

For the system $x' = Ax$, the matrix e^{At} (called the exponential matrix) will give solutions to the system when multiplied by any constant vector \vec{u} .

What is e^{At} ? need to use Taylor series

recall: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} \dots$ etc

so: $e^{At} = I + At + \frac{A^2 t^2}{2} + \frac{A^3 t^3}{6} \dots$ etc

This gives approximation for e^{At} .

Note: this is not necessarily practical for computations (matrices can be very large)

- Multiplying by a constant vector \vec{u} will give a particular solution to $x' = Ax$:

plug in $\hat{x} = u e^{At}$ to the ODE

Homogeneous soln: $\vec{x}' = \vec{u} e^{At} \cdot A$ Rhs: $A \vec{u} e^{At}$