Advanced Factoring (for eigenvalue)

Note: usually won't be this hard

- Factor by grouping

$$\begin{bmatrix} -3 & 3 & 5 \\ 2 & 2 & 0 \\ -4 & 2 & 4 \end{bmatrix}$$

$$|A-\lambda I| = \begin{bmatrix} -3-\lambda & 3 & 5 \\ 2 & 2-\lambda & 0 \\ -4 & 2 & 4-\lambda \end{bmatrix} = -2\begin{vmatrix} 3 & 5 \\ 2 & 4-\lambda \end{vmatrix} + (2-\lambda)\begin{vmatrix} -3-\lambda & 5 \\ -4 & 4-\lambda \end{vmatrix}$$

$$= -2\left(3(4-\lambda)-10\right) + (2-\lambda)\left((-3-\lambda)(4-\lambda)+20\right)$$

$$= -2(2-3\lambda) + (2-\lambda)(\lambda^2-\lambda+8)$$ ← usually we dont keep simplifying here we have to because no common factor.

$$= -4+6\lambda+2\lambda^2-2x+16-\lambda^3+\lambda^2-8\lambda$$

$$0 = [-\lambda^3+3\lambda^2][-4\lambda+12]$$ → can factor by grouping

- look for way to grp terms so they look similar
- factor out what you can from each group.

$$\underline{-\lambda^2(-\lambda+3)} + \underline{4(-\lambda+3)}$$

$$0 = (4-\lambda^2)(-\lambda+3)$$

$$\lambda = \pm 2i \qquad \lambda = 3$$

- Rational group test

(for a polynomial with integer coefficients)
(highest power has coefficient ± 1)

- 2 steps

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 4 & 0 & -4 \\ 2 & 1 & 0 \end{bmatrix} \quad |A-\lambda I| = \begin{bmatrix} 3-\lambda & 4 & -1 \\ 4 & -\lambda & -4 \\ 2 & 1 & -\lambda \end{bmatrix}$$

$$= (3-\lambda)\begin{vmatrix} -\lambda & -4 \\ 1 & -\lambda \end{vmatrix} - 4\begin{vmatrix} 4 & -4 \\ 2 & -\lambda \end{vmatrix} - 1\begin{vmatrix} 4 & -\lambda \\ 2 & 1 \end{vmatrix}$$

$$= (3-\lambda)(\lambda^2+4) - 4(-4\lambda+8) - (4+2\lambda)$$

$$0 = -\lambda^3+3\lambda^2+10\lambda-24$$ ← skipped a factoring step

(can't factor by grouping)
(don't have to multiply both sides by -1)

$0=\lambda^3-3\lambda^2-10\lambda+\underline{24}$ ← our candidates for a root are the factors of 24 ±

[1, 2, 3, 4, 6, 12, 24]

- Now we test these.
- The best way is synthetic division

Synthetic division

*wont find complex roots.

root to check → coefficients of polynomial

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -10 & 24 \\ & \downarrow & +\ 1 & +\ -2 & +\ -12 \\ \hline & 1 & -2 & -12 & \underline{12} \end{array}$$

(multiply by root)

is a root if last num is zero.

(the bottom row is the coefficients of the quotient and the remainder.)

*(if remainder is not 0 try another root)

$$\begin{array}{r|rrrr} -1 & 1 & -3 & -10 & 24 \\ & & -1 & 4 & 6 \\ \hline & 1 & -4 & -6 & \underline{30}\ \times \end{array} \qquad \begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & \underline{0}\ \checkmark \end{array}$$

2 is a root

*(±1 will commonly be a root in this class if this method is needed)

(factored form from synth. div)

→ $(\lambda-2)(\lambda^2-\lambda-12)$

$\underline{\lambda=2}$ $\quad(\lambda-4)(\lambda+3)$

since 2 is a root the factored form is $(\lambda-2)$ because $0=\lambda-2 \rightarrow 2=\lambda$

(this will always work in this class but will not always work outside of it. Will not find complex roots)

* Won't ever have 3 imaginary roots.
  ↳ they come in pairs.