



**Bi-directional Binding Signature**



$$\begin{array}{c}
\rho : \text{Sub}_\Sigma(\Xi, \emptyset) \\
\Gamma, \vec{x}_1 : \Delta_1 \langle \rho \rangle \vdash_{\Sigma, \Omega} t_1 : {}^{\textcolor{violet}{d}_1} A_1 \langle \rho \rangle \quad \cdots \quad \Gamma, \vec{x}_n : \Delta_n \langle \rho \rangle \vdash_{\Sigma, \Omega} t_n : {}^{\textcolor{violet}{d}_n} A_n \langle \rho \rangle \\
\hline
\Gamma \vdash_{\Sigma, \Omega} \text{op}_o(\vec{x}_1 \cdot t_1; \dots; \vec{x}_n \cdot t_n) : {}^{\textcolor{violet}{d}} A_0 \langle \rho \rangle
\end{array}
\text{OP}$$

for  $o : \Xi \triangleright [\Delta_1] A_1^{\textcolor{violet}{d}_1}, \dots, [\Delta_n] A_n^{\textcolor{violet}{d}_n} \rightarrow A_0^{\textcolor{violet}{d}}$  in  $\Omega$

Bidirectional Type System

is either

(synthesis) or

(checking)



*d.*





$$\frac{\Gamma \vdash_{\Sigma, \Omega} t : \Rightarrow B \quad B = A}{\Gamma \vdash_{\Sigma, \Omega} t : \Leftarrow A} \text{SUB} \Leftarrow$$

$$\frac{\Gamma \vdash_{\Sigma, \Omega} t : \Leftarrow A}{\Gamma \vdash_{\Sigma, \Omega} (t \circ A) : \Rightarrow A} \text{ ANNO} \Rightarrow$$

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash_{\Sigma, \Omega} x : \Rightarrow A} \text{VAR} \Rightarrow$$

$$\Gamma \vdash_{\Sigma, \Omega} t :: \textcolor{violet}{d} \ A$$

$$\begin{array}{c}
\rho : \text{Sub}_\Sigma(\Xi, \emptyset) \\
\Gamma, \vec{x}_1 : \Delta_1 \langle \rho \rangle \vdash_{\Sigma, \Omega} t_1 :^{d_1} A_1 \langle \rho \rangle \quad \cdots \quad \Gamma, \vec{x}_n : \Delta_n \langle \rho \rangle \vdash_{\Sigma, \Omega} t_n :^{d_n} A_n \langle \rho \rangle \\
\hline
\Gamma \vdash_{\Sigma, \Omega} \text{op}_o(\vec{x}_1 \cdot t_1; \dots; \vec{x}_n \cdot t_n) :^d A_0 \langle \rho \rangle
\end{array}
\text{OP}$$

for  $o : \Xi \triangleright [\Delta_1]A_1^{d_1}, \dots, [\Delta_n]A_n^{d_n} \rightarrow A_0^d$  in  $\Omega$