3rd and 4th order Linear ODEs with constant coefficients:

Very similar to 2nd order
- find roots of ch. eqn.
- real, complex, repeated roots handled same way
- Y_p is found the same way

Ex. $y^{(4)} - 4y''' + 4y'' = 0$

$r^4 - 4r^3 + 4r^2 = 0$

$r^2(r^2 - 4r + 4) = 0$

$r^2(r-2)(r-2) = 0$

$r = 0, 0, 2, 2$

$y(t) = C_1 + C_2 t + C_3 e^{2t} + C_4 t e^{2t}$

would need 4 initial conditions

$y(0)=$, $y'(0)=$, $y''(0)=$, $y'''(0)=$

$y^{(4)} + 2y'' + y = 0$

$r^4 + 2r^2 + 1 = 0$ (biquadratic)

replace $r^2 = s$

$r^2 = \frac{-2 \pm \sqrt{4-4}}{2}$

$s^2 + 2s + 1 = 0$

$(r^2+1)(r^2+1) = 0$

$r^2 + 1 = 0$ $r^2 + 1 = 0$

$r = \pm i$ $r = \pm i$

$y(t) = C_1 \cos(t) + C_2 \sin(t) + C_3 t\cos(t) + C_4 t\sin(t)$

$y^{(4)} - y = 0$

$r^{(4)} - 1 = 0$ (difference of squares)

$(r^2-1)(r^2+1) = 0$

$(r-1)(r+1)(r^2+1)$

$y(t) = C_1 e^{t} + C_2 e^{-t} + C_3 \cos(t) + C_4 \sin(t)$