Turbulence phenomenology

The Gioia way.

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Velocity component at s scale

$$u_s^2 = \int_0^s E(\sigma)\sigma^{-2}d\sigma = \int_{1/s}^{\infty} E(k)dk.$$

with $E(\sigma) \sim \varepsilon^{2/3}\sigma^{5/3}\mathbf{c_d}\left(\eta/\sigma\right)\mathbf{c_e}\left(R/\sigma\right)$ and $E(k) \sim \varepsilon^{2/3}k^{-5/3}\mathbf{c_d}(\eta k)\mathbf{c_e}\left(Rk\right)$ [1].

$$\begin{cases} \mathbf{c_d}\left(x\right) = \exp\left(-\beta_d x\right) \\ \mathbf{c_e}\left(x\right) = \left(1+\beta_e x^{-2}\right)^{-17/6} \end{cases}.$$

$\mathbf{c_d}$ 形式来自Gioia 等(2006)，$\mathbf{c_e}$ 的选取来自 von Kármán. 在惯性区 $\eta \ll s \ll R$，两个修正项均为 1.

[1] Note: Gioia 等(2006) 中使用 $\mathbf{c_e}\left(\sigma/R\right)$ 形式，注意到 $R/\sigma = Rk$，为了 $\mathbf{c_e}$ 的统一形式将其定义为 $\ldots\left(R/\sigma\right)$

The uniform form of velocity u_s

Let $\xi = sk = s/\sigma$, rewrite u_S in a uniform form:

$$u_s \sim (\varepsilon s)^{1/3} \left[\int_1^\infty \xi^{-5/3} \mathbf{c_d}\left(\frac{\eta}{s}\xi\right) \mathbf{c_e}\left(\frac{R}{s}\xi\right) d\xi \right]^{1/2}.$$

Takeaway msg

- $u_s \sim (\varepsilon s)^{1/3}$ in inertial range ($\eta \ll s \ll R$).
- **提取涡体 (尺度为 s) 特征速度 u_s 的问题转化为修正函数 $\mathcal{I}$ 的讨论**，
  $\mathcal{I}(\eta/s, R/s) = \int_1^\infty \xi^{-5/3} \mathbf{c_d}\left(\frac{\eta}{s}\xi\right) \mathbf{c_e}\left(\frac{R}{s}\xi\right) d\xi$.

The phenomenology big picture

- Energy cascade

$$u_s^3/s \sim u_R^3/R.$$

  - With $\varepsilon \sim u_R^3/R$, $\eta = \left(\nu^3/\varepsilon\right)^{1/4} \sim R \cdot Re^{-\frac{3}{4}}$.
- Local wall shear stress model

$$\tau \sim \rho v_t v_n \sim \rho V u_s.$$

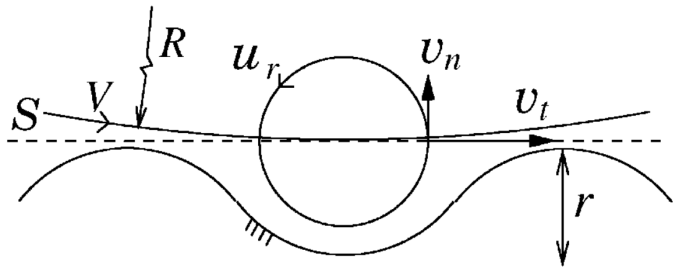

图 1: Schematic of the dominant eddies at immediate vicinity of wall (Gioia 等, 2001).

参考文献 I

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