

From Simply Typed λ -Calculus to a Bidirectional Variant

An annotated term synthesises the given type if the annotation can be checked

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : \Rightarrow A} \text{VAR} \Rightarrow$$

$$\frac{\Gamma \vdash t : \Leftarrow A}{\Gamma \vdash (t \circ A) : \Rightarrow A} \text{ANNO} \Rightarrow$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x. t : A \supset B} \text{ABS}$$

$$\frac{\Gamma \vdash t : A \supset B \quad \Gamma \vdash u : A}{\Gamma \vdash t u : B} \text{APP}$$

From Simply Typed λ -Calculus to a Bidirectional Variant

The type of an argument is checked against the domain of the function

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : \Rightarrow A} \text{VAR} \Rightarrow$$

$$\frac{\Gamma \vdash t : \Leftarrow A}{\Gamma \vdash (t \circ A) : \Rightarrow A} \text{ANNO} \Rightarrow$$

$$\frac{\Gamma, x : A \vdash t : \Leftarrow B}{\Gamma \vdash \lambda x. t : \Leftarrow A \supset B} \text{ABS} \Leftarrow \quad \frac{\Gamma \vdash t : \Rightarrow A \supset B \quad \Gamma \vdash u : \Leftarrow A}{\Gamma \vdash t u : \Rightarrow B} \text{APP} \Rightarrow$$