Name:__________________________________ Section:_________

Laboratory Exercise # 5

Inverted Pendulum Modeling and Identification

ECE/ME 772 & 872: Control Systems

Instructions: Come prepared to your lab section by reviewing lecture notes, references, and this lab assignment. You will only be given access to the lab space during the assigned time slot for your section, so it is important you come prepared. *The lab report is due at 5pm one week after the lab*.

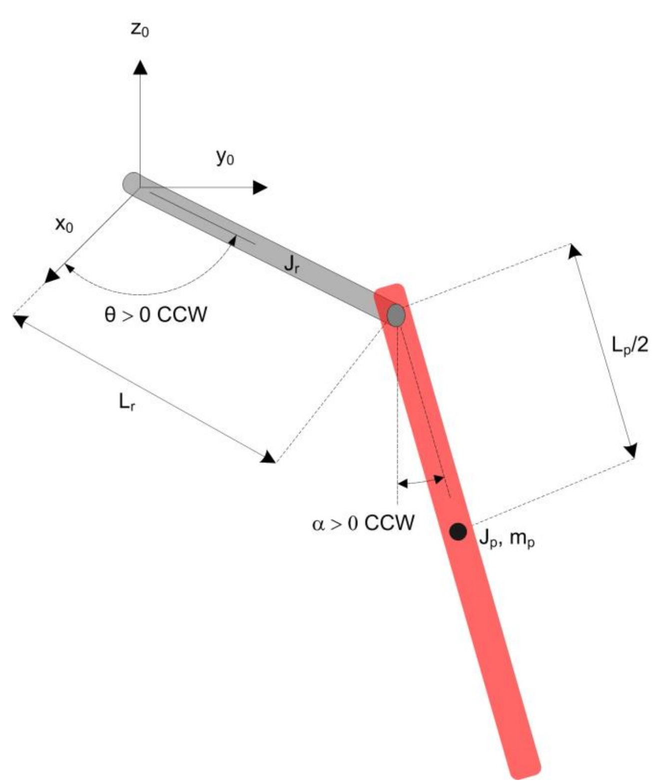
Each student should prepare the lab report as follows. A copy of this lab assignment with your name and section number must be included as the cover page, followed by your typed answers to each question in the assignment. Clearly specify the question number before your answer. Your answers should include *all figures and tables with the appropriate labels and legends*. Points will be given for the overall look of the report, as well as the completeness of your answers.

All groups will need to show the TA initials at the checkpoints marked as "***TA Check and Initials***" before leaving the lab.

Objectives: Derive the mathematical model of the Qube Servo system with the inverted pendulum attachment, and experimentally validate the model.

Part 1: Modeling of the Pendulum

A schematic drawing of the Qube Servo system with the inverted pendulum is shown in the figure below.



The angle of the rotary arm θ, the angle of the pendulum α, and the motor applied torque τ are defined as positive when rotated in the counter-clockwise direction.

The equations of motion of the Qube system are given as

$$\left(m_pL_r^2+\frac{1}{4}m_pL_p^2-\frac{1}{4}m_pL_p^2\cos^2(\alpha)+J_r\right)\ddot{\theta}-\left(\frac{1}{2}m_pL_pL_r\cos(\alpha)\right)\ddot{\alpha}$$
$$+\left(\frac{1}{2}m_pL_p^2\sin(\alpha)\cos(\alpha)\right)\dot{\theta}\dot{\alpha}+\left(\frac{1}{2}m_pL_pL_r\sin(\alpha)\right)\dot{\alpha}^2=\tau-D_r\dot{\theta}$$

$$\frac{1}{2}m_pL_pL_r\cos(\alpha)\ddot{\theta}+\left(J_p+\frac{1}{2}m_pL_p^2\right)\ddot{\alpha}-\frac{1}{4}m_pL_p^2\cos(\alpha)\cos(\alpha)\dot{\theta}^2$$
$$+\frac{1}{2}m_pL_pg\sin(\alpha)=-D_p\dot{\alpha}$$

Model parameters not included in the schematic drawing include the viscous damping constants $D_r=0.0015\,\text{N-m-s/rad}$ and $D_p=0.0005\,\text{N-m-s/rad}$, and gravity g. The applied torque at the base of the rotary arm generated by the servomotor is described by the equation

$$\tau=\frac{k_t(V_m-k_m\dot{\theta})}{R_m}$$

where k_m is the motor back-emf constant, k_t is the torque constant, V_m is the input voltage and R_m is the armature resistance.The above equations of motion are nonlinear. To apply linear control methods, we need to linearize the equations of motion. This can be done using the Taylor expansion approximation about a specific operating point, assuming that we only allow small deviation of the states from this operating point.

For operating point $[\theta_0\ \alpha_0\ \dot{\theta}_0\ \dot{\alpha}_0]=[0\ 0\ 0\ 0]$, the resulting linearized equation of motion becomes

$$(m_pL_r^2+J_r)\ddot{\theta}-\frac{1}{2}m_pL_pL_r\ddot{\alpha}=\tau-D_r\dot{\theta}$$
$$\frac{1}{2}m_pL_pL_r\ddot{\theta}+\left(J_p+\frac{1}{4}m_pL_p^2\right)\ddot{\alpha}+\frac{1}{2}m_pL_pg\alpha=-D_p\dot{\alpha}$$

For operating point $[\theta_0\ \alpha_0\ \dot{\theta}_0\ \dot{\alpha}_0]=[0\ \pi\ 0\ 0]$, the resulting linearized equation of motion becomes

$$(m_pL_r^2+J_r)\ddot{\theta}+\frac{1}{2}m_pL_pL_r\ddot{\alpha}=\tau-D_r\dot{\theta}$$
$$-\frac{1}{2}m_pL_pL_r\ddot{\theta}+\left(J_p+\frac{1}{4}m_pL_p^2\right)\ddot{\alpha}-\frac{1}{2}m_pL_pg\alpha=-D_p\dot{\alpha}$$

Letting $z=[\theta\ \ \alpha]^{\mathrm{T}}$, linearized equation of motion becomes $M\ddot{z}+G\dot{z}+Kz=FV_m$. Finally, let $x=[z^{\mathrm{T}}\ \ \dot{z}^{\mathrm{T}}]^{\mathrm{T}}$ and

$$\dot{x}=\begin{bmatrix}0 & I\\ -M^{-1}K & -M^{-1}G\end{bmatrix}x+\begin{bmatrix}0\\ M^{-1}F\end{bmatrix}V_m$$
$$\dot{x}=Ax+Bu$$

Symbol	Description	Value
k_t	Torque constant	0.042 N-m/A
k_m	Motor back-emf constant	0.042 V/(rad/s)
R_m	Terminal resistance	8.4 Ω
J_r	Rotary arm moment of inertia	$m_r \times L_r^2/12$
L_r	Rotary arm length	0.085 m
m_r	Rotary arm mass	0.095 kg
J_p	Pendulum moment of inertia	$m_p \times L_p^2/12$
L_p	Pendulum length	0.129 m
m_p	Pendulum mass	0.024 kg

Question 1: Find the *M, G, K* and *F* matrices corresponding to the linearized equation of motion at the operating point $[\theta_0\ \alpha_0\ \dot{\theta}_0\ \dot{\alpha}_0] = [0\ 0\ 0\ 0]$

Question 2: Find the *M, G, K* and *F* matrices corresponding to the linearized equation of motion at the operating point $[\theta_0\ \alpha_0\ \dot{\theta}_0\ \dot{\alpha}_0] = [0\ \pi\ 0\ 0]$

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<u>Part 2: State Space Equation of the Pendulum</u>

Using the linearized equation of motion from Part 1, create a state space model of the Qube system in Matlab.

$$\dot{x} = Ax + Bu,$$
$$y = Cx + Du,$$

where $y = [\theta\ \ \alpha]^{\mathrm{T}}$.

1. Define the state space matrices A, B, C and D in Matlab
2. Use the command 'ss(A,B,C,D)' to create the space space model

Question 3: Find the pole locations of the state space model at $[\theta_0\ \alpha_0\ \dot{\theta}_0\ \dot{\alpha}_0] = [0\ 0\ 0\ 0]$. Is the system stable? Does this accurately describe the system behavior? Explain

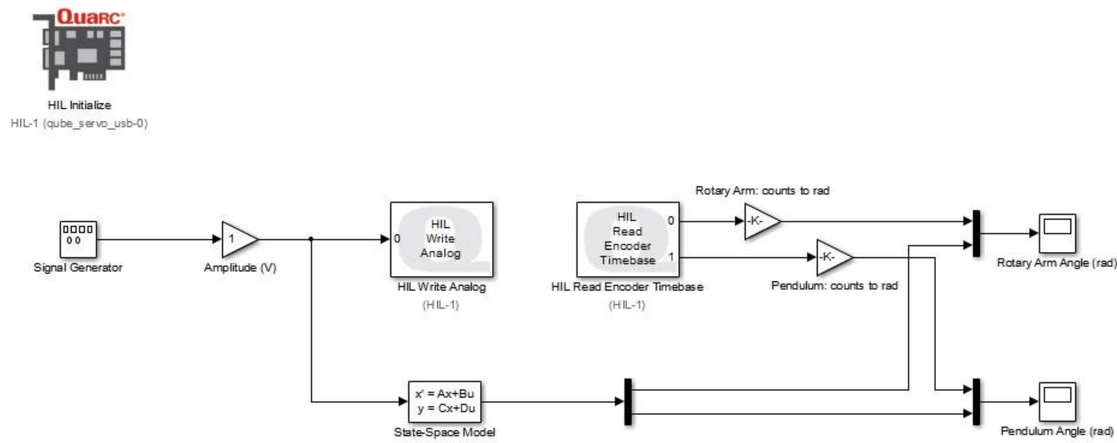
Question 4: Find the pole locations of the state space model at $[\theta_0\ \alpha_0\ \dot{\theta}_0\ \dot{\alpha}_0] = [0\ \pi\ 0\ 0]$. Is the system stable? Does this accurately describe the system behavior? Explain

<u>TA Check and Initials</u>

<u>Part 3: Model Identification</u>

Next we validate the linearized mathematical model of the pendulum.

Build a Quarc model that compares the measured response of the pendulum to the linearized model at $[\theta_0\ \alpha_0\ \dot{\theta}_0\ \dot{\alpha}_0] = [0\ 0\ 0\ 0]$



1. Set Signal Generator to a square from 0-1V and frequency 1Hz
2. Set the gain Amplitude initially to 0.
3. Set the counts to rad Gain to 2π/2048
4. Set State-Space Model to the linearized model at $[\theta_0\ \alpha_0\ \dot{\theta}_0\ \dot{\alpha}_0] = [0\ 0\ 0\ 0]$

   TA Check and Initials

5. Set the gain Amplitude to 1.
6. Run the model for a few seconds. The motor should follow a small oscillation of 1 cycle per second. Collect the angles measured by the encoders and stop model.

Question 5: Compare the model prediction and the measured response of the rotary arm and pendulum angles. Include figures of the model and measured angle responses. Does the model accurately predict the system behavior? Explain the reason of any discrepancies.

Question 6: The viscous damping of each inverted pendulum can vary slightly from system to system. If your model does not accurately represent your specific pendulum system, try modifying the damping coefficients D_r and D_p to obtain a more accurate model. Show your final results by providing the final damping constants, and figures comparing model prediction and measured response.