Wronskian for linear systems:

If y_1, y_2 or more are solutions to $x' = Ax$, they are linearly independent only if $W[y_1, y_2] \neq 0$

To find W construct a matrix with y_1, y_2, etc as the columns and take the determinant.

Ex: $x' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} x$

e. values: $3, -1$

e. vectors: $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $\rightarrow$ we get $y_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{3t} = \begin{bmatrix} 2e^{3t} \\ e^{3t} \end{bmatrix}$ and $y_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{-t} = \begin{bmatrix} -e^{-t} \\ 2e^{-t} \end{bmatrix}$

$$W = \begin{vmatrix} 2e^{3t} & -e^{-t} \\ e^{3t} & 2e^{-t} \end{vmatrix} = 4e^{2t} + e^{2t} = 5e^{2t} \neq 0$$

$\therefore$ y_1, y_2 are linearly independent.

Note: The way we have been finding solutions will always give linearly independent solutions.

$\hookrightarrow$ Also: Wronskian of vectors that aren't solutions will produce meaningless results. Probably not 0.

If we try: $y_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{3t}$ $\quad y_2 = \begin{bmatrix} -4 \\ -2 \end{bmatrix} e^{3t}$

$$W[y_1, y_2] = \begin{vmatrix} 2e^{3t} & -4e^{3t} \\ e^{3t} & -2e^{3t} \end{vmatrix} = -4e^{3t} + 4e^{3t} = \underline{\underline{0}}$$

Note: y_1, y_2 are clearly multiples of eachother, which also indicates that they aren't linearly independent.