An ODE of the form $\frac{dy}{dx} = f(x,y)$ where $f(x,y)$ can be expressed as a func of $\frac{y}{x}$ or $\frac{x}{y}$ is called Homogeneous. (there is a different meaning of Homogeneous used later)

$$\int \frac{1-v}{v^2-4}\,dv$$ Partial fraction Decomp.

$\frac{1-v}{v^2-4}$ factor denom

$$\frac{A}{v+2} + \frac{B}{v-2} = \frac{1-v}{(v-2)(v+2)}$$

$$A(v-2) + B(v+2) = 1-v$$

$$Av - 2A + vB + 2B = 1 - v$$

create system of eqns

- contain v
- don't contain v

$$\boxed{1 = -2A + 2B}$$
$$\boxed{-1 = A + B} \rightarrow A = -1 - B$$

$$1 = -2(-1-B) + 2B$$

$$1 = 2 + 2B + 2B = 2 + 4B$$

$$-1 = 4B \qquad -\frac{1}{4} = B$$

$$A = -1 - \left(-\frac{1}{4}\right) = -\frac{3}{4}$$

$$-\frac{3/4}{v+2} - \frac{1/4}{v-2} = \frac{1-v}{(v-2)(v+2)}$$

$$-\frac{3}{4}\int \frac{1}{v+2}\,dv - \frac{1}{4}\int \frac{1}{v-2}\,dv = \int \frac{1}{x}\,dx$$

$$-\frac{3}{4}\ln|v+2| - \frac{1}{4}\ln|v-2| = \ln|x| + C$$

E.X. $\frac{dy}{dx} = \frac{y-4x}{x-y}$

multiply top and bottom by $\frac{1}{x}$

$$\frac{dy}{dx} = \frac{(y-4x)\frac{1}{x}}{(x-y)\frac{1}{x}} = \frac{\frac{y}{x} - 4}{1 - \frac{y}{x}}$$

sub $v = \frac{y}{x} \rightarrow vx = y$

$$\frac{d}{dx}[vx] = \frac{dy}{dx} \rightarrow \boxed{v + x\frac{dv}{dx} = \frac{dy}{dx}}$$

$$v + x\frac{dv}{dx} = \frac{v-4}{1-v}$$

solve for v. usually looks like

$$x\frac{dv}{dx} = \frac{v-4}{1-v} - v$$

$$x\frac{dv}{dx} = \frac{v-4}{1-v} - \frac{v(1-v)}{1-v} = \frac{v-4-v(1-v)}{1-v}$$

$$x\frac{dv}{dx} = \frac{v^2-4}{1-v}$$ ← Now this is separable.

$\underbrace{\text{v goes with } \tfrac{dv}{dx}}$

$$\frac{1-v}{v^2-4}\frac{dv}{dx} = \frac{1}{x}$$

$$\underbrace{\int \frac{1-v}{v^2-4}\frac{dv}{dx}\,dx}_{\text{Need PFD}} = \underbrace{\int \frac{1}{x}\,dx}_{\ln|x|}$$

$$-\frac{3}{4}\ln\left|\frac{y}{x}+2\right| - \frac{1}{4}\ln\left|\frac{y}{x}-2\right| - \ln|x| = C$$

Ex: $\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$ $\qquad v=\frac{y}{x} \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\frac{dy}{dx} = \frac{x^2}{x^2} + \frac{xy}{x^2} + \frac{y^2}{x^2} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$$

$$v + x\frac{dv}{dx} = 1 + v + v^2$$

$$x\frac{dv}{dx} = 1+v^2 \qquad \frac{1}{1+v^2}\frac{dv}{dx} = \frac{1}{x} \qquad \int \underbrace{\frac{1}{1+v^2}}\,dv = \int \frac{1}{x}\,dx$$

Trig sub:

$\sin^2\theta + \cos^2\theta = 1$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\cos^2\theta - 1 = \sin^2\theta$

3 forms:

$x^2+1 \rightarrow$ use $x = \tan\theta$

$x^2-1 \rightarrow$ use $x = \sec\theta$

$1-x^2 \rightarrow$ use $x = \sin\theta$ (or $\cos\theta$)

$v = \tan\theta \rightarrow \tan^{-1}\theta = \theta$

$dv = \sec^2\theta$

$$\int \frac{1}{1+\tan^2\theta} \cdot \sec^2\theta\,d\theta = \int \frac{1}{\sec^2\theta} \cdot \sec^2\theta\,d\theta = \theta$$

$$\tan^{-1} v = \ln|x| + C$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \ln|x| + C$$

$$\frac{y}{x} = \tan(\ln|x| + C)$$

$$y = x\tan(\ln|x| + C)$$