Turbulence phenomenology

The Gioia way.

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2025 年 7 月 19 日

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Velocity component at s scale

$$u_s^2 = \int_0^s E(\sigma)\sigma^{-2} d\sigma = \int_{1/s}^{\infty} E(k) dk.$$

with $E(\sigma) \sim \varepsilon^{2/3}\sigma^{5/3}\mathbf{c_d}\left(\eta/\sigma\right)\mathbf{c_e}\left(R/\sigma\right)$ and $E(k) \sim \varepsilon^{2/3}k^{-5/3}\mathbf{c_d}(\eta k)\mathbf{c_e}\left(Rk\right)$ [1].

$$\begin{cases} \mathbf{c_d}\left(x\right) = \exp\left(-\beta_d x\right) \\ \mathbf{c_e}\left(x\right) = \left(1 + \beta_e x^{-2}\right)^{-17/6} \end{cases}.$$

$\mathbf{c_d}$ 形式来自Gioia 等(2006)， $\mathbf{c_e}$ 取自 von Kármán. 在惯性区 $\eta \ll s \ll R$， 两个修正项均为 1.

[1] Note: Gioia 等(2006) 中使用 $\mathbf{c_e}\left(\sigma/R\right)$ 形式，注意到 $R/\sigma = Rk$，为了 $\mathbf{c_e}$ 的统一形式将其定义为 $\mathbf{c_e}\left(R/\sigma\right)$

The uniform form of velocity u_s

Let $\xi = sk = s/\sigma$, rewrite u_s in a uniform form:

$$u_s \sim (\varepsilon s)^{1/3} \left[\int_1^{\infty} \xi^{-5/3} \mathbf{c_d} \left(\frac{\eta}{s} \xi \right) \mathbf{c_e} \left(\frac{R}{s} \xi \right) d\xi \right]^{1/2} \triangleq (\varepsilon s)^{1/3} \sqrt{\mathcal{I}}.$$

Takeaway msg

- $u_s \sim (\varepsilon s)^{1/3}$ in inertial range ($\eta \ll s \ll R$).
- 提取涡体 (尺度为 s) 特征速度 u_s 的问题转化为修正函数 $\mathcal{I}$ 的讨论，$\mathcal{I}(\eta/s, R/s) \triangleq \int_1^{\infty} \xi^{-5/3} \mathbf{c_d} \left(\frac{\eta}{s} \xi \right) \mathbf{c_e} \left(\frac{R}{s} \xi \right) d\xi$.

The phenomenology big picture

- Energy cascade

$$u_s^3/s \sim u_R^3/R.$$

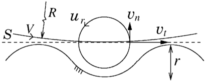
  - With $\varepsilon \sim u_R^3/R$, $\eta = \left(\nu^3/\varepsilon\right)^{1/4} \sim R \cdot Re^{-\frac{3}{4}}$.

The Gioia way: Local shear stress model

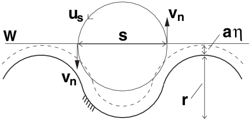
Eddies (u_s, s) that straddle wetted surface $W_y \implies$ shear stress τ,

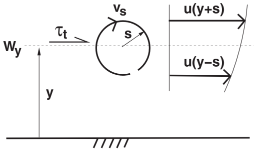
- v_t 代表涡体能够传递的动量大小，取决于湿面积两侧的动量差（$\propto \frac{\partial u}{\partial y} \cdot s$）
- v_n 代表涡体传递动量的速率，取决于涡体的特征速度（$\sim u_s$）

$$\tau \sim \rho v_t v_n.$$



- Local wall shear stress model (Gioia 等, 2006): $v_t \sim V$, $v_n \sim u_{r+a\eta}$
- Local water column shear stress model (Gioia 等, 2010):
  $v_t \sim u(y+s) - u(y-s) \approx 2su'(y) \sim 2y \cdot u'(y)$, $v_n \sim u_y$





Unify the local shear stress model

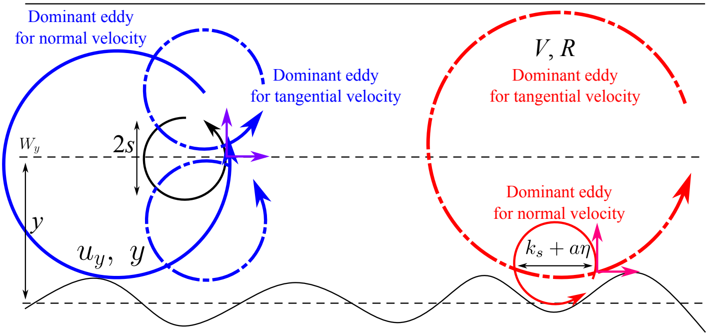


图 1: The unified picture of the Gioia model.

Unify the local shear stress model

- v_n： 近床面位置，特征涡体由粗糙高度 k_s 和黏性底层 $a\eta$ 共同控制 $s = k_s + a\eta$；外区部分的特征涡体完全由床面高度 y 控制[2]
- v_t： Gioia 的贡献。

[2]Townsend 附着涡模型存在类似的特征尺度正比于床面距离的涡体。或许可视作仅考虑外区涡体的特例。

参考文献 I

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- GIOIA G, GUTTENBERG N, GOLDENFELD N, et al., 2010. Spectral Theory of the Turbulent Mean-Velocity Profile[J/OL]. Phys. Rev. Lett., 105: 184501. https://link.aps.org/doi/10.1103/PhysRevLett.105.184501. DOI: 10.1103/PhysRevLett.105.184501.