Level curves (in general)

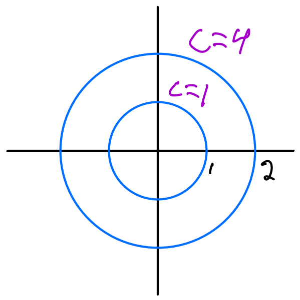
Given by an implicitly defined function

$f(x,y)=C$ (in 2D)

Each value of C gives a different level curve for the function.

Simple example:

$x^2+y^2=C$



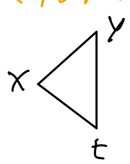
For a 2D system of ODEs:
if $x'(t)$ is a func of just y and $y'(t)$ is a func of just x, then we can find level curves associated with the ODE system.

Note: level curves are not the same as finding a general solution. (but sometimes it can be)
(very similar to solution curve)

Ex: $x'(t)=y$
$y'(t)=x$

this system is "nice" x' relies only on y so we can rewrite this system and solve it using separation of variables.

$$\frac{dy}{dt}=\frac{dy}{dx}\frac{dx}{dt}$$



$$y'(t)=\frac{dy}{dx}x'(t)$$

$$x=\frac{dy}{dx}y$$ ← can solve as an ode in just x and y.

$$\int x=\int\frac{dy}{dx}y\,dx$$

$$\frac{x^2}{2}=\int y\,dy$$

$$\frac{x^2}{2}=\frac{y^2}{2}+C$$

$$x^2-y^2=C$$

Ex: $x'(t)=4-2y$
$y'(t)=12-3x^2$

Approach: use chain rule to get separable ODE $\frac{dy}{dt}=\frac{dy}{dx}\frac{dx}{dt}$

$$12-3x^2=\frac{dy}{dx}(4-2y)$$

$$\int 12-x^2\,dx=\int 4-2y\,dy$$

$$12x-3x^3=4y-y^2+C$$

$$\boxed{12x-x^3-4y+y^2=C}$$