Def: A set of vectors $\vec{x}_1, \dots, \vec{x}_n$ is **linearly dependent** if there exists a set of constants $c_1, c_2 \dots, c_n$ (not all 0) such that $c_1\vec{x}_1, c_2\vec{x}_2, \dots c_n\vec{x}_n = \vec{0}$

If not the vectors are **linearly independent**.

We only talk about linear independence for non-zero vectors.

- The way to check for linear independence is to use augmented matrix where columns are the vectors $\vec{x}_1, \dots, \vec{x}_n$ and the augmented column is the $\vec{0}$ vector.
  - if it reduces to $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ they are linearly independent. (identity matrix)

E.X. Determine if $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 11 \end{bmatrix}$ are linearly independent. If they are find a linear relation between them.

$c_1 x_1 + c_2 x_2 + c_3 x_3 = 0$

this is the same as solving: $\left[\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 2 & 1 & 1 & 0 \\ -1 & 3 & -11 & 0 \end{array}\right]$

Solutions will be all the possible values for $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & -3 & 9 & 0 \\ 0 & 5 & -15 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

the vectors are linearly dependent since this system has infinitely many solutions.

the only way for the vectors to be linearly independent is if the RREF is the identity matrix.

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ $c_1 = 0$, $c_2 = 0$, $c_3 = 0$

$c_1 + 2c_3 = 0 \quad c_1 = -2A$

$c_2 - 3c_3 = 0 \quad c_2 = 3A$

c_3 free $= A$

solutions: $A\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

to find a linear relation between $\vec{x}_1, \vec{x}_2, \vec{x}_3$ take any non-zero value for A and use coresponsing values of c_1, c_2, c_3

take $A=1$ $\quad c_1 = -2$

$c_2 = 3$

$c_3 = 1$

relation is $-2\begin{bmatrix}-1\\2\\-1\end{bmatrix} + 3\begin{bmatrix}2\\1\\3\end{bmatrix} + \begin{bmatrix}-4\\1\\-11\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

If you have a set of 2 vectors then:

if they are scalar multiples of each other they are linearly dependent.

Eg: $\underbrace{\begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}2\\6\end{bmatrix}}_{\times 2}$ or $\underbrace{\begin{bmatrix}-1\\4\\1\end{bmatrix}, \begin{bmatrix}1\\-4\\-1\end{bmatrix}}_{\times -1}$

(only "difficult" to determine 3+ vectors)
