Name:________________________________ Section:_________

Laboratory Exercise # 6

Inverted Pendulum Control

ECE/ME 772 & 872: Control Systems

Instructions: Come prepared to your lab section by reviewing lecture notes, references, and this lab assignment. You will only be given access to the lab space during the assigned time slot for your section, so it is important you come prepared. *The lab report is due at 5pm one week after the lab*.

Each student should prepare the lab report as follows. A copy of this lab assignment with your name and section number must be included as the cover page, followed by your typed answers to each question in the assignment. Clearly specify the question number before your answer. Your answers should include *all figures and tables with the appropriate labels and legends*. Points will be given for the overall look of the report, as well as the completeness of your answers.

All groups will need to show the TA initials at the checkpoints marked as "***TA Check and Initials***" before leaving the lab.

Objectives: Design and implement a feedback control law for the inverted pendulum.

Part 1: Design of Pendulum Balancing Control Law

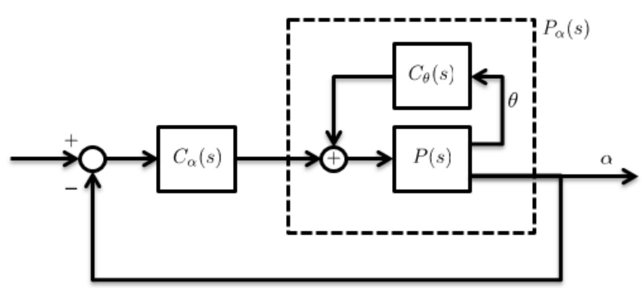
The state space equation of the inverted pendulum are given as

$$P(s): \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + Bu(t) \\ y & = & Cx(t) + Du(t) \end{array}$$

for rotary arm angle θ, inverted pendulum angle α, input $u = V_m$, $x = [\theta \quad \alpha \quad \dot{\theta} \quad \dot{\alpha}]^{\mathrm{T}}$ and $y = [\theta \quad \alpha]^{\mathrm{T}}$. The above state space equation has 2 outputs and 1 input, thus it is a multi-input multi-output (MIMO) system.

Our main focus is to control the pendulum angle α. In order to simplify the controller design problem, we will convert the MIMO system into a single-input single-output (SISO) system by closing the rotary arm position feedback. Your new SISO plant is $P_\alpha(s)$.

The state space equation matrices, $P_\alpha(s)$ and $C_\theta(s)$ can be downloaded from MyCourses.



Question 1: Find a balancing control law $C_\alpha(s)$ using Matlab for closed-loop stability, and closed-loop poles with 1) damping ratio = 0.5, and 2) natural frequency = 40 rad/s. Place pole of compensator

at -70. Include in your solutions the root locus plot, compensator equation, closed-loop poles with damping ratio and natural frequencies, and closed-loop step response. ***TA Check and Initials***

Question 2: Obtain the Nyquist diagram and determine the stability of the feedback system. How robust is your pendulum balancing control system? Find the gain margin, phase margin and delay margin. Show the stability margin measurements from the Bode plots

Part 2: Implementation and Testing

The controller derived in Question 1 will be tested on the pendulum through the provided simution model.

- Implement your control law $C_\alpha(s)$ into the Simulink model. Set the initial pendulum angle to straight up position. ***TA Check and Initials.***
- Run the model. Add a small angle to the pendulum and repeat the simulation.

Question 3: Does the pendulum behave in agreement to your analysis during the controller design process? Explain. If not, revise your controller in Part 1, and try again. Include the measurements of the rotary arm angle, pendulum angle, and control voltage in your solution.

Part 3: MIMO Control Method

In SISO systems, the Root Locus method allow us to design a compensator that places the dominant closed-loop poles at a predefined location. For multi-input multi-output (MIMO) systems with state feedback control, we can arbitrarily place all poles (eigenvalues) of the system under certain system assumptions.

For the MIMO plant $P(s)$, let the control

$$u(t) = -Kx(t), \text{ where } K = [k_1 \quad k_2 \quad k_3 \quad k_4].$$

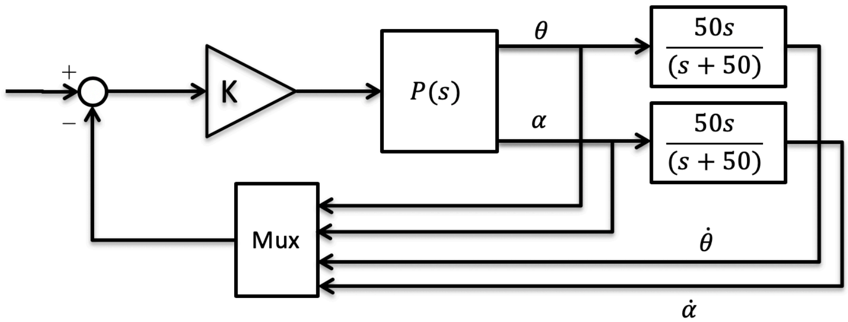
The system state equation of the feedback system reduces to

$$\dot{x}(t) = (A - BK)x(t) = A_c x(t),$$

where the eigenvalues of A_c are the poles (eigenvalues) of the closed-loop system. The eigenvalues of A_c can be placed arbitrarily in this example using the command `place` in Matlab.

Question 4: Find the gain K that places the poles at the four most dominant locations of your closed-loop system from Question 1. Provide the value of the gain K.

Next, we implement the state feedback control in your Simulink model as shown in the figure below. For this you'll have to construct the state vector $x(t) = [\theta \quad \alpha \quad \dot{\theta} \quad \dot{\alpha}]^{\mathrm{T}}$, and the feedback control signal $u(t) = -Kx(t)$. Make sure that the "multiplication" option of the gain block for $-K$ is set to "matrix (K*u)",



Question 5: How does the pendulum behave comparatively to your observations in Question 3? Include the measurements of the rotary arm angle, pendulum angle, and control voltage in your solution.