Bidirectional Type System

$$\frac{\rho\colon \mathsf{Sub}_\Sigma(\Xi, \emptyset) \qquad \Gamma, \vec{x}_1 : \Delta_1\langle\rho\rangle \vdash_{\Sigma,\Omega} t_1 :^{d_1} A_1\langle\rho\rangle \quad \cdots \quad \Gamma, \vec{x}_n : \Delta_n\langle\rho\rangle \vdash_{\Sigma,\Omega} t_n :^{d_n} A_n\langle\rho\rangle}{\Gamma \vdash_{\Sigma,\Omega} \mathsf{op}_o(\vec{x}_1.\, t_1; \ldots; \vec{x}_n.\, t_n) :^{d} A_0\langle\rho\rangle}\ \textsc{Op}$$

$$\text{for } o\colon \Xi \rhd [\Delta_1]A_1^{d_1}, \ldots, [\Delta_n]A_n^{d_n} \to A_0^{d} \text{ in } \Omega$$

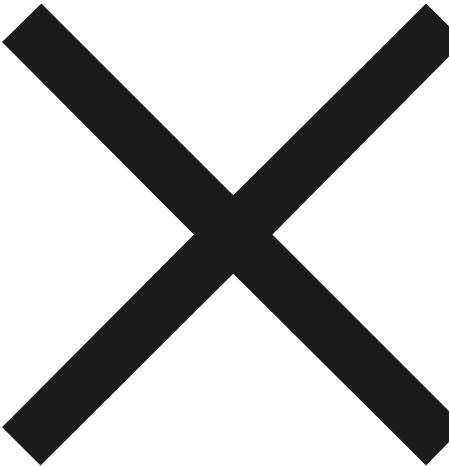
$$\frac{\Gamma \vdash_{\Sigma,\Omega} t :^{\Rightarrow} B \qquad B = A}{\Gamma \vdash_{\Sigma,\Omega} t :^{\Leftarrow} A}\ \textsc{Sub}^{\Leftarrow}$$

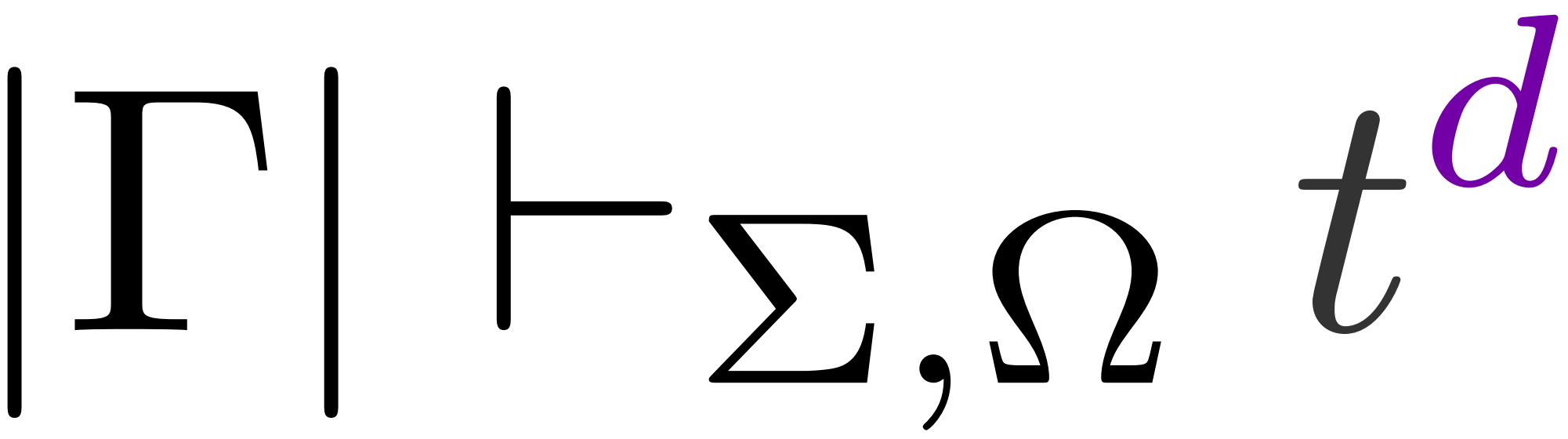
$$\frac{\Gamma \vdash_{\Sigma,\Omega} t :^{\Leftarrow} A}{\Gamma \vdash_{\Sigma,\Omega} (t \mathbin{:\!\!\circ} A) :^{\Rightarrow} A}\ \textsc{Anno}^{\Rightarrow}$$

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash_{\Sigma,\Omega} x :^{\Rightarrow} A}\ \textsc{Var}^{\Rightarrow}$$

Soundness & Completeness

$$\Gamma \vdash_{\Sigma,\Omega} t :^{d} A$$





$$\Gamma \vdash_{\Sigma,\Omega} t : A$$

$$\Gamma \vdash_{\Sigma,\Omega} t :^{d} A$$