Mode-Correctness of Bidirectional Typing

The type information flows without blocking

$$\frac{\Gamma, x : A \vdash t :^{\leftarrow} B}{\Gamma \vdash \lambda x . t :^{\leftarrow} A \supset B} \text{ABS}^{\leftarrow}$$

$$\frac{\Gamma \vdash t : \stackrel{\Rightarrow}{\rightarrow} A \supset B}{\Gamma \vdash t : \stackrel{\Rightarrow}{\rightarrow} B} \xrightarrow{\Gamma \vdash u : \stackrel{\Leftarrow}{\rightarrow} A} \xrightarrow{\text{APP}} A \xrightarrow{} B$$

Mode-Correctness of Bidirectional Typing

In checking mode, the information flow starts from the conclusion

$$\frac{\Gamma, x: \vdash t:^{\Leftarrow}}{\Gamma \vdash \lambda x.t:^{\Leftarrow} \mathsf{Int} \supset \mathsf{Bool}} \mathsf{ABS}^{\Leftarrow} \qquad \frac{\Gamma \vdash t:^{\Rightarrow} A \supset B}{\Gamma \vdash t u:^{\Rightarrow} B} \qquad \frac{\Gamma \vdash u:^{\Leftarrow} A}{\mathsf{APP}^{\Rightarrow}} \mathsf{APP}^{\Rightarrow}$$