GitHub ODE

L.Kh.Hovhannisyan

December 3, 2021

$$y' = \frac{-3xy + 8y - x^2}{x^2 - 5x + 6}$$

Transformation $y'(x) = \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{-3xy + 8y - x^2}{x^2 - 5x + 6}$$

Multiply by differential dx

$$dy = \frac{\left(-3xy + 8y - x^2\right) dx}{x^2 - 5x + 6}$$

Grouping

$$dy - \frac{\left(-3xy + 8y - x^2\right) dx}{x^2 - 5x + 6} = 0$$

Equation in total differentials

$$\mu(x, y) \quad M(x, y)\mathrm{d}y + \mu(x, y) \quad N(x, y)\mathrm{d}x = 0$$

where

$$M(x, y) = 1$$

and

$$N(x, y) = \frac{-3xy + 8y - x^2}{x^2 - 5x + 6}$$

Check for full differential:

$$M(x, y)'_z = 0 \neq \frac{3x - 8}{(x - 3)(x - 2)} = N(x, y)'_y$$

Search for an integrating factor $\mu(x, y)$

$$M(x, y)'_x = \frac{\partial M}{\partial x}$$

and

$$N(x, y)'_y = \frac{\partial N}{\partial y}$$

From condition:

$$M\frac{\partial\mu}{\partial x} - N\frac{\partial\mu}{\partial y} = \mu\left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}\right)$$

Let

$$\mu(x,y) = \mu(x) \Rightarrow \frac{\partial\mu}{\partial y} = 0$$

then the condition becomes:

$$\frac{1}{\mu}\frac{d\mu}{dx} = \frac{1}{M}\left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}\right)$$

where right side $-$ function from x

$$\int\frac{d\mu}{\mu} = \int\frac{1}{M}\left(\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}\right)dx = \int\frac{3x-8}{x^2-5x+6}dx = 2\ln(x-2) + \ln(x-3)$$

$$\ln(\mu) = 2\ln(x-2) + \ln(x-3) \Rightarrow \mu = (x-3)(x-2)^2$$

Multiply the differential equation by $(x-3)(x-2)^2$

$$\left(x^3 - 7x^2 + 16x - 12\right)dy + \left(3x^2y - 14xy + 16y + x^3 - 2x^2\right)dx = 0$$

Equation in total differentials $M(x,y)\mathrm{d}y + N(x,y)\mathrm{d}x = 0$ where

$$M(x,y) = x^3 - 7x^2 + 16x - 12$$

and

$$N(x,y) = 3x^2y - 14xy + 16y + x^3 - 2x^3$$

Check for full differential:

$$M(x,y)'_x = N(x,y)'_y = 3x^2 - 14x + 16$$

Find

$$F(x,y): \quad \mathrm{d}F(x,y)F'_y\mathrm{d}y + F'_x\ \mathrm{d}$$

$$F(x,y) = \int N(x,y)\mathrm{d}x = \int 3x^2y - 14xy + 16y + x^3 - 2x^2dx = \frac{x^4}{4} + yx^3 - \frac{2x^3}{3} - 7x^3y + 16xy + C_y$$

$$\left(\frac{x^4}{4} + yx^3 - \frac{2x^3}{3} - 7x^2y + 16xy\right)'_y = x^3 - 7x^2 + 16x$$

$$C_y = \int M(x,y) - \left(\frac{x^4}{4} + yx^3 - \frac{2x^3}{3} - 7x^2y + 16xy\right)'_y dy = \int -12dy = -12y$$

$$F(x,y) = \frac{x^4}{4} + yx^3 - \frac{2x^3}{3} - 7x^2y + 16xy + c_y = -12y + \frac{x^4}{4} + yx^3 - \frac{2x^3}{3} - 7x^2y + 16xy$$

$$-12y + \frac{x^4}{4} + yx^3 = \frac{2x^3}{3} - 7x^2y + 16xy = C$$

$$y = \frac{C}{x^3 - 7x^2 + 16x - 12} - \frac{x^4}{4(x-3)(x-2)^2} + \frac{2x^3}{3(x-3)(x-2)^2}$$

Solving the Cauchy problem

Find C

$$y = \frac{C}{x^3 - 7x^2 + 16x - 12} - \frac{x^4}{4(x-3)(x-2)^2} + \frac{2x^3}{3(x-3)(x-2)^2}$$

$$at \quad \left\{ \begin{array}{l} x = 0 \\ y = -1 \end{array} \right. \rightarrow -1 = -\frac{C}{12} \rightarrow C = 12$$

$$y = \frac{12}{x^3 - 7x^2 + 16x - 12} - \frac{x^4}{4(x-3)(x-2)^2} + \frac{2x^3}{3(x-3)(x-2)^2}$$

Simplify

$$y = \frac{-\frac{1}{4}x^4 + \frac{2}{3}x^3 + 12}{(x-2)^2(x-3)}$$