Def: the order of differential eqn is the highest derivative that appears.

"higher order" means 3rd order or higher

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modeling

Velocity/acceleration of a falling object

setup eqn for forces acting on obj

$$\underbrace{m\frac{dv}{dt}}_{\text{total force}} = \underbrace{mg}_{\text{force gravity}} - \underbrace{\gamma v}_{\text{force drag}}$$

← chose down to be positive direction

where m is mass, v is velocity, g is gravity, and γ is drag coefficient

say $m = 10\,kg$, $g = 9.8\,m/s^2$, $\gamma = 2\,kg/s$

$$10\frac{dv}{dt} = 98 - 2v$$

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}$$

solve for v

Can't take integral wrt t on right
can't take integral wrt v on left

$$\frac{1}{9.8 - \frac{v}{5}} \cdot \frac{dv}{dt} = 1$$

can now take integral wrt t

$$\int \frac{1}{9.8 - \frac{v}{5}} \frac{dv}{dt}\,dt = \int 1\,dt$$

$$\frac{d}{dt}\left[9.8 - \frac{v(t)}{5}\right] = -\frac{1}{5}\frac{dv}{dt}$$

$$u = 9.8 - \frac{v}{5}$$

$$du = -\frac{1}{5}\frac{dv}{dt}dt$$

$$-5\int \frac{1}{u}\,du = \int 1\,dt$$

$$-5\ln|u| = t + C$$

$$-5\ln|9.8 - \frac{v}{5}| = t + C$$

$$\ln|9.8 - \frac{v}{5}| = -\frac{t}{5} - \frac{C}{5}$$

$-\frac{C}{5}$ is still just arbitrary constant. rename it C

$$\ln|9.8 - \frac{v}{5}| = -\frac{t}{5} + C$$

$$|9.8 - \frac{v}{5}| = e^{-\frac{t}{5} + C}$$

$$|9.8 - \frac{v}{5}| = e^{-\frac{t}{5}} e^{C}$$

$$\left|9.8-\frac{v}{5}\right| = e^{-t/5}C$$

$$9.8-\frac{v}{5} = \pm C e^{-t/5}$$

$$9.8-\frac{v}{5} = ce^{-t/5}$$

$$-\frac{v}{5} = ce^{-t/5}-9.8$$

$$v = -5ce^{-t/5}+49$$

$$v = 49+ce^{-t/5}$$

if we have initial condition we can find a specific solution (particular solution)

$v(0)=0$

$0 = 49+ce^{-0/5}$ $\quad v(t)=49-49e^{-t/5}$

$0=49+A$

$A=-49$

$v(0)=100$

$100=49+ce^{-0/5}$ $\quad v(t)=49+51e^{-t/5}$

$100=49+A$

$A=5$

solution curves/ equilibrium analysis

gen soln: $v(t)=49+ce^{-t/5}$ represents every specific solns. For some ODE's we'll be able to draw gen soln in some form.

One way is with a direction field. (Not easy to draw)

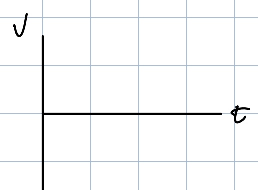
other way is by drawing equilibrium values and solution curves.

Equilibrium analysis is about what happens to the solution as $t \to \infty$

so terminal velocity is

$$\lim_{t\to\infty}(49-49e^{-t/5}) = 49$$

$$\lim_{t\to\infty}(49-ce^{-t/5}) = 49$$



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Direction field

- vector field that shows what the solution of the ODE looks like at any value of t.