Quiz 1:

$ty' + y = t\ln t$

$\frac{dy}{dt} + \frac{1}{t}y = t\ln t$

$\mu(t) = e^{\int \frac{1}{t}dt} = e^{\ln t}$

$ye^{\ln t} = \int \mu(t)$

$yt = \int t\ln t$

$u : t \quad v : \ln t$

$du : 1 \quad dv :$

If we have 1st order ode that isn't exact can we multiply by integrating factor to get exact eqn? sometimes

① $M(x,y) + N(x,y)\frac{dy}{dx} = 0$

is exact if $M_y = N_x$

assume ① is not exact.
multiply by $\mu(x,y)$ to become exact

② $\mu(x,y)M(x,y) + \mu(x,y)N(x,y)\frac{dy}{dx} = 0$

if $\frac{\partial}{\partial y}\left[\mu(x,y)M(x,y)\right] = \frac{\partial}{\partial x}\left[\mu(x,y)N(x,y)\right]$ then ② is exact

$\mu M_y + \mu_y M = \mu N_x + \mu_x N$ ← is PDE we would need to solve for μ which is usually hard.

If $\mu(x,y)$ were actually just a func of x (or y) then we would get an ODE instead which may be easier to solve

③ $\mu(x)M(x,y) + \mu(x)N(x,y)\frac{dy}{dx} = 0$

if $\mu(x)M_y = \mu(x)N_x + \frac{d\mu}{dx}N$ then 3 is exact

$\mu(x)\frac{M_y - N_x}{N} = \frac{d\mu}{dx}$ can solve this ODE is $\frac{M_y - N_x}{N}$ is a func of just x (not y)

- if that doesn't happen then μ doesn't just depend on x.

Ex: $\underbrace{3xy+y^2}_{M}+\underbrace{(x^2+xy)}_{N}y'=0$

$M_y=3x+2y \quad N_x=2x+y \quad \rightarrow$ not exact

if $\dfrac{M_y-N_x}{N}$ is a func of just x we can find $\mu(x)$ to make our ODE exact.

$$\frac{M_y-N_x}{N}=\frac{3x+2y-2x-y}{x(x+y)}=\frac{x+y}{x(x+y)}=\frac{1}{x}$$

$$\mu(x)\cdot\frac{1}{x}=\frac{d\mu}{dx} \qquad \frac{1}{x}=\frac{1}{\mu}\frac{d\mu}{dx}$$

$$\int\frac{1}{x}\,dx=\int\frac{1}{\mu}\frac{d\mu}{dx}\,dx$$

$$\ln|x|=\ln|\mu|+C$$

$$e^{\ln|x|}=e^{\ln|\mu|+C}$$

$$|x|=C\mu$$

$$x=C\mu \qquad \boxed{x=\mu(x)}$$

$$3x^2y+xy^2+(x^3+x^2y)y'=0$$

$\int 3x^2y+xy^2\,dx \qquad \int x^3+x^2y\,dy$

$x^3y+\frac{1}{2}x^2y^2+g(y) \qquad x^3y+\frac{1}{2}x^2y^2+h(x)$

$$x^3y+\frac{1}{2}x^2y^2=C$$

Ex: $1+\left(\frac{x}{y}-\sin(y)\right)y'=0$

$M_y=0 \quad N_x=\frac{1}{y} \quad \rightarrow$ not exact

$$\frac{M_y-N_x}{N}=\frac{0-\frac{1}{y}}{\frac{x}{y}-\sin y}=\frac{-1/y}{\frac{x}{y}-\sin y}=-\frac{1}{x-y\sin y}$$ $\rightarrow$ no $\mu(x)$ to make exact.

is there a $\mu(y)$?

$$\mu(y)M(x,y)+\mu(y)N(x,y)\frac{dy}{dx}=0$$

if $\mu(y)M_x+\frac{d\mu}{dx}M=\mu(y)N_x$

$\dfrac{d\mu}{dy}=\mu(y)\dfrac{N_x-M_y}{M} \leftarrow$ can solve if $\frac{N_x-M_y}{M}$ depends only on y

$$\frac{N_x-M_y}{M}=\frac{1/y-0}{1}=\frac{1}{y} \qquad \frac{d\mu}{dx}=\mu(y)\cdot\frac{1}{y}\longrightarrow \mu(y)=y$$

$y+(x-y\sin y)y'=0 \rightarrow$ exact