# Mathematica Module for ARAP Interpolation

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## Table of Contents

1	In	${f troduction} \ldots 1$
2	$\mathbf{B}$	${f asic}$ Functions
	2.1	MakePolygon
	2.2	HorizontalSnake 2
	2.3	VerticalSnake
	2.4	SnakeTriangle
	2.5	AnimationRange
	2.6	AnimationRange24
	2.7	Polar 4
	2.8	Cartesian 4
	2.9	LinearInterpolate 4
	2.10	LinearInterpolation5
	2.11	PolarInterpolate
	2.12	PolarInterpolation6
	2.13	LinearInterpolateSnake2D6
	2.14	PolarInterpolateSnake2D6
	2.15	PolarDecomposition7
	2.16	PolarDecompositionPlus7
	2.17	RotateAngle7
	2.18	CogTrans
	2.19	PolygonToTriangles8
	2.20	TrianglesToTriangles
	2.21	TrianglesToPolygon10
	2.22	ValNames 10
	2.23	NormF
3	$\mathbf{M}$	Iatrix Functions   12
	3.1	QuadraticFormVariableMatrix
	3.2	Div2if
	3.3	Div2Matrix
	3.4	QuadraticFormMatrix
	3.5	LinearFormVector
	3.6	VtoTriangle
	3.7	VtoTriangles
	3.8	Cog
	3.9	Trans
	3.10	FindMatrix
	3.11	FindMatrix1
	3.12	FindMatrices
	3.13	FindAffineMatrix
		FindAffineMatrices 15

	3.15 3.16 3.17 3.18 3.19 3.20 3.21	F1a       15         F2a       16         EmbedMatrix       16         EmbedMatrix2       17         F1v       17         EmbedVector       18
4	$\mathbf{L}_{\mathbf{c}}$	ocal Interpolations19
	4.1	RotateAngle
	4.2	NewFindMatrices         19
	4.3	LocalLinear
	4.4	LocalPolar
	4.5	LocalAlexa
	4.6	LocalLogExp
	4.7	LocalInterpolations
5	$\mathbf{C}$	onstraint Functions
	5.1	ConstMatrix
	5.2	ConstMatrixM
	5.3	${\tt ConstVectorM}21$
	5.4	ConstfixMatrix
	5.5	ConstfixVector
	5.6	ConstPair
	5.7	ConstPair
	5.8	DoubleMatrix
6	$\mathbf{G}$	${f robal\ Interpolations} \ldots 23$
	6.1	${\tt QuadraticFormAlexa}23$
	6.2	QuadraticFormSim
	6.3	ARAP
7	$\mathbf{D}$	raw Animation
	7.1	ShowStatus
	7.2	${\tt DrawAnimation}24$
	7.3	ListAnimation
Ir	ıdex	x

## 1 Introduction

<< ARAPdata021';

This is a Mathematica Module for investigating an interpolation in Computer Graphics, called the ARAP (as rigid as possible) interpolation. Our module contains about 100 functions of lementary matrix operations, matrix and polygon interpolatins and drawing polygons.

To use this package "ARAPlibv024'" and sample data "ARAPdatav021'" users should set a directory where the modules is stored.

```
[Example]
SetDirectory[FileNameJoin[$HomeDirectory, "--- Some Folder ---"]];
<< ARAPlibv024';</pre>
```

This module was used and introduced in the followings:

- [1] S.Kaji, S.Hirose, S.Sakata, Y.Mizoguchi, Mathematical analysis on affine maps for 2D shape interpolagion (http://dl.acm.org/citation.cfm?id=2422368), SCA '12 Proceedings of the ACM SIGGRAPH/Eurographics Symposium on Computer Animation, pp.71-76.
- Y. Mizoguchi, Mathematical Aspects of Interpolation Technique for Computer Graphics, Forum "Math-for-Industry" 2012, Information Recovery and Discovery, 22 October 2022. http://fmi2012.imi.kyushu-u.ac.jp/
- [3] T.Hirano, A.hirakawa, N.Miyaki, C.Matsufuji and Y.Mizoguchi. Mathematica Module for ARAP Interpolation https://github.com/KyushuUniversityMathematics/MathematicaARAP

## 2 Basic Functions

## 2.1 MakePolygon

MakePolygon[V, tindex]

:: Make polygon

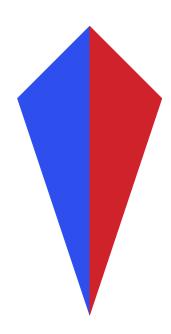
V vertex set

tindex constitution of triangle

return polygon

[Example]

Graphics[MakePolygon[dataS2,dataT2]]



#### 2.2 HorizontalSnake

HorizontalSnake[n]

:: Make vertexes of horizontally long rectangle

n rectangle size

return vertex set of rectangle

[Example]

HorizontalSnake[5]

 $=\{\{0, 0\}, \{0, 1\}, \{1, 0\}, \{1, 1\}, \{2, 0\}, \{2, 1\}, \{3, 0\}, \{3, 1\}, \{4, 0\}, \{4, 1\}\}$ 

#### 2.3 VerticalSnake

## 2.4 SnakeTriangle

#### SnakeTriangle[n]

:: Triangulate Horizontal/Vertex Snake

n rectangle size

return constitution of triangle

[Example]

Graphics[MakePolygon[HorizontalSnake[10], SnakeTriangle[10]]]



Graphics[MakePolygon[VerticalSnake[10], SnakeTriangle[10]]]



## 2.5 AnimationRange

```
AnimationRange[conf]
:: size of Display conforming with polygon

conf { start vertex set , end vertex set}

return coordinates of display
```

```
[Example]
AnimationRange[Configuration2]
={{-2, 4}, {-3, 5}}
```

## 2.6 AnimationRange2

```
AnimationRange2[range1,range2]
:: size of Display

range1

range2

return coordinates of display

[Example]

AnimationRange[Configuration2]

={{-2, 4}, {-3, 5}}
```

## 2.7 Polar

```
Polar[a] :: Convert cartesian coordinates to polar coordinates 
a cartesian coordinates 
return polar coordinates 

[Example] 

Polar[\{0,1\}] 

=\{1,2\pi\}
```

#### 2.8 Cartesian

```
Cartesian[a] :: Convert polar coordinates to cartesian coordinates 
a polar coordinates 
return cartesian coordinate 

[Example] 

Cartesian[\{1,2\pi\}] 

=\{0,1\}
```

## 2.9 LinearInterpolate

```
LinearInterpolate[a,b,t]
:: Perform linear interpolation of point

a start point
```

b an end point

t time

return an interpolated point

[Example]
 LinearInterpolate[{1, 1}, {2, 2}, 0.5]
 ={1.5, 1.5}

## 2.10 LinearInterpolation

LinearInterpolation[p,q,t]

:: Perform linear interpolation of vertex set

p start vertex set

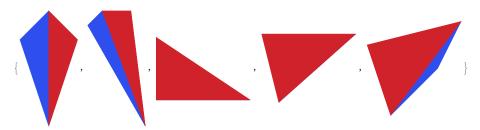
q end vertex set

t time

return interpolated vertex set

[Example]

Graphics[MakePolygon[LinearInterpolation[dataS2, dataE2, #], dataT2]] & /
{0, 0.25, 0.5, 0.75, 1}



## 2.11 PolarInterpolate

 ${\tt PolarInterpolate}[a,b,t]$ 

:: Perform Polar interpolation of point

a a start point

b an end point

time

return an interpolated point

[Example]

PolarInterpolate[{1, 1}, {2, 2}, 0.5]

={1.5, 1.5}

## 2.12 PolarInterpolation

PolarInterpolation[p,q,t]

:: Perform Polar interpolation of vertex set

p start vertex set

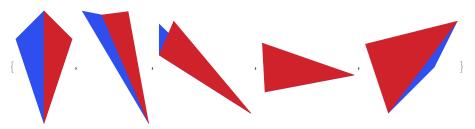
q end vertex set

t time

return interpolated vertex set

#### [Example]

Graphics[MakePolygon[PolarInterpolation[dataS2, dataE2, #], dataT2]] & /
{0, 0.25, 0.5, 0.75, 1}



## 2.13 LinearInterpolateSnake2D

LinearInterpolateSnake2D[n,t]

:: aiueo

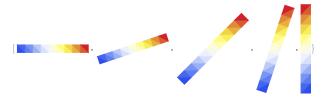
n Snake size

t time  $(0^{-1})$ 

return interpolated snake rectangle

#### [Example]

Graphics[LinearInterpolateSnake2D[10, #]] & / {0, 0.25, 0.5, 0.75, 1}



## 2.14 PolarInterpolateSnake2D

PolarInterpolateSnake2D[n,t]

:: aiueo

n Snake size

t time (0~1)
return interpolated snake rectangle
 [Example]
 Graphics[PolarInterpolateSnake2D[10, #]] & / {0, 0.25, 0.5, 0.75, 1}

## 2.15 PolarDecomposition

```
PolarDecomposition[m]
:: Perform polar decomposition

m matrix

return orthogonal matrix and positive-semidefinite matrix

[Example]
PolarDecomposition[{{1, 1}, {-1, -1}}]
={{{0., 1.}, {-1., 0.}}, {{1., 1.}}}
```

## 2.16 PolarDecompositionPlus

```
PolarDecompositionPlus[m]
:: aiueo

m aiueo

return aiueo

[Example]
PolarDecompositionPlus[m_]
```

## 2.17 RotateAngle

## 2.18 CogTrans

## 2.19 PolygonToTriangles

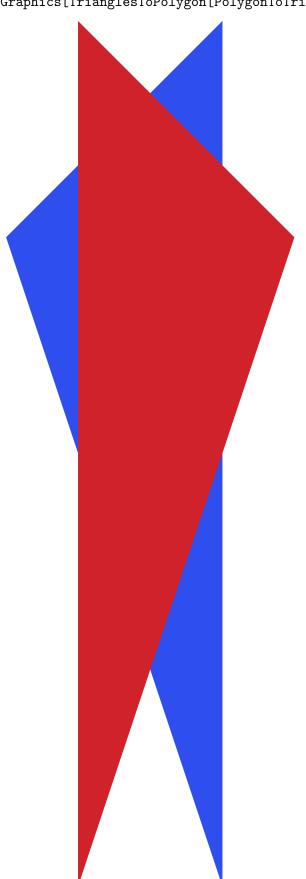
[Example]

```
PolygonToTriangles[1]
:: Set polygon to vertex set of triangle(Center of gravity(0,0))

l Polygon

return vertex of triangles
```

Graphics[TrianglesToPolygon[PolygonToTriangles[MakePolygon[dataS2, dataT2]]]]



## 2.20 TrianglesToTriangles

```
TrianglesToTriangles[offset,cog]
```

:: cog run vertexes in a parallel direction

offset vertex set
cog coordinate
return vertex set

[Example]

TrianglesToTriangles[ $\{\{-1, 1\}, \{0, -2\}, \{0, 2\}, \{1, 1\}\}, \{\{1, 1\}\}\}, \{\{1, -1\}, \{0, 1\}\} = \{\{\{0, 0\}, \{1, -3\}, \{1, 1\}, \{2, 0\}\}, \{\{1, 2\}\}\}$ 

## 2.21 TrianglesToPolygon

TrianglesToPolygon[t1]

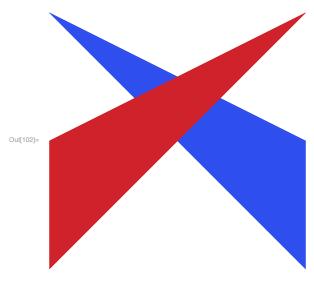
:: Set vertex set of triangle to polygon

tl list of triangle vertex set

return polygon

[Example]

 ${\tt Graphics[TrianglesToPolygon[1, 1, 1, 2, -1, 3, -1, 1, -1, 2, 1, 3]]}$ 



#### 2.22 ValNames

ValNames[n]

:: Name variable

[Example]
NormF[1,2,3,4]

=30

```
n number of variable
return neme of variable
    [Example]
    ValNames[3]
    ={{v1x, v1y}, {v2x, v2y}, {v3x, v3y}}

2.23 NormF
NormF[m] :: Calculate Frobenius norm
    m matrix
return resultant value of Frobenius norm
```

## 3 Matrix Functions

#### 3.1 QuadraticFormVariableMatrix

```
QuadraticFormVariableMatrix[v1]
           :: QuadraticFormVariableMatrix
vl
           list of values
return
           matrix
      [Example]
      QuadraticFormVariableMatrix[{1,2,3}]
      \{\{1, 2, 3\}, \{2, 4, 6\}, \{3, 6, 9\}\}
3.2 Div2if
Div2if[n,1]
           :: divide all elements except n-th element of 1 by 2
n,l
           index, list of values
return
           vector divided all elements except n-th element of 1 by 2
      [Example]
      Div2if[3, {3, 6, 7, 5, 1}]
      \{3/2, 3, 7, 5/2, 1/2\}
3.3 Div2Matrix
Div2Matrix[m]
           :: divide all elements except diagonal ones of m by 2
m
return
           matrix divided all elements except diagonal ones of m by 2
      [Example]
      Div2Matrix[{{1, 3}, {7, 4}}]
      \{\{1, 3/2\}, \{7/2, 4\}\}
3.4 QuadraticFormMatrix
QuadraticFormMatrix[poly,v1]
           :: QuadraticFormMatrix
           polynomial, list of values
poly,vl
           QuadraticFormMatrix
return
      [Example]
      QuadraticFormMatrix[(a*x^2 + b*x + c-y)^2, {a, b, c}]
      \{\{x^4, x^3, x^2\}, \{x^3, x^2, x\}, \{x^2, x, 1\}\}
```

#### 3.5 LinearFormVector

LinearFormVector[poly, v1]

:: LinearFormVector

poly,vl polynominal,list of values

return LinearFormVector

[Example]

LinearFormVector[
$$(a*x^2 + b*x + c - y)^2$$
, {a, b, c}] {-2 x^2 y, -2 x y, -2 y}

## 3.6 VtoTriangle

VtoTriangle[V,t]

:: return coordinates of triangle

V,t list of vertices, list of triangulation

return list of coordinates of vertices of triangle

[Example]

#### 3.7 VtoTriangles

VtoTriangles[V,T]

:: return coordinates of triangles

V,T list of vertices, list of triangulation

return list of coordinates of vertices of triangles

[Example]

```
VtoTriangles[{{-1, 1}, {0, -2}, {1, 1}, {0, 2}}, {{1, 2, 3}, {1, 2, 4}, {2, 3, 4}}] {{{-1, 1}, {0, -2}, {1, 1}}, {{-1, 1}, {0, -2}, {1,1}, {0, 2}}}
```

#### **3.8** Cog

Cog[P] :: return triangle center

P triangle

return triangle center of P

[Example]

$$Cog[{\{-1, 1\}, \{0, -2\}, \{0, 2\}}]$$
 {-(1/3), 1/3}

#### 3.9 Trans

```
Trans[P, 1]
            :: parallel shift by l
P,l
            triangle, vector
return
            triangle
      [Example]
      Trans[\{\{1, 2\}, \{5, -3\}, \{-4, 1\}\}, \{5, 6\}]
      \{\{-4, -4\}, \{0, -9\}, \{-9, -5\}\}
3.10 FindMatrix
FindMatrix[P1,P2]
            :: find matrix converts P1 to P2
P1.P2
            triangles whose center is origin
return
            matrix convert P1 to P2
      [Example]
      P11 = \{\{0, 2\}, \{-3, -1\}, \{3, -1\}\};
      P21 = \{\{-4, 3\}, \{1, -2\}, \{3, -1\}\};
      FindMatrix[P11, P21]
      \{\{1/3, -2\}, \{1/6, 3/2\}\}
3.11 FindMatrix1
FindMatrix1[P1,P2]
            :: find matrix converts P1 to P2
P1.P2
            triangles whose center is origin
            matrix convert P1 to P2
return
      [Example]
      P11 = \{\{0, 2\}, \{-3, -1\}, \{3, -1\}\};
      P21 = \{\{-4, 3\}, \{1, -2\}, \{3, -1\}\};
      FindMatrix1[P11, P21]
      \{\{1/3, -2\}, \{1/6, 3/2\}\}
3.12 FindMatrices
FindMatrices[V1, V2, T]
            :: find matrices converts each triangles represented by V1 and T to ones repre-
            sented by V2 and T
V1, V2, T
            V1,V2:list of vertices T:list of triangulation
            matrices
return
      [Example]
      V1 = \{\{-1, 1\}, \{0, 2\}, \{1, -3\}, \{4, -5\}\};
```

 $\{-(5/27), 2/9, -(1/27)\}, \{-(2/81), -(1/27), 5/81\}\}$ 

```
V2 = \{\{-3, 3\}, \{-2, 5\}, \{2, 1\}, \{3, 1\}\};
      T = \{\{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\};
      FindMatrices[V1, V2, T]
      \{\{\{3/2, -(1/2)\}, \{1, 1\}\},
       {\{12/11, -(1/11)\}, \{10/11, 12/11\}\},\}
       \{\{-(3/13), -(11/13)\}, \{8/13, 12/13\}\}\}
3.13 FindAffineMatrix
FindAffineMatrix[P1,P2]
            :: find affine matrix converts P1 to P2
P1,P2
            triangles
            matrix converts P1 to P2
return
      FindAffineMatrix[\{\{-1, 1\}, \{0, -2\}, \{0, 2\}\}, \{\{-4, 3\}, \{1, -2\}, \{3, 0\}\}]
3.14 FindAffineMatrices
FindAffineMatrices [V1, V2, T]
            :: find affine matrix convert each triangles
V1, V2, T
            V1,V2:list of vertices T:list of triangulation
return
            matrices
      [Example]
      V1 = \{\{-1, 1\}, \{0, 2\}, \{1, -3\}, \{4, -5\}\};
      V2 = \{\{-3, 3\}, \{-2, 5\}, \{2, 1\}, \{3, 1\}\};
      T = \{\{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}\};
      FindAffineMatrices[V1, V2, T]
      \{\{\{3/2, -(1/2), -1\}, \{1, 1, 3\}, \{0, 0, 1\}\},\
      \{\{12/11, -(1/11), -(20/11)\}, \{10/11, 12/11, 31/11\}, \{0, 0, 1\}\},
      \{\{-(3/13), -(11/13), -(4/13)\}, \{8/13, 12/13, 41/13\}, \{0, 0, 1\}\}\}
3.15 F1a
F1a[{\{a1x,a1y\},\{b1x,b1y\},\{c1x,c1y\}\},\{\{m11,m12\},\{m21,m22\}\}}]
            :: compute quadratic form matrix
\{\{a1x,a1y\},\{b1x,b1y\},\{c1x,c1y\}\},\{\{m11,m12\},\{m21,m22\}\}\}
            triangle, matrix
            quadratic form matrix
return
      [Example]
      F1a[{\{-1, 0\}, \{1, 1\}, \{2, -3\}\}, \{\{1, 3\}, \{4, 2\}\}}]
      \{\{17/81, -(5/27), -(2/81)\},\
```

#### **3.16** F2a

```
F1a[{\{a1x,a1y\},\{b1x,b1y\},\{c1x,c1y\}\},\{\{m11,m12\},\{m21,m22\}\}}]
           :: compute quadratic form matrix
\{\{a1x,a1y\},\{b1x,b1y\},\{c1x,c1y\}\},\{\{m11,m12\},\{m21,m22\}\}\}
           triangle, matrix
return
           quadratic form matrix
      [Example]
     F2a[\{\{-1, 0\}, \{1, 1\}, \{2, -3\}\}, \{\{1, 3\}, \{4, 2\}\}]
     \{\{137/2430, 0, -(7/405), -(1/27), -(19/486), 1/27\},
      \{0, 137/2430, 1/27, -(7/405), -(1/27), -(19/486)\},\
      \{-(7/405), 1/27, 4/135, 0, -(1/81), -(1/27)\},\
      \{-(1/27), -(7/405), 0, 4/135, 1/27, -(1/81)\},\
      \{-(19/486), -(1/27), -(1/81), 1/27, 25/486, 0\},\
      \{1/27, -(19/486), -(1/27), -(1/81), 0, 25/486\}
3.17 EmbedMatrix
EmbedMatrix[n,i,j,M]
           :: embed 2-degree matrix M in n-degree 0 matrix
n,i,j,M
           degree,index,index,matrix
return
           matrix
      [Example]
     EmbedMatrix[6, 2, 4, {{1, 2}, {3, 4}}]
     \{\{0, 0, 0, 0, 0, 0, 0\},
      \{0, 1, 0, 2, 0, 0\},\
      \{0, 0, 0, 0, 0, 0\},\
      \{0, 3, 0, 4, 0, 0\},\
      \{0, 0, 0, 0, 0, 0\},\
      \{0, 0, 0, 0, 0, 0, 0\}
3.18 EmbedMatrix
```

```
EmbedMatrix[n,i,j,k,M]
           :: embed 2-degree matrix M in n-degree 0 matrix
n,i,j,k,M
           degree, index, index, index, matrix
return
           matrix
      [Example]
     EmbedMatrix[8, 2, 4, 7, {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
     \{\{0, 0, 0, 0, 0, 0, 0, 0, 0\},\
      \{0, 1, 0, 2, 0, 0, 3, 0\},\
      \{0, 0, 0, 0, 0, 0, 0, 0, 0\},\
      \{0, 4, 0, 5, 0, 0, 6, 0\},\
       \{0, 0, 0, 0, 0, 0, 0, 0\},\
```

```
\{0, 0, 0, 0, 0, 0, 0, 0\},\
\{0, 7, 0, 8, 0, 0, 9, 0\},\
\{0, 0, 0, 0, 0, 0, 0, 0\}
```

```
3.19 EmbedMatrix2
EmbedMatrix2[n,i,j,k,M]
           :: embed 6-degree matrix M in 2n-degree 0 matrix
n,i,j,k,M
           n:size, i,j,k:index M:matrix
return
           matrix
      [Example]
     A = EmbedMatrix[6, 1, 3, 5, \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 9\}\}] +
        EmbedMatrix[6, 2, 4, 6, {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}]
        \{\{1, 0, 2, 0, 3, 0\},\
         \{0, 1, 0, 2, 0, 3\},\
         \{4, 0, 5, 0, 6, 0\},\
         \{0, 4, 0, 5, 0, 6\},\
         \{7, 0, 8, 0, 9, 0\},\
         {0, 7, 0, 8, 0, 9}}
     EmbedMatrix2[4, 1, 3, 4, A]
     \{\{1, 0, 0, 0, 2, 0, 3, 0\},\
      \{0, 1, 0, 0, 0, 2, 0, 3\},\
      \{0, 0, 0, 0, 0, 0, 0, 0, 0\},\
      \{0, 0, 0, 0, 0, 0, 0, 0, 0\},\
      \{4, 0, 0, 0, 5, 0, 6, 0\},\
      \{0, 4, 0, 0, 0, 5, 0, 6\},\
       \{7, 0, 0, 0, 8, 0, 9, 0\},\
       \{0, 7, 0, 0, 0, 8, 0, 9\}
3.20 F1v
F1v[{\{a1x,a1y\},\{b1x,b1y\},\{c1x,c1y\}\},\{\{m11,m12\},\{m21,m22\}\}}]
           :: compute linear form vector
```

```
\{\{a1x,a1y\},\{b1x,b1y\},\{c1x,c1y\}\},\{\{m11,m12\},\{m21,m22\}\}\}
           triangle, matrix
           linear form vector
return
      [Example]
     F1v[{{-1, 0}, {1, 1}, {2, -3}}, {{1, 3}, {4, 2}}]
     \{14/9, 4, -(8/3), -4, 10/9, 0\}
```

## 3.21 EmbedVector

## 4 Local Interpolations

#### 4.1 RotateAngle

m

return

```
RotateAngle[m_,flag_]
           ::controlled rotate angle of m by flag
\mathbf{m}
flag
           1 or 0 or -1, control the rotate angle
return
           angle
4.2 NewFindMatrices
NewFindMatrices[conf_]
           ::find matrices converted each triangles
conf
           configuration
return
           matrices
      [Example]
      NewFindMatrices[Configuration2] =
      \{\{\{-(5/4), -(3/4)\}, \{-1, -1\}\}, \{\{-(7/4), -(3/4)\}, \{2, -1\}\}\}
4.3 LocalLinear
LocalLinear[m_]
           ::compute a local linear interpolations depend on time
           matrices computed by NewFindMatrices[conf_]
m
return
           matrices of local linear interpolation
      [Example]
      LocalLiner[NewFindMatrices[Configuration2]][0.5]=
      \{\{\{-0.125, 0.125\}, \{-0.5, -0.5\}\}, \{\{-0.875, -0.375\}, \{1.5, 0.\}\}\}
4.4 LocalPolar
LocalPolar[m_]
           ::compute a local polar interpolations depend on time
           matrices computed by NewFindMatrices[conf_]
m
return
           matrices of local polar interpolation
4.5 LocalAlexa
LocalAlexa[m_]
           ::compute a local ARAP interpolations depend on time
```

matrices computed by NewFindMatrices[conf\_]

matrices of local alexa interpolation

## 4.6 LocalLogExp

```
LocalLogExp[m_]
```

::compute a local log-exp interpolations depend on time

m matrices computed by NewFindMatrices[conf\_]

return matrices of local log-exp interpolation

## 4.7 LocalInterpolations

#### LocalInterpolations[local\_,conf\_]

::compute local interpolations that you choice

local LocalLinear/LocalPolar/LocalAlexa/LocalLogExp

conf configuration

return

#### [Example]

```
LocalInterpolations[LocalPolar,Configuration2][t]= \{\{(1+0.352786\ t)\ Cos[3.03094\ t]+0.855844\ t\ Sin[3.03094\ t],\ 0.855844\ t\ Cos[3.03094\ t]+(1-0.0889399\ t)\ Sin[3.03094\ t]\},\ \{0.855844\ t\ Cos[3.03094\ t]-1.\ (1+0.352786\ t)\ Sin[3.03094\ t],\ (1-0.0889399\ t)\ Cos[3.03094\ t]-0.855844\ t\ Sin[3.03094\ t]\}\},\ \{\{(1+1.65165\ t)\ Cos[2.35619\ t]+0.176777\ t\ Sin[2.35619\ t]\},\ -0.176777\ t\ Cos[2.35619\ t]-1.\ (1+0.237437\ t)\ Sin[2.35619\ t]\},\ \{-0.176777\ t\ Cos[2.35619\ t]+(1+1.65165\ t)\ Sin[2.35619\ t]\}\}
```

## 5 Constraint Functions

#### 5.1 ConstMatrix

ConstMatrix[m\_,st\_]

::Compute a matrix part of a constraint function of a specific vertex you choice.

m vertex

st list of start coordinates

return matrix

#### 5.2 ConstMatrixM

 ${\tt ConstMatrixM}[st\_,en\_,tri\_,t\_]$ 

::Compute a matrix part of a constraint function of barycenter .

st list of start coordinates

en list of end coordinates

tri list of triangulation

t parameter of time(0...1)

return matrix

[Example]

ConstMatrixM[dataS2,dataE2,dataT2,0.5]

#### 5.3 ConstVectorM

ConstVectorM[st\_,en\_,tri\_,t\_]

::Compute a vector part of a constraint function of barycenter.

st list of start coordinates

en list of end coordinates

tri list of triangulation

t parameter of time(0...1)

return vector

[Example]

ConstVectorM[dataS2,dataE2,dataT2,0.5]

#### 5.4 ConstfixMatrix

ConstfixMatrix $[n_{-},k_{-},l_{-},st_{-}]$ 

::Compute a matrix part of a constraint function fixing user - specified vector.

n weight of constraint function

k,l Choice two numbers you want to fix.

st list of start coordinates
return matrix
[Example]
ConstfixMatrix[2,1,2,dataS2]

#### 5.5 ConstfixVector

ConstfixVector  $[n_-, k_-, l_-, st_-]$ ::Compute a vector part of a constraint function fixing user - specified vector. n weight of constraint function

Choice two numbers you want to fix.

st list of start coordinates

return vector
[Example]
ConstfixVector[2,1,2,dataS2]

#### 5.6 ConstPair

ConstPair[m\_]

k,l

::Compute a pair of matrix and vector of a constraint function of a specific vertex you choice.

m choice of vertex return {matrix, vector}

[Example]
ConstPair[1]

#### 5.7 ConstPair

ConstPair[m\_,n\_]

::Compute a pair of matrix and vector of a constraint function of a specific two vertices you choice.

m,n choice of vertex return {matrix, vector}

[Example]
ConstPair[1,2]

#### 5.8 DoubleMatrix

DoubleMatrix[m\_]
::
m matrix
return matrix
[Example]
DoubleMatrix[]

## 6 Grobal Interpolations

#### 6.1 QuadraticFormAlexa

```
QuadraticFormAlexa[local_,conf_]
::Compute a error function using Alexa's method.

local choice of local interpolation

conf configuration

return {matrix,vector}

6.2 QuadraticFormSim

QuadraticFormSim[local_,conf_]
```

#### quadracici ormsim [locar\_, coni\_]

::Compute a error function using similarity preserving method.

local choice of local interpolation

conf configuration return {matrix, vector}

#### **6.3** ARAP

```
ARAP[local_,energy_,const_,conf_]
           ::Compute a global error function in matrix form.
local
           choice of local interpolation
           choice of energy function
energy
           choice of constraint function
const
           configuration
conf
           {matrix, vector}
return
     [Example]
     ARAP [LocalPolar, QuadraticFormAlexa, ConstPair [1], Configuration2] [1]
     \{\{\{2, 0, -(1/4), 0, 0, 0, -(3/4), 0\},
     \{0, 2, 0, -(1/4), 0, 0, 0, -(3/4)\},\
     \{-(1/4), 0, 1/4, 0, -(1/4), 0, 1/4, 0\},\
     \{0, -(1/4), 0, 1/4, 0, -(1/4), 0, 1/4\},\
     \{0, 0, -(1/4), 0, 1, 0, -(3/4), 0\},\
     \{0, 0, 0, -(1/4), 0, 1, 0, -(3/4)\},\
     \{-(3/4), 0, 1/4, 0, -(3/4), 0, 5/4, 0\},\
     \{0, -(3/4), 0, 1/4, 0, -(3/4), 0, 5/4\}\},\
     \{-6.5, -6., -1., 0.5, 3.5, -4., 8.88178*10^{-16}, 5.5\}
```

## 7 Draw Animation

#### 7.1 ShowStatus

ShowStatus[st,en,tri,plotrange]

:: Draw start and end of Graphic

st start state of vertex set

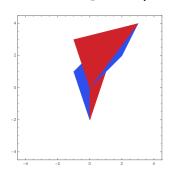
en end state of vertex set

tri constitution of triangle

plotrange Display range

return graph of start and end status

[Example] ShowStatus[dataS2, dataE2, dataT2, 4.5]



#### 7.2 DrawAnimation

DrawAnimation[local, energy, const, conf]

:: aiueo

local Choice of local interpolation

energy Choice of energy interpolation

const Choice of constraint interpolation

conf {start vertex set,end vertex set,constitution of triangle}

return animation

[Example]

DrawAnimation[LocalAlexa, FrobeniusEnergy, Const2, Configuration2]

#### 7.3 ListAnimation

ListAnimation[k,local,energy,const,conf]

:: aiueo

k number of frame division

local Choice of local interpolation

energy Choice of energy interpolation

const Choice of constraint interpolation

conf {start vertex set,end vertex set,constitution of triangle}

return list of graph

[Example]

ListAnimation[k\_,n\_,c\_,e\_,st\_,en\_,tri\_,plotrange]



Index 27

# $\mathbf{Index}$

$\mathbf{A}$	$\mathbf{M}$
AnimationRange       3         AnimationRange2       4         ARAP       23	MakePolygon2
ARAF	N.T.
$\mathbf{C}$	N
Cartesian       4         Cog       13         CogTrans       8	NewFindMatrices         19           NormF         11
ConstfixMatrix	<b></b>
ConstfixVector	P
ConstMatrix       21         ConstMatrixM       21         ConstPair       22         ConstVectorM       21	Polar         4           PolarDecomposition         7           PolarDecompositionPlus         7           PolarInterpolate         5
D	PolarInterpolateSnake2D
Div2if	PolarInterpolation 6 PolygonToTriangles 8
DrawAnimation	$\mathbf{Q}$
E	QuadraticFormAlexa23
EmbedMatrix       16         EmbedMatrix2       17         EmbedVector       18	QuadraticFormMatrix       12         QuadraticFormSim       23         QuadraticFormVariableMatrix       12
$\mathbf{F}$	R
F1a	
$ \begin{array}{cccc} \texttt{F1v} & & & 17 \\ \texttt{F2a} & & & 16 \\ \texttt{FindAffineMatrices} & & & 15 \\ \end{array} $	RotateAngle
FindAffineMatrix	C
FindMatrices	$\mathbf{S}$
FindMatrix       14         FindMatrix1       14	ShowStatus         24           SnakeTriangle         3
Н	
HorizontalSnake	${f T}$
L	Trans         14           TrianglesToPolygon         10           TrianglesToTained         10
LinearFormVector         13           LinearInterpolate         4	TrianglesToTriangles
LinearInterpolation         5           ListAnimation         25	<b>1</b> 7
LocalAlexa	V
${\tt LocalInterpolations$	ValNames
LocalLinear	VerticalSnale
LocalLogExp         20           LocalPolar         19	VtoTriangle13VtoTriangles13