

Determinant of a square matrix:

$|A|$ is determinant of A

$$2 \times 2: \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

$$3 \times 3: \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

one method: go along top row and each entry multiply by 2×2 determinant given by covering up the row and column of the entry. Alternating \pm for each. (doesn't need to be top row)

$$= a_{11} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12} \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13} \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix} = 1 \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} - (-1) \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} + 0 \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$
$$1(6-1) + 1(4-0) = 5+4=9$$

- Alternate 3×3 method. Instead of using a top row, use a different row or a column. Be careful of \pm signs in the sum, they need to follow correct pattern.

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \quad (\text{can extend to larger matrices})$$

→ Better to use row 2 for zeroes

$$\text{Ex: } \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 4 \\ -1 & 1 & 2 \end{bmatrix} = -4 \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} = -4(1+2) = -12$$

4×4 and above: Repeat process

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = a_{11} \begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{bmatrix} - a_{12} \begin{bmatrix} a_{21} & a_{23} & a_{24} \\ a_{31} & a_{33} & a_{34} \\ a_{41} & a_{43} & a_{44} \end{bmatrix} + a_{13} \begin{bmatrix} a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{44} \end{bmatrix} - a_{14} \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$