ECE653 A3 Q1

$$\frac{(n \geqslant 0) \wedge (r = 0) \wedge (i = 0) \wedge (p = 1) \Rightarrow \mathbf{Inv}[0/i, 0/r, 1/p] \quad \frac{\mathbf{Inv} \wedge (i \neq n) \Rightarrow \mathbf{Inv}[i+1/i, r+p/r, 2p/p]}{\{\mathbf{Inv} \wedge b\} b \{\mathbf{Inv}\}} \quad \mathbf{Inv} \neg b \Rightarrow \{r = 2^n - 1\}}{\{n \geqslant 0 \wedge r = 0 \wedge i = 0 \wedge p = 1\} P \{r = 2^n - 1\}}$$

The three constrains are:

$$n \geqslant 0 \Rightarrow \mathbf{Inv}[0/r, 0/i, 1/p]$$

$$\mathbf{Inv} \wedge i \neq n \Rightarrow \mathbf{Inv}[(r+p)/r, 2p/p, (i+1)/i]$$

$$\mathbf{Inv} \wedge i = n \Rightarrow r = 2^n - 1$$

Let $\mathbf{Inv} = \left(p = 2^i \wedge r = 2^i - 1 \wedge i \leqslant n\right)$, then the constrains become:

$$n \geqslant 0 \Rightarrow p = 1 \wedge r = 0 \wedge n \geqslant 0$$

$$p = 2^i \wedge r = 2^i - 1 \wedge i \leqslant n \wedge i \neq n \Rightarrow p = 2^{(i+1)} - 1 \wedge (i+1) \leqslant n \wedge (i+1) \neq n$$

$$p = 2^i \wedge r = 2^i - 1 \wedge i \leqslant n \wedge i = n \Rightarrow r = 2^n - 1$$