Mode-Correctness of Bidirectional Typing

In checking mode, the information flow starts from the conclusion

$$\frac{\Gamma, x: \vdash t:^{\Leftarrow}}{\Gamma \vdash \lambda x.t:^{\Leftarrow} \mathsf{Int} \supset \mathsf{Bool}} \mathsf{ABS}^{\Leftarrow} \qquad \frac{\Gamma \vdash t:^{\Rightarrow} A \supset B}{\Gamma \vdash t u:^{\Rightarrow} B} \qquad \frac{\Gamma \vdash u:^{\Leftarrow} A}{\mathsf{APP}^{\Rightarrow}} \mathsf{APP}^{\Rightarrow}$$

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$$\frac{\Gamma \vdash t :^{\Rightarrow} A \supset B}{\Gamma \vdash t : u :^{\Rightarrow} B} \xrightarrow{\Gamma \vdash u :^{\Leftarrow} A} A_{PP}^{\Rightarrow}$$