t time

return an interpolated point

```
[Example]
LinearInterpolate[{1, 1}, {2, 2}, 0.5]
={1.5, 1.5}
```

2.10 LinearInterpolation

LinearInterpolation[*p*,*q*,*t*]
:: Perform linear interpolation of vertex set

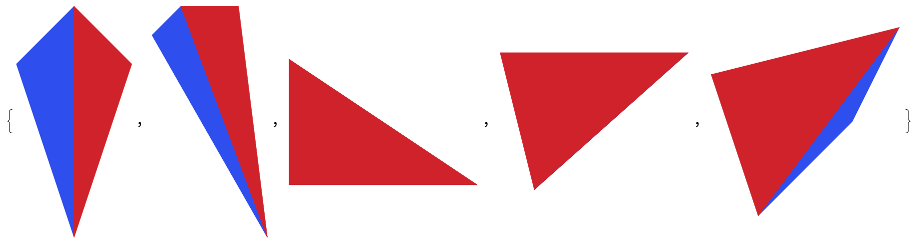
p start vertex set

q end vertex set

t time

return interpolated vertex set

```
[Example]
Graphics[MakePolygon[LinearInterpolation[dataS2, dataE2, #], dataT2]] & /
{0, 0.25, 0.5, 0.75, 1}
```



2.11 PolarInterpolate

PolarInterpolate[a,*b*,*t*]
:: Perform Polar interpolation of point

a a start point

b an end point

t time

return an interpolated point

```
[Example]
PolarInterpolate[{1, 1}, {2, 2}, 0.5]
={1.5, 1.5}
```

2.12 PolarInterpolation

PolarInterpolation[*p*,*q*,*t*]
:: Perform Polar interpolation of vertex set

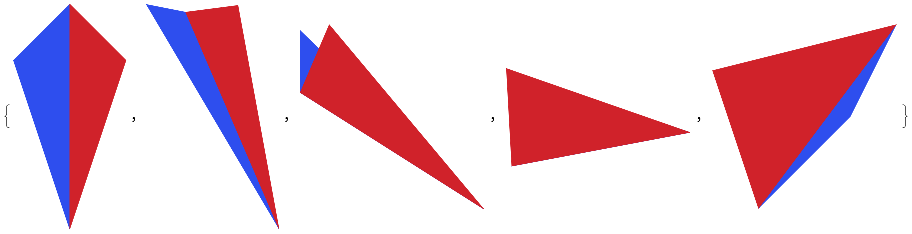
p start vertex set

q end vertex set

t time

return interpolated vertex set

```
[Example]
Graphics[MakePolygon[PolarInterpolation[dataS2, dataE2, #], dataT2]] & /
{0, 0.25, 0.5, 0.75, 1}
```



2.13 LinearInterpolateSnake2D

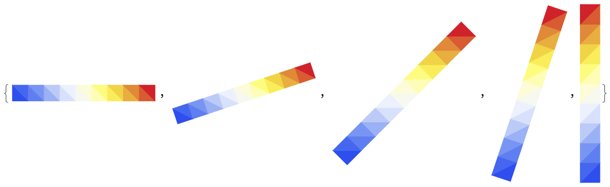
LinearInterpolateSnake2D[*n*,*t*]
:: aiueo

n Snake size

t time (0˜1)

return interpolated snake rectangle

```
[Example]
Graphics[LinearInterpolateSnake2D[10, #]] & / {0, 0.25, 0.5, 0.75, 1}
```



2.14 PolarInterpolateSnake2D

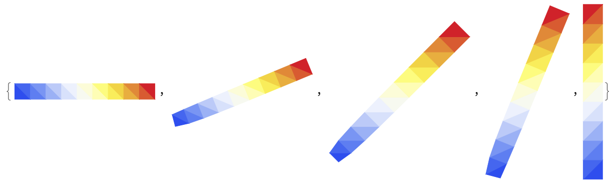
PolarInterpolateSnake2D[*n*,*t*]
:: aiueo

n Snake size

t time (0˜1)

return interpolated snake rectangle

```
[Example]
Graphics[PolarInterpolateSnake2D[10, #]] & / {0, 0.25, 0.5, 0.75, 1}
```



2.15 PolarDecomposition

`PolarDecomposition[`*m*`]`

:: Perform polar decomposition

m matrix

return orthogonal matrix and positive-semidefinite matrix

```
[Example]
PolarDecomposition[{{1, 1}, {-1, -1}}]
={{{0., 1.}, {-1., 0.}}, {{1., 1.}, {1., 1.}}}
```

2.16 PolarDecompositionPlus

`PolarDecompositionPlus[`*m*`]`

:: aiueo

m aiueo

return aiueo

```
[Example]
PolarDecompositionPlus[m_]
```

2.17 RotateAngle

`RotateAngle[`*m*`]`

:: Compute angle of Rotation matrix that is performed polar decomposition

m matrix

return angle

```
[Example]
RotateAngle[{{1, 2}, {3, 4}}]
=ArcTan[3]
```

2.18 CogTrans

`CogTrans[pl]`
:: move to triangles (trigangles center og gravity(0,0))

pl list of a coordinate triangle

return list of a coordinate triangle

```
[Example]
CogTrans[{{0,3},{3,3},{0,-3}}]
={{-1, 2}, {2, 2}, {-1, -4}}
```

2.19 PolygonToTriangles

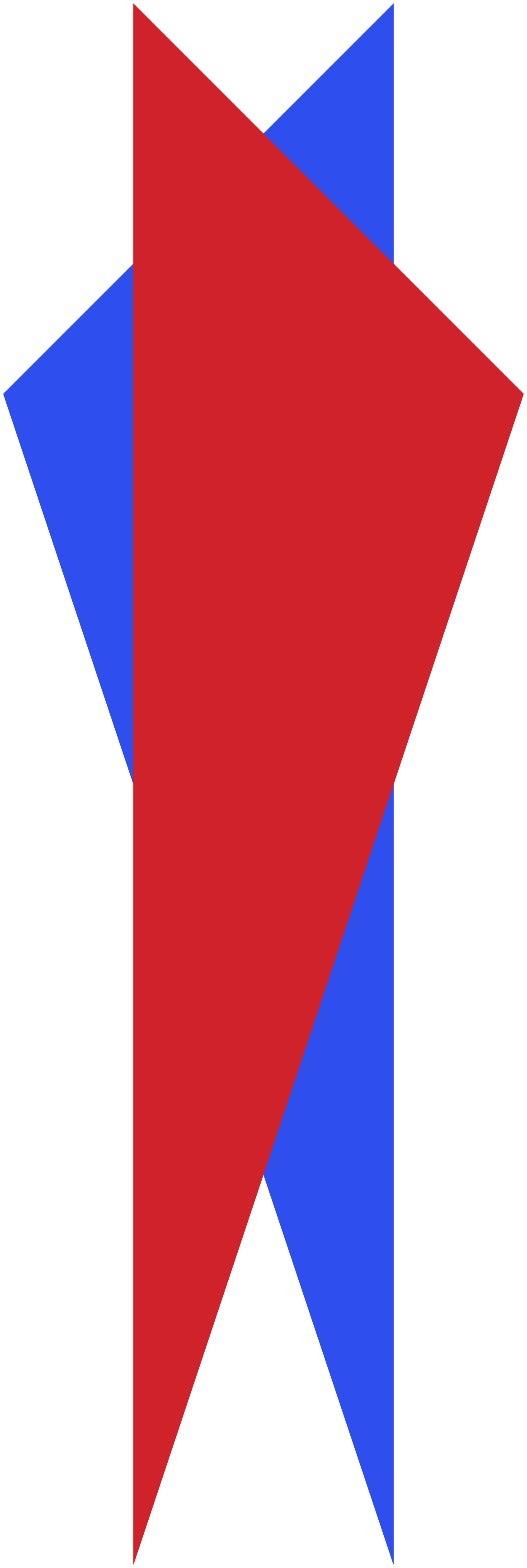
`PolygonToTriangles[l]`
:: Set polygon to vertex set of triangle(Center of gravity(0,0))

l Polygon

return vertex of triangles

```
[Example]
```

```
Graphics[TrianglesToPolygon[PolygonToTriangles[MakePolygon[dataS2, dataT2]]]]
```



2.20 TrianglesToTriangles

`TrianglesToTriangles[`*`offset`*`,`*`cog`*`]`

:: *cog* run vertexes in a parallel direction

offset vertex set

cog coordinate

return vertex set

```
[Example]
TrianglesToTriangles[{{{-1, 1}, {0, -2}, {0, 2}, {1, 1}}, {{1, 1}}}, {{1, -1}, {0, 1}}
={{{0, 0}, {1, -3}, {1, 1}, {2, 0}}, {{1, 2}}}
```

2.21 TrianglesToPolygon

`TrianglesToPolygon[`*`tl`*`]`

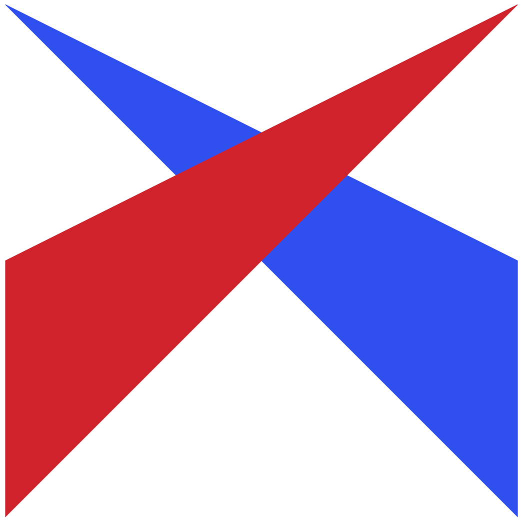
:: Set vertex set of triangle to polygon

tl list of triangle vertex set

return polygon

```
[Example]
Graphics[TrianglesToPolygon[1, 1, 1, 2, -1, 3, -1, 1, -1,2, 1, 3]]
```

Out[102]=



2.22 ValNames

`ValNames[`*`n`*`]`

:: Name variable

n number of variable

return neme of variable

```
[Example]
ValNames[3]
={{v1x, v1y}, {v2x, v2y}, {v3x, v3y}}
```

2.23 NormF

`NormF[`*m*`]` :: Calculate Frobenius norm

m matrix

return resultant value of Frobenius norm

```
[Example]
NormF[1,2,3,4]
=30
```

3 Matrix Functions

3.1 QuadraticFormVariableMatrix

`QuadraticFormVariableMatrix[vl]`
:: QuadraticFormVariableMatrix

vl list of values

return matrix

```
[Example]
QuadraticFormVariableMatrix[{1,2,3}]
{{1, 2, 3}, {2, 4, 6}, {3, 6, 9}}
```

3.2 Div2if

`Div2if[n,l]`
:: divide all elements except n-th element of l by 2

n,l index,list of values

return vector divided all elements except n-th element of l by 2

```
[Example]
Div2if[3, {3, 6, 7, 5, 1}]
{3/2, 3, 7, 5/2, 1/2}
```

3.3 Div2Matrix

`Div2Matrix[m]`
:: divide all elements except diagonal ones of m by 2

m matrix

return matrix divided all elements except diagonal ones of m by 2

```
[Example]
Div2Matrix[{{1, 3}, {7, 4}}]
{{1, 3/2}, {7/2, 4}}
```

3.4 QuadraticFormMatrix

`QuadraticFormMatrix[poly,vl]`
:: QuadraticFormMatrix

poly,vl polynomial,list of values

return QuadraticFormMatrix

```
[Example]
QuadraticFormMatrix[(a*x^2 + b*x + c-y)^2, {a, b, c}]
{{x^4, x^3, x^2}, {x^3, x^2, x}, {x^2, x, 1}}
```

3.5 LinearFormVector

LinearFormVector[*poly*,*vl*]
:: LinearFormVector

poly,*vl* polynominal,list of values

return LinearFormVector

```
[Example]
LinearFormVector[(a*x^2 + b*x + c - y)^2, {a, b, c}]
{-2 x^2 y, -2 x y, -2 y}
```

3.6 VtoTriangle

VtoTriangle[*V*,*t*]
:: return coordinates of triangle

V,*t* list of vertices,list of triangulation

return list of coordinates of vertices of triangle

```
[Example]
VtoTriangle[{{-1, 1}, {0, -2}, {1, 1}, {0, 2}}, {1, 2, 4}]
{{-1, 1}, {0, -2}, {0, 2}}
```

3.7 VtoTriangles

VtoTriangles[*V*,*T*]
:: return coordinates of triangles

V,*T* list of vertices,list of triangulation

return list of coordinates of vertices of triangles

```
[Example]
VtoTriangles[{{-1, 1}, {0, -2}, {1, 1}, {0, 2}}, {{1, 2, 3}, {1, 2, 4}, {2, 3, 4}}]
{{{-1, 1}, {0, -2}, {1, 1}}, {{-1, 1}, {0, -2}, {0, 2}}, {{0, -2}, {1,1}, {0, 2}}}
```

3.8 Cog

Cog[*P*] :: return triangle center

P triangle

return triangle center of P

```
[Example]
Cog[{{-1, 1}, {0, -2}, {0, 2}}]
{-(1/3), 1/3}
```

3.9 Trans

`Trans[`*P*`,`*l*`]`

:: parallel shift by l

P,l triangle,vector

return triangle

```
[Example]
Trans[{{1, 2}, {5, -3}, {-4, 1}}, {5, 6}]
{{-4, -4}, {0, -9}, {-9, -5}}
```

3.10 FindMatrix

`FindMatrix[`*P1*`,`*P2*`]`

:: find matrix converts P1 to P2

P1,P2 triangles whose center is origin

return matrix convert P1 to P2

```
[Example]
P11 = {{0, 2}, {-3, -1}, {3, -1}};
P21 = {{-4, 3}, {1, -2}, {3, -1}};
FindMatrix[P11, P21]
{{1/3, -2}, {1/6, 3/2}}
```

3.11 FindMatrix1

`FindMatrix1[`*P1*`,`*P2*`]`

:: find matrix converts P1 to P2

P1,P2 triangles whose center is origin

return matrix convert P1 to P2

```
[Example]
P11 = {{0, 2}, {-3, -1}, {3, -1}};
P21 = {{-4, 3}, {1, -2}, {3, -1}};
FindMatrix1[P11, P21]
{{1/3, -2}, {1/6, 3/2}}
```

3.12 FindMatrices

`FindMatrices[`*V1*`,`*V2*`,`*T*`]`

:: find matrices converts each triangles represented by V1 and T to ones represented by V2 and T

V1,V2,T V1,V2:list of vertices T:list of triangulation

return matrices

```
[Example]
V1 = {{-1, 1}, {0, 2}, {1, -3}, {4, -5}};
```

```
V2 = {{-3, 3}, {-2, 5}, {2, 1}, {3, 1}};
T = {{1, 2, 3}, {1, 2, 4}, {2, 3, 4}};
FindMatrices[V1, V2, T]
{{{3/2, -(1/2)},{1, 1}},
 {{12/11, -(1/11)}, {10/11, 12/11}},
 {{-(3/13), -(11/13)},{8/13, 12/13}}}
```

3.13 FindAffineMatrix

`FindAffineMatrix[`*`P1`*`,`*`P2`*`]`
:: find affine matrix converts P1 to P2

P1,P2 triangles

return matrix converts P1 to P2

```
[Example]
FindAffineMatrix[{{-1, 1}, {0, -2}, {0, 2}}, {{-4, 3}, {1, -2}, {3, 0}}]
```

3.14 FindAffineMatrices

`FindAffineMatrices[`*`V1`*`,`*`V2`*`,`*`T`*`]`
:: find affine matrix convert each triangles

V1,V2,T V1,V2:list of vertices T:list of triangulation

return matrices

```
[Example]
V1 = {{-1, 1}, {0, 2}, {1, -3}, {4, -5}};
V2 = {{-3, 3}, {-2, 5}, {2, 1}, {3, 1}};
T = {{1, 2, 3}, {1, 2, 4}, {2, 3, 4}};
FindAffineMatrices[V1, V2, T]
{{{3/2, -(1/2), -1}, {1, 1, 3}, {0, 0, 1}},
{{12/11, -(1/11), -(20/11)}, {10/11, 12/11, 31/11}, {0, 0, 1}},
{{-(3/13), -(11/13), -(4/13)}, {8/13, 12/13, 41/13}, {0, 0, 1}}}
```

3.15 F1a

`F1a[{{`*`a1x`*`,`*`a1y`*`},{`*`b1x`*`,`*`b1y`*`},{`*`c1x`*`,`*`c1y`*`}},{{`*`m11`*`,`*`m12`*`},{`*`m21`*`,`*`m22`*`}}]`
:: compute quadratic form matrix

{{a1x,a1y},{b1x,b1y},{c1x,c1y}},{{m11,m12},{m21,m22}}
triangle,matrix

return quadratic form matrix

```
[Example]
F1a[{{-1, 0}, {1, 1}, {2, -3}}, {{1, 3}, {4, 2}}]
{{17/81, -(5/27), -(2/81)},
 {-(5/27), 2/9, -(1/27)},
 {-(2/81), -(1/27), 5/81}}
```

3.16 F2a

`F1a[{{`*`a1x`*`,`*`a1y`*`},{`*`b1x`*`,`*`b1y`*`},{`*`c1x`*`,`*`c1y`*`}},{{`*`m11`*`,`*`m12`*`},{`*`m21`*`,`*`m22`*`}}]`
:: compute quadratic form matrix

{{a1x,a1y},{b1x,b1y},{c1x,c1y}},{{m11,m12},{m21,m22}}
triangle,matrix

return quadratic form matrix

```
[Example]
F2a[{{-1, 0}, {1, 1}, {2, -3}}, {{1, 3}, {4, 2}}]
{{137/2430, 0, -(7/405), -(1/27), -(19/486), 1/27},
 {0, 137/2430, 1/27, -(7/405), -(1/27), -(19/486)},
 {-(7/405), 1/27, 4/135, 0, -(1/81), -(1/27)},
 {-(1/27), -(7/405), 0, 4/135, 1/27, -(1/81)},
 {-(19/486), -(1/27), -(1/81), 1/27, 25/486, 0},
 {1/27, -(19/486), -(1/27), -(1/81), 0, 25/486}}
```

3.17 EmbedMatrix

`EmbedMatrix[`*`n`*`,`*`i`*`,`*`j`*`,`*`M`*`]`
:: embed 2-degree matrix M in n-degree 0 matrix

n,i,j,M degree,index,index,matrix

return matrix

```
[Example]
EmbedMatrix[6, 2, 4, {{1, 2}, {3, 4}}]
{{0, 0, 0, 0, 0, 0},
 {0, 1, 0, 2, 0, 0},
 {0, 0, 0, 0, 0, 0},
 {0, 3, 0, 4, 0, 0},
 {0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0}}
```

3.18 EmbedMatrix

`EmbedMatrix[`*`n`*`,`*`i`*`,`*`j`*`,`*`k`*`,`*`M`*`]`
:: embed 2-degree matrix M in n-degree 0 matrix

n,i,j,k,M degree,index,index,index,matrix

return matrix

```
[Example]
EmbedMatrix[8, 2, 4, 7, {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
{{0, 0, 0, 0, 0, 0, 0, 0},
 {0, 1, 0, 2, 0, 0, 3, 0},
 {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 4, 0, 5, 0, 0, 6, 0},
 {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 7, 0, 8, 0, 0, 9, 0},
 {0, 0, 0, 0, 0, 0, 0, 0}}
```

3.19 EmbedMatrix2

`EmbedMatrix2[`*n*`,`*i*`,`*j*`,`*k*`,`*M*`]`

:: embed 6-degree matrix M in 2n-degree 0 matrix

n,i,j,k,M n:size, i,j,k:index M:matrix

return matrix

```
[Example]
A = EmbedMatrix[6, 1, 3, 5, {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}] +
  EmbedMatrix[6, 2, 4, 6, {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]

  {{1, 0, 2, 0, 3, 0},
   {0, 1, 0, 2, 0, 3},
   {4, 0, 5, 0, 6, 0},
   {0, 4, 0, 5, 0, 6},
   {7, 0, 8, 0, 9, 0},
   {0, 7, 0, 8, 0, 9}}

EmbedMatrix2[4, 1, 3, 4, A]

{{1, 0, 0, 0, 2, 0, 3, 0},
 {0, 1, 0, 0, 0, 2, 0, 3},
 {0, 0, 0, 0, 0, 0, 0, 0},
 {0, 0, 0, 0, 0, 0, 0, 0},
 {4, 0, 0, 0, 5, 0, 6, 0},
 {0, 4, 0, 0, 0, 5, 0, 6},
 {7, 0, 0, 0, 8, 0, 9, 0},
 {0, 7, 0, 0, 0, 8, 0, 9}}
```

3.20 F1v

`F1v[{{`*a1x*`,`*a1y*`},{`*b1x*`,`*b1y*`},{`*c1x*`,`*c1y*`}},{{`*m11*`,`*m12*`},{`*m21*`,`*m22*`}}]`

:: compute linear form vector

{{a1x,a1y},{b1x,b1y},{c1x,c1y}},{{m11,m12},{m21,m22}}

triangle,matrix

return linear form vector

```
[Example]
F1v[{{-1, 0}, {1, 1}, {2, -3}}, {{1, 3}, {4, 2}}]
{14/9, 4, -(8/3), -4, 10/9, 0}
```

3.21 `EmbedVector`

`EmbedVector[`*n*`,`*i*`,`*j*`,`*k*`,`*V*`]`
:: embed vector V in 2n-degree 0 vector

n,i,j,k,V n:size i,j,k:index V:vector

return vector

```
[Example]
EmbedVector[5, 1, 3, 4, {-6, 7, 28, 19, -4, 31}]
{-6, 7, 0, 0, 28, 19, -4, 31, 0, 0}
```

4 Local Interpolations

4.1 RotateAngle

`RotateAngle[m_,flag_]`
::controlled rotate angle of m by flag

m matrix

flag 1 or 0 or -1, control the rotate angle

return angle

4.2 NewFindMatrices

`NewFindMatrices[conf_]`
::find matrices converted each triangles

conf configuration

return matrices

```
[Example]
NewFindMatrices[Configuration2]=
{{{-(5/4), -(3/4)}, {-1, -1}}, {{-(7/4), -(3/4)}, {2, -1}}}
```

4.3 LocalLinear

`LocalLinear[m_]`
::a local linear interpolations depend on time

m matrices computed by NewFindMatrices[*conf_*]

return matrices of local linear interpolation

```
[Example]
LocalLiner[NewFindMatrices[Configuration2]][0.5]=
{{{-0.125, 0.125}, {-0.5, -0.5}}, {{-0.875, -0.375}, {1.5, 0.}}}
```

4.4 LocalAlexa

`LocalAlexa[m_]`
::a local ARAP interpolations depend on time

m matrices computed by NewFindMatrices[*conf_*]

return matrices of local alexa interpolation

4.5 LocalLogExp

`LocalLogExp[m_]`
::a local log-exp interpolations depend on time

m matrices computed by NewFindMatrices[*conf_*]

return matrices of local log-exp interpolation

4.6 LocalInterpolations

LocalInterpolations[*local_*,*conf_*]
::local interpolations that you choice

local LocalLinear/LocalPolar/LocalAlexa/LocalLogExp

conf configuration

return

```
[Example]
LocalInterpolations[LocalPolar,Configuration2][t]=
{{{(1 + 0.352786 t) Cos[3.03094 t] + 0.855844 t Sin[3.03094 t],
0.855844 t Cos[3.03094 t] + (1 - 0.0889399 t) Sin[3.03094 t]},
{0.855844 t Cos[3.03094 t] - 1. (1 + 0.352786 t) Sin[3.03094 t],
(1 - 0.0889399 t) Cos[3.03094 t] - 0.855844 t Sin[3.03094 t]}},
{{(1 + 1.65165 t) Cos[2.35619 t] + 0.176777 t Sin[2.35619 t],
-0.176777 t Cos[2.35619 t] - 1. (1 + 0.237437 t) Sin[2.35619 t]},
{-0.176777 t Cos[2.35619 t] + (1 + 1.65165 t) Sin[2.35619 t],
(1 + 0.237437 t) Cos[2.35619 t] - 0.176777 t Sin[2.35619 t]}}}
```

5 Constraint Functions

5.1 ConstMatrix

`ConstMatrix[m_,st_]`

::a matrix part of a constraint function of a specific vertex you choice

m vertex

st list of start coordinates

return matrix

5.2 ConstMatrixM

`ConstMatrixM[st_,en_,tri_,t_]`

::a matrix part of a constraint function of barycenter

st list of start coordinates

en list of end coordinates

tri list of triangulation

t parameter of time(0...1)

return matrix

```
[Example]
ConstMatrixM[dataS2,dataE2,dataT2,0.5]=
{{1/16, 0, 1/16, 0, 1/16, 0, 1/16, 0},
{0, 1/16, 0, 1/16, 0, 1/16, 0, 1/16},
{1/16, 0, 1/16, 0, 1/16, 0, 1/16, 0},
{0, 1/16, 0, 1/16, 0, 1/16, 0, 1/16},
{1/16, 0, 1/16, 0, 1/16, 0, 1/16, 0},
{0, 1/16, 0, 1/16, 0, 1/16, 0, 1/16},
{1/16, 0, 1/16, 0, 1/16, 0, 1/16, 0},
{0, 1/16, 0, 1/16, 0, 1/16, 0, 1/16}}
```

5.3 ConstVectorM

`ConstVectorM[st_,en_,tri_,t_]`

::a vector part of a constraint function of barycenter

st list of start coordinates

en list of end coordinates

tri list of triangulation

t parameter of time(0...1)

return vector

```
[Example]
ConstVectorM[dataS2,dataE2,dataT2,0.5]=
{-0.0625, -0.171875, -0.0625, -0.171875, -0.0625, -0.171875, -0.0625, -0.171875}
```

5.4 ConstfixMatrix

`ConstfixMatrix[`*`n_`*`,`*`k_`*`,`*`l_`*`,`*`st_`*`]`
::a matrix part of a constraint function fixing user - specified vector

n weight of constraint function

k,l Choice two numbers you want to fix.

st list of start coordinates

return matrix

```
[Example]
ConstfixMatrix[2,1,2,dataS2]=
{{2, 0, -2, 0, 0, 0, 0, 0},
{0, 2, 0, -2, 0, 0, 0, 0},
{-2, 0, 2, 0, 0, 0, 0, 0},
{0, -2, 0, 2, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}}
```

5.5 ConstfixVector

`ConstfixVector[`*`n_`*`,`*`k_`*`,`*`l_`*`,`*`st_`*`]`
::a vector part of a constraint function fixing user - specified vector

n weight of constraint function

k,l Choice two numbers you want to fix.

st list of start coordinates

return vector

```
[Example]
ConstfixVector[2,1,2,dataS2]={2, -6, -2, 6, 0, 0, 0, 0}
```

5.6 ConstPair

`ConstPair[`*`m_`*`]`
::a pair of matrix and vector of a constraint function of a specific vertex you choice

m choice of vertex

return {matrix,vector}

```
[Example]
ConstPair[1][Configuration,s]=
{{{1, 0, 0, 0, 0, 0, 0, 0},
{0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}},
{-2 (-1 + 3 s), -2 (1 + s), 0, 0, 0, 0, 0, 0}}
```

5.7 ConstPair

`ConstPair[`*m*`_,`*n*`_]`

::a pair of matrix and vector of a constraint function of a specific two vertices you choice

m,*n* choice of vertex

return {matrix,vector}

```
[Example]
ConstPair[1,2][Configuration2,s]=
{{{1, 0, 0, 0, 0, 0, 0, 0},
{0, 1, 0, 0, 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0, 0, 0},
{0, 0, 0, 1, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0}},
{-2 (-1 + 3 s), -2 (1 + s), -6 s, -2 (-2 (1 - s) + 4 s), 0, 0, 0, 0}}
```

5.8 DoubleMatrix

`DoubleMatrix[`*m*`_]`

::Return matrix appearing elements and 0 in turn.

m matrix

return matrix

```
[Example]
DoubleMatrix[{{1, 2}, {3, 4}}]=
{{1, 0, 2, 0}, {0, 1, 0, 2}, {3, 0, 4, 0}, {0, 3, 0, 4}}
```

6 Grobal Interpolations

6.1 QuadraticFormAlexa

QuadraticFormAlexa[*local_*,*conf_*]

::a error function using Alexa's method

local	choice of local interpolation
conf	configuration
return	{matrix,vector}

6.2 QuadraticFormSim

QuadraticFormSim[*local_*,*conf_*]

::a error function using similarity preserving method

local	choice of local interpolation
conf	configuration
return	{matrix,vector}

6.3 ARAP

ARAP[*local_*,*energy_*,*const_*,*conf_*]

::a global error function in matrix form

local	choice of local interpolation
energy	choice of energy function
const	choice of constraint function
conf	configuration
return	{matrix,vector}

```
[Example]
ARAP[LocalPolar,QuadraticFormAlexa,ConstPair[1],Configuration2][1]=
{{{2, 0, -(1/4), 0, 0, 0, -(3/4), 0},
{0, 2, 0, -(1/4), 0, 0, 0, -(3/4)},
{-(1/4), 0, 1/4, 0, -(1/4), 0, 1/4, 0},
{0, -(1/4), 0, 1/4, 0, -(1/4), 0, 1/4},
{0, 0, -(1/4), 0, 1, 0, -(3/4), 0},
{0, 0, 0, -(1/4), 0, 1, 0, -(3/4)},
{-(3/4), 0, 1/4, 0, -(3/4), 0, 5/4, 0},
{0, -(3/4), 0, 1/4, 0, -(3/4), 0, 5/4}},
{-6.5, -6., -1., 0.5, 3.5, -4., 8.88178*10^-16, 5.5}}
```

7 Draw Animation

7.1 ShowStatus

`ShowStatus[`*`st`*`,`*`en`*`,`*`tri`*`,`*`plotrange`*`]`
:: Draw start and end of Graphic

st start state of vertex set

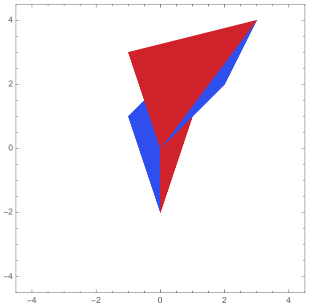
en end state of vertex set

tri constitution of triangle

plotrange Display range

return graph of start and end status

```
[Example]
ShowStatus[dataS2, dataE2, dataT2, 4.5]
```



7.2 DrawAnimation

`DrawAnimation[`*`local`*`,`*`energy`*`,`*`const`*`,`*`conf`*`]`
:: aiueo

local Choice of local interpolation

energy Choice of energy interpolation

const Choice of constraint interpolation

conf {start vertex set,end vertex set,constitution of triangle}

return animation

```
[Example]
DrawAnimation[LocalAlexa, FrobeniusEnergy, Const2, Configuration2]
```

7.3 ListAnimation

`ListAnimation[`*`k`*`,`*`local`*`,`*`energy`*`,`*`const`*`,`*`conf`*`]`

:: aiueo

k number of frame division

local Choice of local interpolation

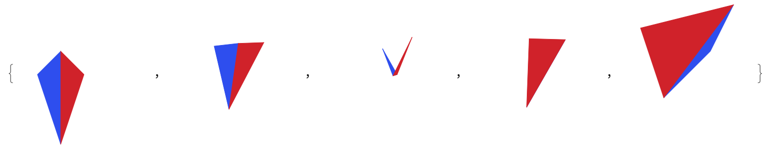
energy Choice of energy interpolation

const Choice of constraint interpolation

conf {start vertex set,end vertex set,constitution of triangle}

return list of graph

```
[Example]
ListAnimation[k_,n_,c_,e_,st_,en_,tri_,plotrange]
```



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