From Simply Typed λ-Calculus to a Bidirectional Variant

The type of a variable can be looked up in the context Γ

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x :^{\Rightarrow} A}\ \textsc{Var}^{\Rightarrow}$$

$$\frac{\Gamma \vdash t : A}{\Gamma \vdash (t \Rightarrow A) : A}\ \textsc{Anno}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x.t : A \supset B}\ \textsc{Abs}$$

$$\frac{\Gamma \vdash t : A \supset B \qquad \Gamma \vdash u : A}{\Gamma \vdash t\ u : B}\ \textsc{App}$$

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An annotated term synthesises the given type if the annotation can be checked

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x :^{\Rightarrow} A}\ \textsc{Var}^{\Rightarrow} \qquad \frac{\Gamma \vdash t :^{\Leftarrow} A}{\Gamma \vdash (t \ni A) :^{\Rightarrow} A}\ \textsc{Anno}^{\Rightarrow}$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x.t : A \supset B}\ \textsc{Abs} \qquad \frac{\Gamma \vdash t : A \supset B \qquad \Gamma \vdash u : A}{\Gamma \vdash t\ u : B}\ \textsc{App}$$