

# Simply Typed Language $\Gamma \vdash_{\Sigma, \Omega} t : A$

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash_{\Sigma, \Omega} x : A} \text{VAR}$$

$$\frac{\Gamma \vdash_{\Sigma, \Omega} t : A}{\Gamma \vdash_{\Sigma, \Omega} (t \circ A) : A} \text{ANNO}$$

$$\frac{\rho : \text{Sub}_{\Sigma}(\Xi, \emptyset) \quad \Gamma, \vec{x}_1 : \Delta_1 \langle \rho \rangle \vdash_{\Sigma, \Omega} t_1 : A_1 \langle \rho \rangle \cdots \Gamma, \vec{x}_n : \Delta_n \langle \rho \rangle \vdash_{\Sigma, \Omega} t_n : A_n \langle \rho \rangle}{\Gamma \vdash_{\Sigma, \Omega} \text{op}_o(\vec{x}_1.t_1; \dots; \vec{x}_n.t_n) : A_0 \langle \rho \rangle} \text{OP}$$

for  $o : \Xi \triangleright [\Delta_1]A_1, \dots, [\Delta_n]A_n \rightarrow A_0$  in  $\Omega$

# Bidirectional Type System $\Gamma \vdash_{\Sigma, \Omega} t :^d A$

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash_{\Sigma, \Omega} x :^{\Rightarrow} A} \text{VAR}^{\Rightarrow}$$

$$\frac{\Gamma \vdash_{\Sigma, \Omega} t :^{\Rightarrow} B \quad B = A}{\Gamma \vdash_{\Sigma, \Omega} t :^{\Leftarrow} A} \text{SUB}^{\Leftarrow}$$

$$\frac{\Gamma \vdash_{\Sigma, \Omega} t :^{\Leftarrow} A}{\Gamma \vdash_{\Sigma, \Omega} (t \circ A) :^{\Rightarrow} A} \text{ANNO}^{\Rightarrow}$$

$$\frac{\begin{array}{c} \rho : \text{Sub}_{\Sigma}(\Xi, \emptyset) \\ \Gamma, \vec{x}_1 : \Delta_1 \langle \rho \rangle \vdash_{\Sigma, \Omega} t_1 :^{d_1} A_1 \langle \rho \rangle \quad \dots \quad \Gamma, \vec{x}_n : \Delta_n \langle \rho \rangle \vdash_{\Sigma, \Omega} t_n :^{d_n} A_n \langle \rho \rangle \end{array}}{\Gamma \vdash_{\Sigma, \Omega} \text{op}_o(\vec{x}_1.t_1; \dots; \vec{x}_n.t_n) :^d A_0 \langle \rho \rangle} \text{OP}$$

for  $o : \Xi \triangleright [\Delta_1]A_1^{d_1}, \dots, [\Delta_n]A_n^{d_n} \rightarrow A_0^d$  in  $\Omega$