The equations for this particular Padé are given in Appendix A. of Journal of Low Temperature Physics 29, 179 (1977) which is a copy paste of part of chapter 8 of the book G. A. Baker Jr., Essentials of Padé Approximants (Academic Press, New York, 1975). I suggest you check out that book, it contains many different possible Padé fitting. We have that, for this specific Padé approximant $P_N(z) = A_N(z)/B_N(z)$, once we have obtained the a_n coefficients of the continued fraction expansion (calculated from the unmodified algorithm given in the ref above)

$$\begin{aligned} A_{n+1}(z) &= A_n(z) + (z - z_n)a_{n+1}A_{n-1}(z) \\ B_{n+1}(z) &= B_n(z) + (z - z_n)a_{n+1}B_{n-1}(z) \end{aligned} \tag{1}$$

and

$$A_0 = 0, \quad A_1 = a_1, \quad B_0 = B_1 = 1. \tag{2}$$

We want a numerically stable approach to apply this algorithm. We rewrite $P_N(z)$ as

$$\begin{aligned} P_N(z) &= \frac{A_N(z)/A_{N-1}(z)}{B_N(z)/B_{N-1}(z)} \frac{A_{N-1}(z)/A_{N-2}(z)}{B_{N-1}(z)/B_{N-2}(z)} \cdots \frac{A_2(z)/A_1(z)}{B_2(z)/B_1(z)} \frac{A_1(z)}{B_1(z)} \\ &\equiv \frac{\bar{A}_N(z)}{\bar{B}_N(z)} \frac{\bar{A}_{N-1}(z)}{\bar{B}_{N-1}(z)} \cdots \frac{\bar{A}_2(z)}{\bar{B}_2(z)} \frac{A_1(z)}{B_1(z)}. \end{aligned} \tag{3}$$

From (3) we have

$$\begin{aligned} P_1(z) &= \frac{A_1(z)}{B_1(z)} = a_1, \\ \bar{A}_2(z) &= \frac{A_2(z)}{A_1(z)} = 1, \\ \bar{B}_2(z) &= \frac{B_2(z)}{B_1(z)} = 1 + a_2(z - z_1)\frac{B_0(z)}{B_1(z)} = 1 + a_2(z - z_1), \\ P_2(z) &= P_1(z)\frac{\bar{A}_2(z)}{\bar{B}_2(z)} \end{aligned} \tag{4}$$

and then

$$\begin{aligned} \bar{A}_{n+1}(z) &= 1 + a_{n+1}(z - z_n)\frac{A_{n-1}(z)}{A_n(z)} = 1 + \frac{a_{n+1}(z - z_n)}{\bar{A}_n}, \\ \bar{B}_{n+1}(z) &= 1 + a_{n+1}(z - z_n)\frac{B_{n-1}(z)}{B_n(z)} = 1 + \frac{a_{n+1}(z - z_n)}{\bar{B}_n}, \\ P_{n+1}(z) &= P_n(z)\frac{\bar{A}_{n+1}(z)}{\bar{B}_{n+1}(z)}. \end{aligned} \tag{5}$$

Since $\bar{A}_{n+1}(z)$ and $\bar{B}_{n+1}(z)$ are defined as deviation from 1, they will always be well defined in finite precision. The ratio between the two remains well defined and goes to 1 as n increases and therefore $P_{n+1}(z)$ is always stable no matter the value of n.