ECE 653 Assignment 2

Question 1

- 1. $1 \to 2 \to 3 \to 4 \to 9 \to 11 \to 12 \to 13 \to 17$
  2. $1 \to 2 \to 3 \to 4 \to 9 \to 11 \to 14 \to 15 \to 16 \to 17$
  3. $1 \to 2 \to 5 \to 6 \to 7 \to 9 \to 11 \to 12 \to 13 \to 17$
  4. $1 \to 2 \to 5 \to 6 \to 7 \to 9 \to 11 \to 14 \to 15 \to 16 \to 17$
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Edge	PV	PC
$1 \to 2$	$x \to X_0,\ y \to Y_0$	$true$
$2 \to 3$	$x \to X_0,\ y \to Y_0$	$X_0 + Y_0 > 15$
$3 \to 4$	$x \to X_0 + 7,\ y \to Y_0$	$X_0 + Y_0 > 15$
$4 \to 9$	$x \to X_0 + 9,\ y \to Y_0 - 12$	$X_0 + Y_0 > 15$
$9 \to 11$	$x \to X_0 + 9,\ y \to Y_0 - 12$	$X_0 + Y_0 > 15$
$11 \to 12$	$x \to X_0 + 9,\ y \to Y_0 - 12$	$X_0 + Y_0 > 15 \wedge 2(X_0 + Y_0) > 27$
$12 \to 13$	$x \to 3(X_0 + 9),\ y \to Y_0 - 12$	$X_0 + Y_0 > 15 \wedge 2(X_0 + Y_0) > 27$
$13 \to 17$	$x \to 3(X_0 + 9),\ y \to 2(Y_0 - 12)$	$X_0 + Y_0 > 15 \wedge 2(X_0 + Y_0) > 27$

Edge	PV	PC
$1 \to 2$	$x \to X_0,\ y \to Y_0$	$true$
$2 \to 3$	$x \to X_0,\ y \to Y_0$	$X_0 + Y_0 > 15$
$3 \to 4$	$x \to X_0 + 7,\ y \to Y_0$	$X_0 + Y_0 > 15$
$4 \to 9$	$x \to X_0 + 7,\ y \to Y_0 - 12$	$X_0 + Y_0 > 15$
$9 \to 11$	$x \to X_0 + 9,\ y \to Y_0 - 12$	$X_0 + Y_0 > 15$
$11 \to 14$	$x \to X_0 + 9,\ y \to Y_0 - 12$	$X_0 + Y_0 > 15 \wedge 2(X_0 + Y_0) \leqslant 27$
$14 \to 15$	$x \to X_0 + 9,\ y \to Y_0 - 12$	$X_0 + Y_0 > 15 \wedge 2(X_0 + Y_0) \leqslant 27$
$15 \to 16$	$x \to 4(X_0 + 9),\ y \to Y_0 - 12$	$X_0 + Y_0 > 15 \wedge 2(X_0 + Y_0) \leqslant 27$
$16 \to 17$	$x \to 4(X_0 + 9),\ y \to 3Y_0 + 4X_0$	$X_0 + Y_0 > 15 \wedge 2(X_0 + Y_0) \leqslant 27$

Edge	PV	PC
$1 \to 2$	$x \to X_0,\ y \to Y_0$	$true$
$2 \to 5$	$x \to X_0,\ y \to Y_0$	$X_0 + Y_0 \leqslant 15$
$5 \to 6$	$x \to X_0,\ y \to Y_0$	$X_0 + Y_0 \leqslant 15$
$6 \to 7$	$x \to X_0,\ y \to Y_0 + 10$	$X_0 + Y_0 \leqslant 15$
$7 \to 9$	$x \to X_0 - 2,\ y \to Y_0 + 10$	$X_0 + Y_0 \leqslant 15$
$9 \to 11$	$x \to X_0,\ y \to Y_0 + 10$	$X_0 + Y_0 \leqslant 15$
$11 \to 12$	$x \to X_0,\ y \to Y_0 + 10$	$X_0 + Y_0 \leqslant 15 \wedge 2(X_0 + Y_0) > 1$
$12 \to 13$	$x \to 3X_0,\ y \to Y_0 + 10$	$X_0 + Y_0 \leqslant 15 \wedge 2(X_0 + Y_0) > 1$
$13 \to 17$	$x \to 3X_0,\ y \to 2Y_0 + 20$	$X_0 + Y_0 \leqslant 15 \wedge 2(X_0 + Y_0) > 1$

Edge	PV	PC
$1 \to 2$	$x \to X_0,\ y \to Y_0$	$true$
$2 \to 5$	$x \to X_0,\ y \to Y_0$	$X_0 + Y_0 \leqslant 15$
$5 \to 6$	$x \to X_0,\ y \to Y_0$	$X_0 + Y_0 \leqslant 15$
$6 \to 7$	$x \to X_0,\ y \to Y_0 + 10$	$X_0 + Y_0 \leqslant 15$
$7 \to 9$	$x \to X_0 - 2,\ y \to Y_0 + 10$	$X_0 + Y_0 \leqslant 15$
$9 \to 11$	$x \to X_0,\ y \to Y_0 + 10$	$X_0 + Y_0 \leqslant 15 \wedge 2(X_0 + Y_0) \leqslant 1$
$11 \to 15$	$x \to X_0,\ y \to Y_0 + 10$	$X_0 + Y_0 \leqslant 15 \wedge 2(X_0 + Y_0) \leqslant 1$
$15 \to 16$	$x \to 4X_0,\ y \to Y_0 + 10$	$X_0 + Y_0 \leqslant 15 \wedge 2(X_0 + Y_0) \leqslant 1$
$16 \to 17$	$x \to 4X_0,\ y \to 3Y_0 + 4X_0 + 30$	$X_0 + Y_0 \leqslant 15 \wedge 2(X_0 + Y_0) \leqslant 1$

- 1. Path a: Feasible, $X_0 = 10, Y_0 = 20$.
  2. Path b: Not feasible.
  3. Path c: Feasible, $X_0 = 3, Y_0 = 6$.
  4. Path d: Feasible, $X_0 = 0, Y_0 = 0$.

Question 2

- a. If we want at most one variable is true, we need to make sure that every combination is false, which means, each two variables cannot be true at the same time, we have:

  $$\neg((a_1 \wedge a_2) \vee (a_1 \wedge a_3) \vee (a_1 \wedge a_4) \vee (a_2 \wedge a_3) \vee (a_2 \wedge a_4) \vee (a_3 \wedge a_4))$$

  We can transform this equation into CNF clauses:

  $$(\neg a_1 \wedge \neg a_2) \vee (\neg a_1 \wedge \neg a_3) \vee (\neg a_1 \wedge \neg a_4) \vee (\neg a_2 \wedge \neg a_3) \vee (\neg a_2 \wedge \neg a_4) \vee (\neg a_3 \wedge \neg a_4)$$

- b. For the left side:

  $$Left = \forall x \cdot \exists y \cdot P(x) \vee Q(y) \Rightarrow \forall x \cdot (\exists y \cdot P(x)) \vee (\exists y \cdot Q(y))$$

  Since y has no relationship with $P(x)$, we can eliminate y.

$$\Rightarrow \forall x \cdot (P(x)) \vee (\exists y \cdot Q(y))$$

$$\Rightarrow \forall x \cdot P(x) \vee \forall x \cdot \exists y \cdot Q(y) \Rightarrow \forall x \cdot P(x) \vee \exists y \cdot Q(y) = Right$$

For the right side:

$$Right = \forall x \cdot P(x) \vee \exists y \cdot Q(y) \Rightarrow \forall x \cdot P(x) \vee \forall x \cdot \exists y \cdot Q(y)$$

$$\Rightarrow \forall x \cdot (P(x) \vee \exists y \cdot Q(y))$$

$$\Rightarrow \forall x \cdot (\exists y \cdot P(x) \vee \exists y \cdot Q(y))$$

$$\Rightarrow \forall x \cdot \exists y \cdot (P(x) \vee Q(y)) = Left$$

- c.
- d. a.
- d. b. Since $\exists x \exists y \exists z (P(x,y) \wedge P(z,y) \wedge P(x,z) \wedge \neg P(z,x))$ and $P_2 = \{(x, x+1) | x \in \mathbb{N}\}$, which means we need to find a set that satisfied $y = x+1 \wedge y = z+1 \wedge z = x+1$, which is $\emptyset$. So it is violate the formula.
- d. c. Since $P_3 = \{(A,B) | A, B \in \mathcal{P}(\mathbb{N}) \wedge A \subseteq B\}$, which means we need to find 3 sets x, y, and z that satisfied: $x \subseteq y \wedge z \subseteq y \wedge x \subseteq z \wedge z \nsubseteq x$ and all of the elements of x, y, and z are natural numbers.

  Suppose $x = \{1\}$, $y = \{1,2,3\}$, and $z = \{1,2\}$, we can see:

  $$x \subseteq y : \{1\} \subseteq \{1,2,3\}$$

  $$z \subseteq y : \{1,2\} \subseteq \{1,2,3\}$$

  $$x \subseteq z : \{1\} \subseteq \{1,2\}$$

  $$z \nsubseteq x : \{1,2\} \nsubseteq \{1\}$$

  So, it is satisfied the formula.
- e.