Separable ODEs

<u>Idea:</u> "separate" the terms with dependent var from terms with independent var.
often non-linear

<u>Need form:</u> $\underbrace{f(y)\frac{dy}{dx}}_{\text{multiplication only}} = G(x)$

explicit form: ← default
$y(x) = \dots$
implicit form:
$\dots = C$

E.x: $\frac{dy}{dx} = \frac{2x}{y-1}$

$(y-1)\frac{dy}{dx} = 2x \rightarrow \int (y-1)\frac{dy}{dx}dx = \int 2x\ dx$

$u = y$

$du = \frac{dy}{dx}dx$

$\int y-1\,dy = \int 2x\ dx$

$\frac{1}{2}y^2 - y = x^2 + C$

$\frac{1}{2}y^2 - y - x^2 = C$

$\frac{dy}{dx} = x + y$ ← not separable

$\frac{dy}{dx} = 2y^2 + xy^2 \qquad y(0) = 1$

$\frac{1}{y^2}\frac{dy}{dx} = 2 + x$

$\int y^{-2}\,dy = \int 2 + x\ dx$

$-\frac{1}{y} = 2x + \frac{1}{2}x^2 + C$

$-\frac{1}{1} = 0 + 0 + C \qquad C = -1$

$-\frac{1}{y} = 2x + \frac{1}{2}x^2 - 1$

$y(x) = -\frac{1}{2x + \frac{1}{2}x^2 - 1}$

$u'(t) = -K(u(t) - T)$

$\frac{du}{dt} = -K(u(t) - T)$

$\frac{du}{dt}\frac{1}{u(t) - T} = -K$

$u_0 - t$