SC42155 Modelling of Dynamical Systems (year 2025-26).

Mini Project

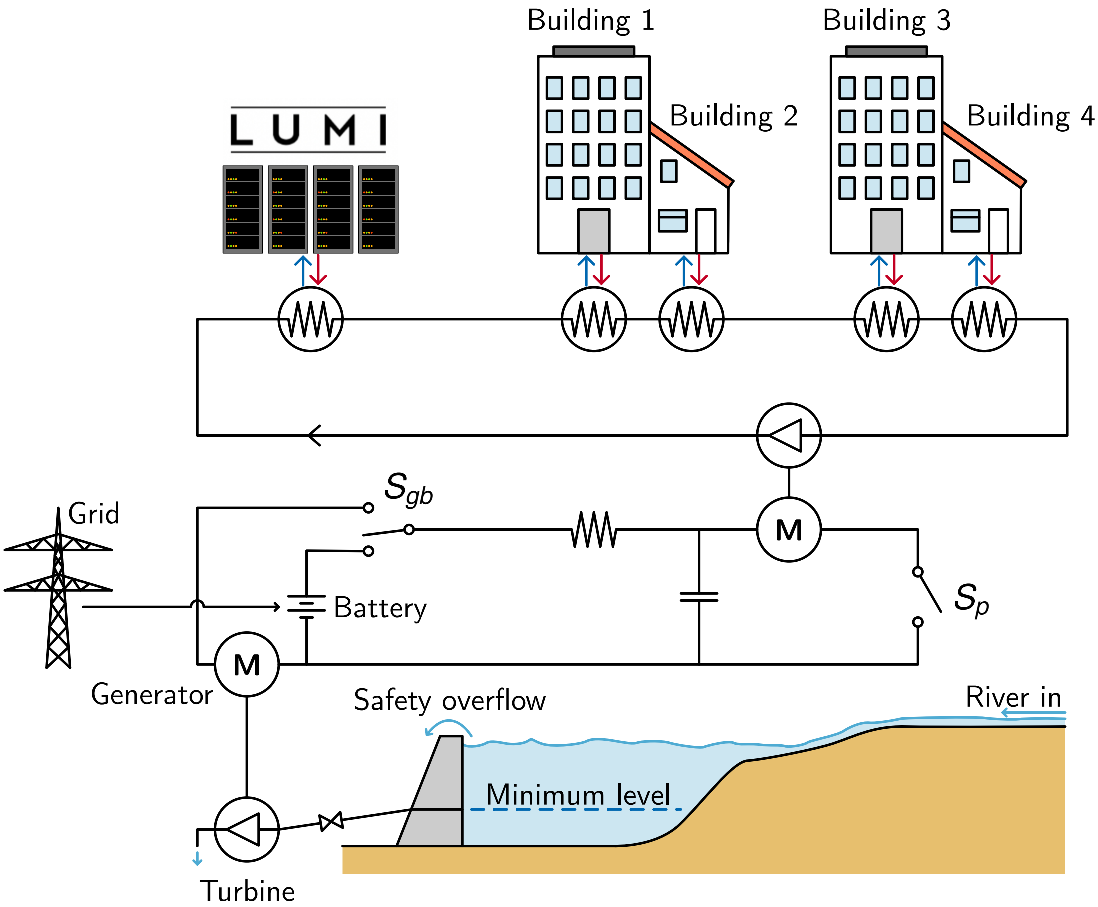

Figure 1: A piping and wiring diagram for the district heating system, including the hydroelectric power source

1 Introduction

In this exercise, we consider a hypothetical district heating system in Finland (Fig. 1). The excess heat from a supercomputer, Lumi, is used to heat water that is sent via a district heating network to individual buildings[1]. A pump recirculates water in the heating network, sending it first through a heat exchanger that extracts heat from the cooling water of Lumi. Then the district heating water goes through a series of heat exchangers, one for

[1] "LUMI's waste heat is used to heat up hundreds of households annually in the city of Kajaani." `https://www.lumi-supercomputer.eu/sustainable-future/`

each building, where it cedes heat to each building's internal heating water circuit. Some of the buildings are contiguous, such that we can assume there is some heat conduction through the buildings' walls. The pump is actuated by an electric motor, which is fed by either a hydropower plant or, when there is no hydropower available, by a backup generator. The backup generator is powered by a battery and can operate only for a limited time.

In the following, we provide a detailed description of each component, and then we comment on the goals of this modelling exercise. Actual questions to be answered follow in the next sections.

1.1 Heat source: Lumi

Lumi is a supercomputer that needs to be continuously cooled. We assume that it generates a heat amount denoted by $Q_{\mathrm{LUMI}}(t)$ which changes over time, and depends on its computation load. This heat is transferred with 100% efficiency to its dedicated cooling circuit comprising a volume $V_{\mathrm{water,LUMI}}$ of water, which moves through the circuit with a flow rate $w_{\mathrm{water,LUMI}}$. Via LUMI's heat exchanger, this water can exchange heat with the district heating fluid. The heat exchanger is modelled as a thermal resistance $R_{\mathrm{ex,LUMI}}$.

1.2 Buildings

We assume there are 4 buildings, where buildings 1 and 2 share a common wall, and 3 and 4 share a common wall. Buildings 1 and 3 are office buildings, while 2 and 4 are residential ones. The temperature of the i–th building is denoted by $T_{\mathrm{B},i}$, its thermal capacity by $C_{\mathrm{B},i}$, and the thermal resistance towards the external environment as $R_{\mathrm{B},i,0}$. Internally, each building has a heating circuit containing a volume $V_{\mathrm{water,B}}$ of water, a recirculation pump and a radiator with a thermal resistance R_{rad} between its internal fluid and the building. Finally, the building heat exchanger with the district network has a thermal resistance $R_{\mathrm{ex,B}}$. The recirculation pump can be on or off. When it is on it holds $R_{\mathrm{rad}} = R_{\mathrm{rad,low}}$ and $R_{\mathrm{ex,B}} = R_{\mathrm{ex,B,low}}$, while when it is off it holds $R_{\mathrm{rad}} = R_{\mathrm{rad,high}}$ and $R_{\mathrm{ex,B}} = R_{\mathrm{ex,B,high}}$. This because we assume the heat exchange is easier when fluid is moving.

1.3 District heating piping and pump

The district heating piping has a flow resistance of $R_{\mathrm{dh,flow}}$, while the pump has a linear relationship between its rotation speed and the flow it can produce $w_{\mathrm{dh}} = \kappa_{\mathrm{pump}}\omega_{\mathrm{pump}}$. The inertia of the pump and of the fluid in the circuit can be neglected in this exercise. The piping has a linear thermal conductance of $G_{\mathrm{dh,thermal}}$ towards the environment, measured in $\mathrm{W} \cdot \mathrm{K}^{-1} \cdot \mathrm{m}^{-1}$, and the piping total length is L_{dh}. The fluid has a thermal conductance κ_{dh} which is very low.

1.4 Pump electrical circuit

The district heating pump is moved by a motor with constant $K_{\mathrm{dh,motor}}$, which is part of an DC electric circuit with a series resistance $R_{\mathrm{dh,motor}}$ and a parallel capacitor $C_{\mathrm{dh,motor}}$. The circuit has a switch S_{p} in order to turn the motor on or off and a three-way switch S_{gb} to connect either the hydropower generator or the backup battery to the circuit..

1.5 Hydropower plant and backup generator

Power for turning the district heating pump is provided by a small-scale, local hydropower plant. The plant is fed by a water reservoir, into which a river brings a flow of water $w_{\mathrm{hydro,in}}$. The reservoir has a maximum height of $h_{\mathrm{hydro,max}}$ and a penstock located at a height of $h_{\mathrm{hydro,min}}$ draws water from it for feeding the turbine (located at a height of 0). The flow resistance of the penstock and turbine is $R_{\mathrm{hydro,flow}}$, while the turbine has an efficiency $\epsilon_{\mathrm{hydro,turbine}}$ in converting power from the flow into mechanical power to the generator. The generator can be modelled as a DC electric motor acting as generator, with a constant $K_{\mathrm{hydro,generator}}$ such that when the turbine spins at its nominal velocity $\omega^{\star}_{\mathrm{hydro,generator}}$ the voltage produced is equal to 600 V. Furthermore, the turbine has a regulation system, not described here, that makes sure that it spins constantly at its nominal speed when the water flow is present. The backup generator, which can output the same voltage of 600V, is powered by a rechargeable battery with a total capacity $C_{\mathrm{hydro,battery}}$. The battery can be recharged only using power coming from the electric grid[2].

In case the water flow is not available, then the backup generator can step in until its battery is completely depleted. To this end, two switches are present in the electrical circuit: S_{p} turns the pump electric motor on or off; S_{gb} connects/disconnects the generator and disconnects/connects the battery to the electric circuit. The way this last switch is built is such that it is not possible to connect both the generator and the battery to the circuit at the same time.

2 Physical Modelling (10 pts.)

Please answer the following sub-question

1. Derive a distributed parameter model of the district heating piping.
2. Derive lumped parameter dynamical models of all the other components (subsections 1.1 to 1.5) using first principles.
3. Convert each lumped parameters model into a state space representation, indicate clearly what are the allowed ranges of states, inputs, or outputs where needed.

[2]It would have been quite interesting and more realistic to let excess power from the hydro plant to recharge the battery. Anyway, for the sake of ease of solution, we will not consider this possibility here.

4. Choose realistic parameter values (may need to iterate this and the next steps) and present them in tables.

5. Simulate each component (including the piping from subquestion 1) individually under realistic conditions and present results using plots, tables, and discussions.

6. Derive a complete model of the system connecting the models for all components

7. Simulate the complete model under realistic conditions. Choose the conditions such that the hydropower reservoir will not deplete below the minim level, and all the pumps (in the building and in the district heating piping) will stay constantly on. Present results using plots, tables, and discussions.

8. Discuss the assumption to neglect the inertia of the pump, of the district heating piping water, of the turbine, and of the water in the penstock. Which difference would have made it not to neglect it? Is the assumption then justified?

9. Discuss if the velocity regulation system for the hydro turbine violates the principle of conservation of energy. How do you think such a system is implemented, mechanically and electrically, in real life?

3 Discrete Dynamics Model (7 pts.)

In this part of the assignment, you have to model diverse parts of the system in Figure 1 as automata, finite-state transducers, or Markov (Decision) Processes. Namely, you will model the following subsystems:

- LUMI's heat generation as an effect of its activity;
- State of the building's pumps (on/off);
- Supervisory controller for the heat distribution network pumping.

3.1 LUMI heat generation

Consider a simplified heat generation dynamics of the LUMI by assuming that the generated heat can only take three values $q^1_{LUMI} < q^2_{LUMI} < q^3_{LUMI}$, corresponding to low, middle, and high activity in the data center. Consider the following additional assumptions and a priori knowledge about the data-center activity:

- Levels of activity suffer significant variations only every 4 hours, allowing to divide the day into 6 periods: 6 am - 10 am; 10 am - 2 pm; 2 pm - 6 pm; 6 pm - 10 pm; 10 pm - 2 am; 2 am - 6 am.
- During office hours (6 am - 6 pm), the activity is either high or medium level. Statistically, one observes a high activity more often.
- After office hours and before nighttime (6 pm - 10 pm), the activity is either medium or low level. Statistically, one observes a medium activity more often.
- In the nighttime (10 pm - 6 am), all types of activity are observed, with a higher frequency for medium level of activity, followed by low activity, and rarely (but sometimes) observing a high activity.

Questions:

1. Select an appropriate modeling framework and apply it to describe the LUMI heat generation dynamics.
2. Simulate 30 days of heat generation traces and provide statistics verifying the correctness of your model. Illustrate with appropriate figures your answer.

3.2 Building's pumps

As indicated earlier, buildings 1 and 3 are office buildings, while 2 and 4 are residential ones. Each of these types of buildings employs a different control strategy to regulate its temperature. The state of the building pump p_i is switched following the events: `clock`, $T_i > T_r$, and $T_i \leq T_r$, where T_i denotes building i's temperature, T_r is a constant, and the event `clock` is triggered every 4 hours following the periods described in Section 3.1.

Residential buildings (2, 4):

- The pump remains `off` always between 10 am - 6 pm, and between 2 am - 6 am.
- Outside those hours, the pump turns `off` if the pump of the office building attached to the residential one is `on`.
- If the temperature $T_i > T_r$ the pump turns `off`.
- If the temperature $T_i \leq T_r$ the pump turns `on`, unless in conflict with the previous rules.

Office buildings (1, 3):

- The pump remains `off` always between 10 pm - 6 am.
- Outside those hours, the pump turns `on` if the temperature $T_i \leq T_r$.
- If the temperature $T_i > T_r$ the pump turns `off`.

Questions:

3. Select an appropriate modeling framework and apply it to describe the control of the pumps in buildings 1 and 2 (or 3 and 4 alternatively).
4. Select a value for T_r and select some trajectories of temperature evolution of the buildings for a 24h cycle[3]. For those temperature trajectories, produce the corresponding trajectory of the pump states. Illustrate with appropriate plots.

3.3 Heat distribution supervisor

The supervisor in charge of the heat distribution controls two actuators: i) the switch S_{P} in Figure 1, controlling whether the pump is activated or not[4]; ii) the switch S_{gb}, determining whether the back-up battery or the hydroelectric power source is powering the pump. The switch S_p can be `open` or `closed`, while S_{bg} can take the values `b` (operating with back-up battery), or `g` when running on the hydroelectric power.

[3]Not necessarily generated by the complete physical model of question 1, but reasonable. You may use a simplified model for this, e.g., a linear increase or decrease of temperature depending on the pump state.

[4]Please note that after switching off the pump may still spin for some time due to the discharge of the capacitor

The controller operates these switches every 3 hours, enforcing the following properties, depending on the state of the building's pumps p_i, $i = 1, \ldots, 4$, and the generator power $P_h \in \{\texttt{low}, \texttt{high}\}$. The actual power generated, being above or below a prescribed value P_r, determines whether the power is high or low (respectively)[5].

- **Battery (dis)charge:** When engaged, the battery can provide power for up to 6 h. The battery requires at least 12h to recharge completely from empty (after 6h of use), while if only used for 3h (from a full state), it takes 9h to recharge completely. The battery is always charged to full state before being allowed to be engaged.
- **Generator/Backup:** When $S_p = \texttt{closed}$, i.e. the pump is demanding energy, The generator is always the default choice, i.e. $S_{bg} = \texttt{g}$, unless it is not generating enough power, i.e. $P_h = \texttt{low}$, in which case the battery is activated if not prevented by the previous rule.
- **Pump activation:** The pump is activated, i.e. $S_p = \texttt{closed}$, whenever there is at least one building pump on, i.e. whenever $|\{p_i \mid p_i = \texttt{on}\}| \geq 1$. The pump is deactivated otherwise, or whenever the battery cannot be activated and the generator does not generate enough power, i.e. $P_h = \texttt{low}$.

Questions:

5. Construct an automaton that determines whether the battery may be activated at a given 3h interval.
6. Select an appropriate model formalism and capture the complete heat distribution supervisor.
7. Select some reasonable sequences of pump states p_i, and power generated level P_h (at the 3h sampling times)[6] and simulate your model for 36 hours.

4 Hybrid system (3 pts.)

1. Describe as a jump-flow system the dynamics of the hydroelectric plant, taking the river flow as an exogenous input. The model should be able to capture situations where the water level overflows or falls below the minimum level.
2. Describe using the Hybrid Automaton formalism the entire system, including the buildings' pump logic. Consider the heat distribution supervisor actions S_{bg} and S_p, and the LUMI power generated Q_{LUMI} as exogenous inputs to the model.

[5] The students should decide what a reasonable value of P_r is, based on the choices for the other parameters and the simulations done in Question 1.

[6] As explained in footnote 3, these do not need to be generated from the complete model of question 1, but you can just create synthetic, albeit realistic ones.

3. Select values for the system parameters: $T_r, P_r, \ldots,$ **etc.**. Simulate the complete system, including the heat distribution supervisor and LUMI heat generation. Provide a few illustrative plots of the system's behavior.