

Initial Value Problems:

1. $x' = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & -2 & 3 \end{bmatrix}x \quad x(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Gen. solution:

$$x(t) = C_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t} + C_2 e^t \left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \cos 2t - \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \sin 2t \right) + C_3 e^t \left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \sin 2t + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \cos 2t \right)$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + C_3 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 1 & 1 & 3 \end{bmatrix} \rightarrow \text{SOLN. gives } C_1, C_2, C_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1/2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 5/2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5/2 \\ 0 & 0 & 1 & 1/2 \end{bmatrix} \quad C_1 = 1 \quad C_2 = 5/2 \quad C_3 = 1/2$$

(Should NOT have a row of all zeroes when solving for constants)

Particular solution:

$$x(t) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t} + \frac{5}{2} e^t \left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \cos 2t - \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \sin 2t \right) + \frac{1}{2} e^t \left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \sin 2t + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \cos 2t \right)$$