ME41055
Multibody Dynamics B

Spring Term 2018, Tue 13:45-15:30, room 3mE-CZ C, 4 ECTS credits.

Homework Assignment 9 (HW9)

A simple mechanical model of the human arm consists of two rigid bodies connected by three hinges. The space fixed coordinate system is, seen from a human perspective looking straight ahead North, the z-axis up, the y-axis North, and the x-axis East. The arm is an open loop structure with, starting from the torso, a hinge with an angle α about the x-axis, a hinge with an angle β about the y-axis, the upper arm with length $d = 30$ cm in the minus z-direction, a hinge with an angle γ about the x-axis, and finally the lower arm with length $e = 40$ cm in the plus y-direction. The location of the imaginary hand at the endpoint is now $(0, e, -d)$ with all angles α, β, and γ equal to zero. The upper arm has a concentrated mass of $m_d = 3$ kg at a distance $d/3$ from the shoulder whereas the lower arm has a concentrated mass of $m_e = 3$ kg at $e/2$ from the elbow. In a first approximation, we neglect the mass moments of inertia of the rigid bodies. We assume gravity to work in the minus z-direction with a field strength of $g = 9.81$ N/kg.

a. Make a sketch of the model, use cans-in-series to depict the hinges.

b. Derive the equations of motion for the arm in terms of the independent generalised degrees of freedom α, β, and γ, and check your results for some simple configurations where you can predict the resulting accelerations of the degrees of freedom.

c. Picture a ball catch posture given by $(\alpha, \beta, \gamma) = (110°, -20°, -20°)$. Determine the three hinge Torques necessary to maintain this posture (equilibrium).

d. Check your result by means of a forward dynamic analysis of the system for a time period of 5 seconds (copy and paste the torques from item (b)). Clearly, after initially being at rest, the arms start moving. Explain why this happens.

e. What happens when you start your 5 second simulation from a different equilibrium posture, namely $(\alpha, \beta, \gamma) = (30°, -20°, -20°)$? Explain the results. (First calculate the the Torques to maintain this new posture and then copy and paste these in the equations of motion for the forward dynamic analysis).

f. In real life the arms have a mass moment of inertia at the centre of mass. Describe the procedure to incorporate the effect of these mass moments of inertia on the derivation of the equations of motion as described in item b. Assume the mass moment of inertia matrices at the cm, expressed in their local body fixed coordinate system where x' is along the length of the arms, are denoted by the two diagonal matrices respectively $\mathbf{I}'_d = \mathrm{diag}(0.004, 0.025, 0.025)$ kgm^2 and $\mathbf{I}'_e = \mathrm{diag}(0.002, 0.040, 0.040)$ kgm^2.

g. Determine the new mass matrix for the equations of motion for the arm in terms of the independent generalised degrees of freedom α, β, and γ, in the configuration as described in item c and compare it to the mass matrix from item b.