From Simply Typed λ-Calculus to a Bidirectional Variant

A synthesised type can be used to check against a give type

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x :^{\Rightarrow} A}\ \textsc{Var}^{\Rightarrow} \qquad \frac{\Gamma \vdash t :^{\Leftarrow} A}{\Gamma \vdash (t \ni A) :^{\Rightarrow} A}\ \textsc{Anno}^{\Rightarrow} \qquad \frac{\Gamma \vdash t :^{\Rightarrow} B \qquad B = A}{\Gamma \vdash t :^{\Leftarrow} A}\ \textsc{Sub}^{\Leftarrow}$$

$$\frac{\Gamma, x : A \vdash t :^{\Leftarrow} B}{\Gamma \vdash \lambda x.t :^{\Leftarrow} A \supset B}\ \textsc{Abs}^{\Leftarrow} \qquad \frac{\Gamma \vdash t :^{\Rightarrow} A \supset B \qquad \Gamma \vdash u :^{\Leftarrow} A}{\Gamma \vdash t\ u :^{\Rightarrow} B}\ \textsc{App}^{\Rightarrow}$$

Bidirectional Typing Judgements

$$\Gamma \vdash t :^{\Leftarrow} A$$

Γ, t, and A are all inputs

$$\Gamma \vdash t :^{\Rightarrow} A$$

Γ and t are inputs and A is the output