Recall: Logistic Model for Population of one species:

$x'(t) = x(r - ax)$ or $x'(t) = r(1-\frac{x}{K})x$

where x is pop., r is intrinsic growth rate, K is carrying capacity, and x' is rate of pop. change.

For 2 species we will first look at a model for competing species.

$x'(t) = x(r_1 - a_1x - b_1y)$

$y'(t) = y(r_2 - a_2y - b_2x)$

x, y: pop. of each species

$a_1, a_2, b_1, b_2, r_1, r_2 > 0$

r: intrinsic growth

Ex:

1 $x'(t) = x(1-x-y)$

2 $y'(t) = y(\frac{3}{4} - y - \frac{1}{2}x)$

1) $x=0$ → $y=0$: $(0,0)$; $\frac{3}{4} - y = 0$, $y=\frac{3}{4}$: $(0,\frac{3}{4})$

$1-x-y=0$, $y=1-x$ → $y=0$, $x=1$: $(1,0)$; $3/4 - y - 1/2x = 0$, $y=1/2$, $x=1/2$: $(1/2, 1/2)$

$x'(t) = x - x^2 - xy$

$y'(t) = \frac{3}{4}y - y^2 - \frac{1}{2}xy$

$$J(x,y) = \begin{bmatrix} 1-2x-y & -x \\ -\frac{1}{2}y & \frac{3}{4}-2y-\frac{1}{2}x \end{bmatrix}$$

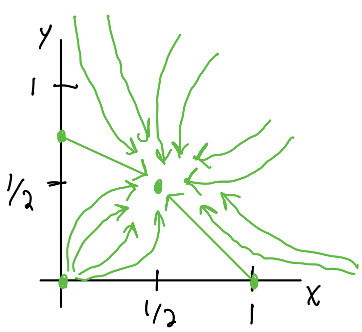
$J(0,0) = \begin{bmatrix} 1 & 0 \\ 0 & 3/4 \end{bmatrix}$ $\lambda = 1, 3/4$ nodal source

$J(0,3/4) = \begin{bmatrix} 1/4 & 0 \\ -3/8 & -3/4 \end{bmatrix}$ $\lambda = \frac{1}{4}, -\frac{3}{4}$ saddle unstable

$J(1,0) = \begin{bmatrix} -1 & -1 \\ 0 & 1/4 \end{bmatrix}$ $\lambda = -1, 1/4$ saddle unstable

$J(\frac{1}{2},\frac{1}{2}) = \begin{bmatrix} -1/2 & -1/2 \\ -1/4 & -1/2 \end{bmatrix}$ $\lambda = \frac{-1 \pm \sqrt{1/2}}{2}$ → $\frac{-1-\sqrt{1/2}}{2}$ −, $\frac{-1+\sqrt{1/2}}{2}$ −

nodal sink

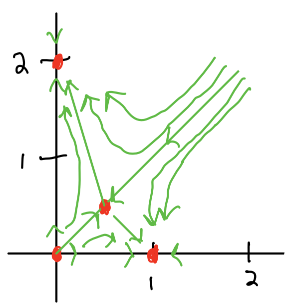


If we start with a non zero pop. of both species they eventually reach equilibrium at $(.5, .5)$ meaning they coexist.

Ex: $x'(t) = x(y - x - 1)$

$y'(t) = y(\frac{1}{2} - \frac{1}{4}y - \frac{3}{4}x)$

Equilibria: $(0,0)$ → nodal source; $(0,2)$ → nodal sink; $(1,0)$ → nodal sink; $(1/2, 1/2)$ → saddle



if we start with non zero pop. of both species we almost certainly have 1 species die off.