# Converting a 2nd order ODE to system of 1st order ODE's (linear)

Idea: If we have an ODE $u'' + au' + bu = 0$

(linear, homogeneous, 2nd order)

and we want to re-write it as $x' = Ax$ for some $A$

Define: $x_1 = u$ then $x_1' = u' = x_2$ and $x_2' = u''$

$x_2 = u'$ $\quad = 0x_1 + 1 \cdot x_2$

we can find by solving the ODE $u'' = bu - au'$

$= -bx_1 - ax_2$

so $x' = Ax$ will be

$$x' = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} x$$

**Ex.** $2u'' - 3u' + 4u = 0 \quad u(0) = -1 \quad u'(0) = 2$

Convert IVP to a system of 1st order ODE's

First find correct form:

notice $a = -\frac{3}{2}$ $\quad -a = \frac{3}{2}$

$b = 2$ $\quad -b = -2$

$u'' \underbrace{-\frac{3}{2}}_{a} u' + \underbrace{2}_{b} u = 0$

$u'' = \frac{3}{2}u' - 2u$

$x_1 = u$ $\quad x_1' = x_2$

$x_2 = u'$ $\quad x_2' = u''$

$u'' = -2x_1 + \frac{3}{2}x_2 \longrightarrow x' = \begin{bmatrix} 0 & 1 \\ -2 & 3/2 \end{bmatrix} x$

$$x(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} u(0) \\ u'(0) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

## What if 2nd order ODE isn't homogeneous?

$u'' + u' + u = \sin t$

note: system we convert to will be nonhomogeneous (haven't seen before) $x' = A\vec{x} + \vec{b}$

nonhomogeneous part

$x' - A\vec{x} = \vec{b}$ nonhomogeneous

$x' - A\vec{x} = 0$ homogeneous

$x_1 = u$ $\quad x_1' = x_2$

$x_2 = u'$ $\quad x_2' = u''$

$u'' = -x_1 - x_2 + \sin t$

$u'' = -u - u' + \sin t$

so system is $x' = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ \sin t \end{bmatrix}$ always 0