(Q)

define set

A1)

-collection of objects
-no duplicates
-unordered

(Q)

give an example of a set

A = {1, 2, 3, 4, 5}
[denoted by capital letter]

(Q)

what is 'membership'
denoted by?
Give an example.

denoted by: '∈'
e.g. $1 \in A$
e.g. $1 \notin B$ (doesn't belong)

(Q)

what are the 2 ways to define sets? (with examples)

1) enumerating / listing all elements:

- $B = \{2, 6, 5, 7\}$

2) providing a property that all members satisfy:

- $\{x | x \in A \text{ and } P(x)\}$

[p(x) is the property]

(Q)

what are the 4 set operations?
(describe each)

1) Union
   - -'u'
   - -or
   - -A u B = {x|x ∈ A or x ∈ B}

2) Intersection
   - -'n'
   - -and
   - -A n B = {x|x ∈ A and x ∈ B}

3) difference
   - - '-'
   - -belongs to first set and not the second set
   - -A - B = {x|x ∈ A and x ∉ B}

4) Cartesian product
   - -'x'
   - -all possible combinations of pairs between 2 sets
   - - A x B = {, , ... , }

(Q) describe
ordered pairs

-objects that have order associated with each other can be paired

-e.g. <x, y>

-two ordered pairs <a, b> and <c,d> are equal if a = c and b = d

(Q)

Describe empty sets

-contains no objects
-denoted by Ø
-e.g. A = {}

(Q)

Describe disjoint sets

-two sets are disjoint if
they have no
elements in common
-intersection is empty

(Q)

Describe equal sets

-2 sets are equal if they have the same elements
-elements can be in different orders

(Q)

define cardinality

-the number of elements in the set
-denoted by |set_name|
-e.g. |A| = 5

(Q)

What do you call a set with one element?

Singleton set

(Q)

What is the difference between a subset and a proper subset?

subset =
-every element of A is in B;
-$A \subseteq B$

proper subset =
-every element of A is in B,
-but A is not equal to B,
-there is at least one item in B that is not in A
-$A \subset B$

(Q)

describe supersets

-opposite of subset

(Q)

Describe a universal set

-non-empty set
-denoted by U
-all sets under consideration are subsets of the universal set

(Q)

describe complement
sets

-is the difference between the universal set and a given set
-denoted by: comp(A) = U - A

(Q)

What is a binary relation?

-denoted by 'R'
-defined from one set to another
-equivalent to querying to sets and getting an output
-e.g.
A = {0, 1, 2, 3}
B = {0, 1, 2, 3, 4}
<a, b> ∈ R if b + a = 3
R = {<0, 3>, <1, 2>, <2, 1>, <3, 0>}

(Q)

describe
ordered n-tuples

-equal if all the same elements exist in the same indexes

(Q)

Describe a binary set

-a relation between 2 sets
-first item is in set A
-second item is in set B
-e.g. A = {1, 2, 3}
B = {A, B, C}
binary relation = {<1, A>, <2, B>,
<3, C>, <2, A> ... etc}

(Q)

Describe an n-ary relation

-'R'
-a set of ordered tuples
-e.g. <a1, a2, ... , aN>
-a relation is stated that forms an n-ary relation from a set
-e.g. A = {0, 1, 2, 3, 4, 5}
-R on AxAxA [sets] where <a, b, c> [tuples] and a < b < c [relation]
-R = {<0, 1, 2>, <1, 2, 3>, ... ,<2, 3, 4>}

(Q)

What is a
subrelation?

-R1 is a subrelation of R2 if:
-every ordered tuple in R1 is also in R2
-$R1 \subseteq R2$

(Q)

What is a directed graph?

-Diagram
-points represent a particular element
-arrows show binary relations between points
-curled arrow shows <a, a> relation

(Q)

describe symmetry

if <a,b> exists, <b, a> exists

(Q)

describe transitivity

-<a,b> exists & <b, c> exists;
-then <a,c> exists

(Q) describe reflexitivity

if <a, a> exists for every element 'a' in R

(Q)

What is equivalence

When a set has all 3 properties of relations;

-reflexive

-symmetric

-transitive

(Q)

Describe functions

-Special type of binary relation
-associates each element of a set with a unique element of another set
-f : A $\rightarrow$ B
-domain[A] = preimage
-codomain [B] = image
-range = subset of codomain; actual output

(Q)

What is a surjective function

-codomain = range
-for every element 'y' in set B, there is at least one element 'x' in A, such that $f(x) = y$

(Q) What is an injective function

1-to-1 function

(Q)

What is a bijective function

A function that is injective and surjective

(Q)

What are the 5 operations which can be performed on functions and what do they mean?

1) sum
2) difference
3) product
4) quotient
5) composition

(Q)

Describe the sum of functions

-add the functions together
-(f + g)(x) = f(x) + g(x)
e.g.
f = {<-3, 2>, <-2, 4>, <-1, 6>, <0, 8>}
g = {<-2, 5>, <0, 7>, <2, 9>}
f + g;
"for each common 'x' value, find the sum of the 'y' values, and pair these together"

1) intercection of domains = {-2, 0}

2) add the codomain results of each of the values in the intercectino together;
-f(-2) + g(-2) = 4 + 5 = 9
-f(0) + g(0) = 8 + 7 = 15

3) combine these into a list;
f + g = {<-2, 9>, <0, 15>}

(Q)

Describe the difference of functions

-first function minus second function
-same as sum, but instead of adding the 'y' values, take the second away from the first

(Q)

Describe the product of functions

-multiply together
-same as sum but you multiply together

(Q)

Describe the quotient of functions

-divide first by second
-same as sum but you divide first by last
-dont include any divide by zeros

(Q)

Describe the composition of functions

-$(f \circ g)(x) = f(g(x))$

-must check that the output of [g(x)] is inside the domain of [f(x)]

e.g.

f = {<-2, 5>, <0, 7>, <2, 9>}

g = {<-3, 0>, <-2, 1>}

//can use -3 as a domain as 0 is in the domain of 'f'

f(g(-3)) = 7 so:

$f \circ g$ = {<-3, 7>}

(Q)

Define atomic proposition

- Proposition whose truth/falsity does not depend on the truth/falsity of any other proposition
- Aka; not affected by other propositions

(Q)

Define compound proposition

Propositions.

- constructed from:
  - atomic propositions by
    - combining them with connectives

(Q)

What are the 6 fundamental connectives?

- AND $\wedge$
- OR $\vee$
- XOR $\oplus$, or $\veebar$
- NOT $\sim$, or $\neg$
- Conditional $\rightarrow$, or $\Rightarrow$
- Biconditional $\leftrightarrow$, $\Leftrightarrow$, or $\equiv$

P	Q	$P \rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T
antecedent	consequent	conditional

Conditional – fundamental connective that combines two propositions P and Q into a third proposition called conditional or implication which is false when the antecedent P is true and its consequent Q is false; otherwise, a conditional is true ($P \rightarrow Q$)

P	Q	$P \leftrightarrow Q$
F	F	T
F	T	F
T	F	F
T	T	T

Biconditional – fundamental connective that combines two propositions P and Q into a third proposition called biconditional which is true when both P and Q have the same truth value, and it is false if P and Q have different truth values ($P \leftrightarrow Q$)

(Q)

What is the bodmas for fundamental connectives?

Operator precedence for propositional logic:

- $\neg$
- $\wedge$
- $\vee$
- $\rightarrow$
- $\leftrightarrow$

(Q)

Which ones of the following are propositions?

a) “x +1 = 1”

b) “1 + 2 = 3”

c) “2 + 2“

d) “What time is it?”

e) “Paris is in France”

f) “Let’s playxercise!”

a) Not a proposition
b) Proposition
c) Not a proposition
d) Not a proposition
e) Proposition
f) Not a proposition

Proposition - a claim about how things are that is either true or false, but not both.

(Q)

Describe logical reasoning

Argument:

- Sequence of propositions that end with a conclusion

Premise:

- Basis on which a conclusion is established

Conclusion:

- Claim that we establish as true through reasoning

If premise & conclusion are true, argument is valid.

Written:

Premise 1
Premise 2…
Premise n
___
∴ Conclusion

Premise 1
Premise 2…
Premise n
___
∴ Conclusion

(Q)

Describe propositional logic

Made of:

- Atomic propositions
- Compound propositions
- Fundamental connectives

Rules:

- Inference rules
  - Used to build valid arguments
- Transformation/replacement rules
  - Rules for manipulating propositions
  - By replacing it with logical expressions

(Q)

Describe rules of inference and name the 7 rules of inference.

- Highlights logical reasoning
- As a sequence of steps
- Which justify the conclusion

- $[P \wedge (P \rightarrow Q)] \Rightarrow Q$ modus ponens
- $[(P \rightarrow Q) \wedge \sim Q] \Rightarrow \sim P$ modus tollens

- $P \Rightarrow (P \vee Q)$ addition
- $(P \wedge Q) \Rightarrow P$ simplification

- $[(P \rightarrow Q) \wedge (Q \rightarrow R)] \Rightarrow (P \rightarrow R)$ hypothetical syllogism
- $[(P \vee Q) \wedge \sim P] \Rightarrow Q$ disjunctive syllogism

- $(P \rightarrow Q) \Rightarrow P \rightarrow (P \wedge Q)$ absorption

Logical formula:

P	**premise**
$(P \rightarrow Q)$	**premise**
$\therefore$ Q	**conclusion**

P = "cat fur was found at the scene of the crime"
Q = "dog fur was found at the scene of the crime"
A = "officer Thompson had an allergy attack"
B = "Moriarty is responsible for the crime"

P XOR Q
Q -> A
P -> B
~A
~Q
P

B

$[(Q \rightarrow A) \wedge \sim A] \Rightarrow \sim Q$ modus tollens

Q -> A	*If dog fur was found at the scene of the crime, then officer Thompson had an allergy attack*
~A	*Officer* Thompson did not have an allergy attack
~Q	*Dog fur was not found at the scene of the crime*

$[(P \veebar Q) \wedge \sim Q] \Rightarrow P$ disjunctive syllogism

P XOR Q	*Either cat fur was found at the scene of the crime, or dog fur was found at the scene of the crime*
~Q	*Dog fur was not found at the scene of the crime*
P	*Cat fur was found at the scene of the crime*

(Q)

What are the 3 logical properties of individual propositions?

&

What is the 1 logical property of. Two propositions?

Logical properties of individual propositions:

- Tautologies
- Contradictions
- Contingencies

Logical properties of two propositions:

- Logical equivalence

Could make table with these in tbh

(Q)

Describe tautologies

Propositions which are always true

e.g.

Q	~Q	$Q \vee \sim Q$
T	F	T
F	T	T

(Q)

Describe contradictions

Propositions which are always false

e.g.

Q	~Q	$Q \wedge \sim Q$
T	F	F
F	T	F

(Q)

Describe contingencies

Propositions which are not always true/false

e.g.

P	~P
T	F
F	T

(Q)

Describe logical equivalnce

2 Propositions which have the same truth value under all circumstances

: $P \equiv Q$

(Q)

Describe and name the
12 rules of
replacement

#	law	description	and	or
1	de morgans	add brackets, negate all, swap operator	~A ^ ~B = ~(A v B)	~A v ~ B = ~(A ^ B)
2	commutative	order doesn’t matter	A ^ B = B ^ A	A v B = B v A
3	associative	same operators, brackets can be (re)moved	A ^ (B ^ C) = (A ^ B) ^ C	A v (B v C) = (A v B) v C
4	distributive	expanding brackets	A ^ (B v C) = (A ^ B) v (A ^ C)	A v (B ^ C) = (A v B) ^ (A v C)
5	absoption	common letter, changing sign = com.let.	A v (A ^ B) = A	A ^ (A v B) = A
6	identity	and TRUE = 1, or FALSE = 0	A ^ 1 = 1	A v 0 = 0
7	idempotence	clones are equal	A ^ A = A	A v A = A
8	negation	^ = contradiction; v = tautology	A ^ ~A = 0	A v ~A = 1
9	double negation	double negation = normal	~(~A) = A	~(~A) = A
10	implication		n/a	A -> B = ~A v B
11	contraposition		n/a	A -> B = ~B -> ~A
12	equivalence		A <-> B = (A -> B) ^ (B -> A)	n/a