Mode-Correctness

- To synthesise the types of premises of a rule,
  - for a checking premise, all types need to be known by the checking site;
  - for a synthesis premise, only types in the context need to be known, and the synthesised meta-type variables become known.

$$
\begin{aligned}
\mathsf{MC}_{as}(\cdot) &= \top \\
\mathsf{MC}_{as}\left(\overrightarrow{[\Delta_i]A_i^{d_i}}, [\Delta_n]A_n^{\Leftarrow}\right) &= \mathit{fv}(\Delta_n, A_n) \subseteq \left(S \cup \mathit{fv}^{\Rightarrow}\left(\overrightarrow{[\Delta_i]A_i^{d_i}}\right)\right) \land \mathsf{MC}_{as}\left(\overrightarrow{[\Delta_i]A_i^{d_i}}\right) \\
\mathsf{MC}_{as}\left(\overrightarrow{[\Delta_i]A_i^{d_i}}, [\Delta_n]A_n^{\Rightarrow}\right) &= \mathit{fv}(\Delta_n) \subseteq \left(S \cup \mathit{fv}^{\Rightarrow}\left(\overrightarrow{[\Delta_i]A_i^{d_i}}\right)\right) \land \mathsf{MC}_{as}\left(\overrightarrow{[\Delta_i]A_i^{d_i}}\right)
\end{aligned}
$$

Decidability of Bidirectional Type Checking and Synthesis

See our paper for the algorithm/proof

- For a mode-correct bidirectional type system specified by (Σ, Ω),
  - if $|\Gamma| \vdash_{\Sigma,\Omega} t^{\Rightarrow}$, then it is decidable whether $\Gamma \vdash_{\Sigma,\Omega} t :^{\Rightarrow} A$ for some A;
  - if $|\Gamma| \vdash_{\Sigma,\Omega} t^{\Leftarrow}$, then it is decidable for any A whether $\Gamma \vdash_{\Sigma,\Omega} t :^{\Leftarrow} A$;