Def: A diagonal matrix is a square matrix with all the non-zero entries on the diagonal. $[\diagdown]$

eg. $\begin{bmatrix}1&0&0\\0&-2&0\\0&0&3\end{bmatrix}$ $\begin{bmatrix}0&0&0\\0&-2&0\\0&0&6\end{bmatrix}$ Not $\rightarrow$ $\begin{bmatrix}1&0&0\\0&2&0\\0&1&3\end{bmatrix}$

Def: A Block diagonal matrix is a square matrix with all the non-zero entries in "blocks" on the diagonal.

eg. $\begin{bmatrix}\boxed{1}&0&0\\0&2&3\\0&4&5\end{bmatrix}$ $\begin{bmatrix}0&1&0\\1&0&0\\0&0&\boxed{1}\end{bmatrix}$ $\begin{bmatrix}1&2&0&0\\3&4&0&0\\0&0&1&2\\0&0&3&4\end{bmatrix}$ $\begin{bmatrix}\boxed{1}&0&0&0\\0&4&3&0\\0&2&1&0\\0&0&0&\boxed{4}\end{bmatrix}$ $\begin{bmatrix}\boxed{1}&0&0&0\\0&4&3&5\\0&2&1&8\\0&8&2&4\end{bmatrix}$

Not: $\begin{bmatrix}\boxed{1}&0&\boxed{1}^{*}\\0&2&4\\0&3&5\end{bmatrix}$ $\begin{bmatrix}1&3&0\\8&2&4\\0&3&5\end{bmatrix}$ blocks can't overlap

Why do these matter?

the determinant of a block diagonal matrix is equal to the products of the determinants of the blocks.

the determinant of a 1x1 block is just the value of the entry.

Ex. $\begin{bmatrix}2&0&0\\0&1&3\\0&4&5\end{bmatrix} = 2\left((5\cdot1)-(3\cdot4)\right)$

$\begin{bmatrix}1&2&0&0\\3&4&0&0\\0&0&1&2\\0&0&3&4\end{bmatrix} = (4-6)\cdot(4-6) = 4$

Def: inverse matrices

for a square matrix A, the inverse of A (written as A^{-1}) is the matrix such that $AA^{-1}=I$ and $A^{-1}A=I$. (I is the identity matrix) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

(you don't need to know)

to compute A^{-1}, create a big augmented matrix $[A \mid I]$ and find RREF. If A is invertable (has inverse) then you get $[I \mid A^{-1}]$

If A doesn't have an inverse then A is singular. Then you get all zeroes in the RREF.

Do need to Know

That process works with any size square matrix A. If A is a 2x2 matrix there is a trick for finding A^{-1}.

notice if $|A|=0$ then A^{-1} does not exist.

if $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad A^{-1}=\frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

ex: $\begin{bmatrix} -1 & 3 \\ 4 & 2 \end{bmatrix}$ $\quad A^{-1}=\frac{1}{-2-12}\begin{bmatrix} 2 & -3 \\ 4 & -1 \end{bmatrix}=\begin{bmatrix} 2/-14 & 3/14 \\ 2/7 & 1/14 \end{bmatrix}$

Def: Trace of a matrix

the trace of a square matrix A is the sum of the diagonal entries of A.

Ex. $tr(A)$ for $A=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 0 \\ 2 & 3 & 5 \end{bmatrix}=1+4+5=10$

Invertable Matrix theorem: (Either All are True or All are false)

For a square nxn matrix A, the following properties are equivalent:

- A is invertable (A^{-1} exists)
- A has a RREF of I (identity matrix)
- The system $A\vec{x}=\vec{b}$ has exactly 1 solution (for any b)
- The columns of A are linearly independent
- The determinant of A is non zero ($|A|\neq 0$)
- 0 is Not an eigenvalue of A.

Eigenvalues and Eigenvectors:

For a square matrix A, $\vec{v}\,(\neq\vec{0})$ is called an eigenvector of A with eigenvalue λ if $A\vec{v}=\lambda\vec{v}$ (note: both sides are a vector)

Ex: $\begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}=\begin{bmatrix} -5+2 \\ -7+4 \end{bmatrix}=\begin{bmatrix} -3 \\ -3 \end{bmatrix}$

so $A\vec{v}=\lambda\vec{v}$ $\quad \begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}=-3\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Important fact:

Eigenvalues are always defined up to a scalar multiple.

i.e. if you take a non zero scalar multiple of $\vec{v}$ we don't consider it a new eigenvector, but a different way to express the same eigenvector.

we actually want to find all the linearly independent eigenvectors.