Newtons law of cooling

S – temp of obj
t – time

$\frac{dS}{dt} = K(T-S)$ or $-K(S-T)$

Proportionality constant

T – temp of surroundings

if ambient is 70°F
and rate constant is 0.005

$\frac{dS}{dt} = 0.005(70-S)$

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method of solving ODE's
· Integrating factors
– dont need constant coefficients

a first order linear ODE can always be written in the form: $\frac{dy}{dt} + p(t)y = g(t)$

or $P(t)\frac{dy}{dt} + Q(t)y = G(t)$

really convient problem

$(4+t^2)\frac{dy}{dt} + 2ty = 4t$

LHS looks like it could have come from the product rule:

$\frac{d}{dt}[(4+t^2)y] = 4t$

so we can take integral w.r.t. t right away.

$\int \frac{d}{dt}[(4+t^2)y] = \int 4t$

$(4+t^2)y = 2t^2 + C$

$y = \frac{2t+C}{4t^2}$

in method of integrating factors goal is to make ode look like it came from product rule.

$\frac{dy}{dt} + \frac{1}{2}y = \frac{1}{2}e^{t/3}$

can always multiply both sides by something.
– dont know what that something is use $\mu(t)$

$\mu(t)\frac{dy}{dt} + \frac{1}{2}\mu(t)y(t) = \frac{1}{2}\mu(t)e^{t/3}$

goal $\frac{d}{dt}[\mu(t)y(t)] = \mu(t)\frac{dy}{dt} + \frac{1}{2}\mu(t)y(t)$

$\mu(t)\frac{dy}{dt} + \frac{d\mu(t)}{dt}y(t) = \mu(t)\frac{dy}{dt} + \frac{1}{2}\mu(t)y(t)$

need $\frac{d\mu}{dt} = \frac{1}{2}\mu$

$\frac{1}{\mu}\frac{d\mu}{dt} = \frac{1}{2}$

$\int \frac{1}{\mu}\frac{d\mu}{dt}dt = \int \frac{1}{2}dt$

$u = \mu \quad du = \frac{d\mu}{dt}dt$

$\int \frac{1}{u}du = \int \frac{1}{2}dt$

$\ln|u| = \frac{1}{2}t + C$

$|u| = e^{t/2 + C} = Ce^{t/2}$

$u = Ce^{t/2} \quad \mu(t) = Ce^{t/2}$

Pick any value of c would give me the product rule in og eqn.

$c=1$ is convenient

$$e^{t/2}\frac{dy}{dt}+\frac{1}{2}e^{t/2}y=\frac{1}{2}e^{t/2}e^{t/3}$$

$$\frac{d}{dt}\left[e^{t/2}y\right]=\frac{1}{2}e^{5t/6}$$

$$\int\frac{d}{dt}\left[e^{t/2}y\right]dt=\int\frac{1}{2}e^{5t/6}$$

$$e^{t/2}y=\frac{1}{2}\frac{6}{5}e^{5t/6}+C$$

$$y=\frac{3}{5}e^{t/3}+Ce^{-t/2}$$

from: $\frac{dy}{dt}+P(t)y=g(t)$

$$\left[\begin{array}{l} \mu(t)y=\int\mu(t)g(t)dt \\ \mu(t)=e^{\int P(t)dt} \end{array}\right]$$