Examples:

1) g $x'(t) = x + y^2$ find and classify all
h $y'(t) = y - 1$ equilibrium points.

h $y - 1 = 0$
$y = 1$ $(-1, 1)$

g $x + 1 = 0$
$x = -1$

$$J = \begin{bmatrix} 1 & 2y \\ 0 & 1 \end{bmatrix} \quad J(-1,1) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad \lambda = 1$$

algebraic mult. 2

$\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

geometric mult. 1

answer:
improper nodal source

2) g $x'(t) = (x - y)(x + 2)$
h $y'(t) = (x - y)(y - 1)$

$x^2 - xy + 2x - 2y$
$xy - y^2 - x + y$

1. find eq. pts.
2 cases:

g $x - y = 0$, $x = y$ $\qquad$ $x + 2 = 0$, $x = -2$

$x - y = 0$
$-2 - y = 0$
$y = -2$
$(-2, -2)$

$y - 1 = 0$
$y = 1$
$(-2, 1)$

h $x - x = 0$
$^*0 = 0$ $\qquad$ $y - 1 = 0$, $y = 1$, $(1, 1)$

$$J(x,y) = \begin{bmatrix} 2x - y + 2 & -x - 2 \\ y - 1 & x - 2y + 1 \end{bmatrix}$$

$$J(-2,1) = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \quad \lambda = -3$$
$\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

alg. mult. 2
geo. mult. 2 $\Big\}$ proper nodal sink

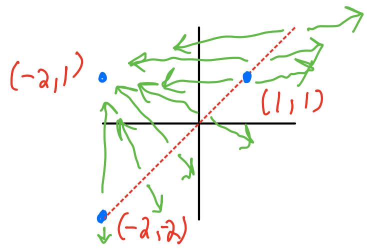
$$J(1,1) = \begin{bmatrix} 3 & -4 \\ 0 & 0 \end{bmatrix} \quad \lambda = 0, 3$$
$\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

alg. mult. 1
geo. mult. 1

so there should be a nullcline through $(1,1)$ along the eigenvector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$J(-2,-2) = \begin{bmatrix} 0 & 0 \\ -3 & 3 \end{bmatrix} \quad \lambda = 0, 3 \to \vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Note: if you find $0 = 0$ when finding eq. points this means there are infinitely many solutions.
specifically $x = y$ is always an equilibrium (nullcline)



Ex: $\begin{cases} x'(t) = y \\ y'(t) = \sin x \end{cases}$ find all equilibrium pts. and classify

$y = 0$

$\sin x = 0$ ← infinitely many solns. $x = n\pi$ for $n \in \mathbb{Z}$

infinitely many equilibria at $(n\pi, 0)$

$\cos n\pi = \begin{cases} 1 & \text{if even} \\ -1 & \text{if odd} \end{cases}$

$J(f,g) = \begin{bmatrix} 0 & 1 \\ -\cos x & 0 \end{bmatrix}$ $J(n\pi, 0) = \begin{bmatrix} 0 & 1 \\ -\cos(n\pi) & 0 \end{bmatrix}$

n even $J(n\pi, 0) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$(-\lambda)(-\lambda) + 1$

$\lambda = \pm i$

center for all n even.
stable

n odd $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$(-\lambda)(-\lambda) - 1$

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$

saddle for all n odd.
unstable