Quiz next class.
3x4 augmented matrix
row reduce to RREF.

EX: $\left[\begin{array}{ccc|c} 0 & 1 & -1 & 3 \\ 1 & 0 & 2 & 2 \\ 0 & -3 & 3 & -9 \end{array}\right]$ swap R_1/R_2 $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & 3 \\ 0 & -3 & 3 & -9 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$ add $3x$ R_2 to R_3

is now RREF

What is the soln of this system?

$x_1 + 2x_3 = 2$

$x_2 - x_3 = 3$

A free variable is a variable that can have any value.

x_3 is a free variable because there is no pivot in the third column.

$\therefore$ $x_1 + 2c = 2$ $\quad x_1 = -2c + 2$

$x_2 - c = 3$ $\quad x_2 = c + 3$

$x_3 = c$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2c+2 \\ c+3 \\ c \end{bmatrix} = \begin{bmatrix} -2c \\ c \\ c \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \underbrace{c\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}}$$

This represents all ∞ solutions.
with a specific value of c would be one individual soln.

Sometimes the matrix will contain complex entries.

$\left[\begin{array}{ccc|c} -1+i & 0 & 0 & 0 \\ -1 & -1+i & 1 & 0 \\ -2 & -2 & 1+i & 0 \end{array}\right]$ divide R1 by $-1+i$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & -1+i & 1 & 0 \\ -2 & -2 & 1+i & 0 \end{bmatrix}$

add R_1 to R_2 and $2R_1$ to R_3

(you can do 2 row op's at once if they don't effect the same row)

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1+i & 1 & 0 \\ 0 & -2 & 1+i & 0 \end{bmatrix}$ swap R_2 w R_3 $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 1+i & 0 \\ 0 & -1+i & 1 & 0 \end{bmatrix}$ divide R_2 by -2

$i = \sqrt{-1}$

$i^2 = -1$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{2}(1+i) & 0 \\ 0 & -1+i & 1 & 0 \end{bmatrix}$ Add $-1+i$ times R_2 to R_3 $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{2}(1+i) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$-1+i \cdot \left(-\frac{1}{2}(1+i)\right) + 1$

$\frac{1}{2}(-1+i)(1+i)$

$\frac{1}{2}(-1-i+i+i^2)+1$

$\frac{1}{2}(-1-1)+1$

$-1+1=0$

$x_1 = 0$

$x_2 - \frac{1}{2}(1+i)x_3 = 0$ $\quad x_2 = \frac{1}{2}(1+i)c$

$x_3 = c$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = c\begin{bmatrix} 0 \\ \frac{1}{2}+\frac{1}{2}i \\ 1 \end{bmatrix}$$

3/7 Why does RREF work?

- the 3 elem. row ops. do not change what the solutions to the system are.

1. swap 2 rows
2. multiply a row by a scalar (except 0)
3. add n times a row to another.

What will the solution look like

- unique soln. pivot in every row

looks like $\left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right]$ soln. is $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ uncommon in this class

- Could get no soln. get a row with all zeroes except for the last.

$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 7 \end{array}\right] \rightarrow 0x_1 + 0x_2 + 0x_3 = 7$

$0 = 7$

* shouldn't happen in this class. Prob. made mistake

- Could get infinitely many solutions: one row is all zeroes. case we will almost always have in this class.

$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow 1x_1 + 2x_3 = 1$

$1x_2 + 3x_3 = -1$

$x_3 = A$ (constant)

$x_2 + 3A = -1$

$x_2 = -3A - 1$

$x_1 = 1 - 2A$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

Pivots can be in a different layout.

$\begin{array}{c} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ \begin{array}{ccc} x_1 & \underbrace{x_2} & x_3 \end{array} \end{array}$

$x_1 - 2x_2 = 5$

$x_3 = 1$

$x_2 = A$

$x_1 = 2A + 5$

no pivot in column 2 $\therefore$ free var is x_2

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

- can have more than 1 row of all zeroes

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 - 4x_2 + 2x_3 = 7$ $\qquad$ $x_1 = 4A - 2B + 7$

$$\left.\begin{array}{l} x_2 = A \\ x_3 = B \end{array}\right\} \text{different constants}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + B \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix}$$