A+

多元统计第一章实验

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一、基于例1.2.1的一些基本函数及运算

In [79]:

```
import numpy as np
x=np.array([1,3,4,1,2,5])
x
```

Out[79]:

```
array([1, 3, 4, 1, 2, 5])
```

In [80]:

```
len(x)    #向量的长度
```

Out[80]:

```
6
```

In [81]:

```
A= np.reshape(x, (2,3),order='F')
A
```

Out[81]:

```
array([[1, 4, 2],
       [3, 1, 5]])
```

In [82]:

```
A.dtype    #数据的类型
```

Out[82]:

```
dtype('int32')
```

In [83]:

```
A.shape    #矩阵的维数
```

Out[83]:

```
(2, 3)
```

In [84]:

```
A.T    #矩阵转置
```

Out[84]:

```
array([[1, 3],
       [4, 1],
       [2, 5]])
```

In [85]:

```
np.sum(A)    #矩阵求和
```

Out[85]:

```
16
```

In [86]:

```
np.sum(A,axis=1)    #按行求和
```

Out[86]:

```
array([7, 9])
```

In [87]:

```
np.sum(A,axis=0)    #按列求和
```

Out[87]:

```
array([4, 5, 7])
```

In [88]:

```
np.mean(A)    #矩阵求均值
```

Out[88]:

```
2.6666666666666665
```

In [89]:

```
np.mean(A,axis=1)    #按行求均值
```

Out[89]:

```
array([2.33333333, 3.        ])
```

In [90]:

```
np.mean(A,axis=0)    #按列求均值
```

Out[90]:

```
array([2. , 2.5, 3.5])
```

In [91]:

```
B=np.reshape([6, 0, 2, 3, 1, 4],(2,3))
B
```

Out[91]:

```
array([[6, 0, 2],
       [3, 1, 4]])
```

In [92]:

```
A+B    #矩阵相加
```

Out[92]:

```
array([[7, 4, 4],
       [6, 2, 9]])
```

In [93]:

```
A-B    #矩阵相减
```

Out[93]:

```
array([[-5,  4,  0],
       [ 0,  0,  1]])
```

In [94]:

```
C=np.reshape([1, 2, 1, 3],(2,2),order='F')
C
```

Out[94]:

```
array([[1, 1],
       [2, 3]])
```

In [95]:

```
np.dot(C,A)    #矩阵相乘
```

Out[95]:

```
array([[ 4,  5,  7],
       [11, 11, 19]])
```

In [96]:

```
A*B    #元素级相乘
```

Out[96]:

```
array([[ 6,  0,  4],
       [ 9,  1, 20]])
```

二、计算方阵的一些函数值

In [97]:

```
A=np.reshape([1,2,3,4,5,2,4,7,8,9,3,7,10,15,20,4,8,15,30,20,5,9,20,20,40],(5,5))
A
```

Out[97]:

```
array([[ 1,  2,  3,  4,  5],
       [ 2,  4,  7,  8,  9],
       [ 3,  7, 10, 15, 20],
       [ 4,  8, 15, 30, 20],
       [ 5,  9, 20, 20, 40]])
```

In [98]:

```
np.diag(A)    #对角线元素
```

Out[98]:

```
array([ 1,  4, 10, 30, 40])
```

In [99]:

```
np.diag(np.diag(A))    #对角线元素创建的对角矩阵
```

Out[99]:

```
array([[ 1,  0,  0,  0,  0],
       [ 0,  4,  0,  0,  0],
       [ 0,  0, 10,  0,  0],
       [ 0,  0,  0, 30,  0],
       [ 0,  0,  0,  0, 40]])
```

In [100]:

```
np.eye(5)    #5阶单位矩阵
```

Out[100]:

```
array([[1., 0., 0., 0., 0.],
       [0., 1., 0., 0., 0.],
       [0., 0., 1., 0., 0.],
       [0., 0., 0., 1., 0.],
       [0., 0., 0., 0., 1.]])
```

In [101]:

```
np.linalg.inv(A)    #矩阵的逆
```

Out[101]:

```
array([[ 9.78873239e+00, -2.18309859e+00, -1.85915493e+00,
         1.12676056e-01,  1.40845070e-01],
       [-2.18309859e+00,  7.74647887e-01,  7.88732394e-01,
        -1.69014085e-01, -2.11267606e-01],
       [-1.85915493e+00,  7.88732394e-01,  3.94366197e-02,
        -8.45070423e-03,  3.94366197e-02],
       [ 1.12676056e-01, -1.69014085e-01, -8.45070423e-03,
         7.32394366e-02, -8.45070423e-03],
       [ 1.40845070e-01, -2.11267606e-01,  3.94366197e-02,
        -8.45070423e-03,  3.94366197e-02]])
```

In [102]:

```
np.linalg.det(A)    #矩阵的行列式
```

Out[102]:

```
-355.00000000000006
```

In [103]:

```
np.linalg.eig(A)    #矩阵的特征值和特征向量
```

Out[103]:

```
(array([70.33488803, 14.44024095,  1.997606  ,  0.09374538, -1.86648037]),
 array([[ 0.10513926,  0.00733125,  0.26673691,  0.95627367, -0.05730686],
        [ 0.20596656,  0.05549834,  0.82858975, -0.22629386,  0.46554035],
        [ 0.39707684, -0.02585507,  0.32382661, -0.18402887, -0.83840992],
        [ 0.5462168 ,  0.78569385, -0.25851683,  0.01391472,  0.13155919],
        [ 0.70035756, -0.61553463, -0.26569873,  0.0164776 ,  0.24443626]]))
```

In [104]:

```
np.trace(A)    #矩阵的迹
```

Out[104]:

```
85
```

In [105]:

```
a,b=np.linalg.eig(A)
r=a[0]    #第一个特征值
x=b[:,:1]    #第一个特征向量
r,x
```

Out[105]:

```
(70.33488803054398,
 array([[0.10513926],
        [0.20596656],
        [0.39707684],
        [0.5462168 ],
        [0.70035756]]))
```

In [106]:

```
Ax=np.around(np.dot(A,x),2)    #所有数保留两位小数
rx=np.around(np.dot(r,x),2)
(Ax==rx).all()    #验证Ax=rx
```

Out[106]:

```
True
```

三、例1.6.6中的奇异值分解

In [107]:

```
A=np.reshape([1,1,2,-2,2,2],(2,3),order='F')
A
```

Out[107]:

```
array([[ 1,  2,  2],
       [ 1, -2,  2]])
```

In [109]:

```
np.linalg.svd(A)    #奇异值分解
```

Out[109]:

```
(array([[ 0.70710678, -0.70710678],
        [ 0.70710678,  0.70710678]]),
 array([3.16227766, 2.82842712]),
 array([[ 4.47213595e-01, -8.84271441e-16,  8.94427191e-01],
        [-5.18104078e-16, -1.00000000e+00, -8.14163551e-16],
        [-8.94427191e-01,  6.18327757e-17,  4.47213595e-01]]))
```

集 美 大 学 学 院

作 业 日 期 ____ 年 ____ 月 ____ 日

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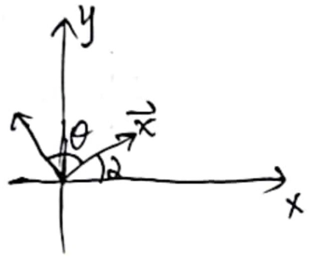
① 1.7 $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

证明：设 $\vec{x} = (r\cos\alpha, r\sin\alpha)$

取 $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ 　　则 $A\vec{x} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{pmatrix} r\cos\alpha \\ r\sin\alpha \end{pmatrix}$



$$= \begin{pmatrix} r\cos\alpha\cos\theta - r\sin\alpha\sin\theta \\ r\cos\alpha\sin\theta + r\sin\alpha\cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} r\cos(\alpha+\theta) \\ r\sin(\alpha+\theta) \end{pmatrix}$$

② 计算 $A = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ 的特征值和特征向量，其中 $|\rho| \neq 1$.

解：$|A - \lambda I| = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = (1-\lambda)^2 - \rho^2$

故A的特征值是 $\lambda_1 = 1+\rho$ 和 $\lambda_2 = 1-\rho$.

由 $Ax_1 = \lambda_1 x_1$，有：$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix} = (1+\rho)\cdot\begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix}$

即 $\begin{cases} x_{11} + \rho x_{21} = (1+\rho)x_{11} \\ \rho x_{11} + x_{21} = (1+\rho)x_{21} \end{cases}$ 　解得：$x_{11} = x_{21}$

则 $\lambda_1 = 1+\rho$ 相对应的单位特征向量 $\vec{x}_1 = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})'$

由 $Ax_2 = \lambda_2 x_2$，有：$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix} = (1-\rho)\begin{pmatrix} x_{11} \\ x_{21} \end{pmatrix}$

即 $\begin{cases} x_{11} + \rho x_{21} = (1-\rho)x_{11} \\ \rho x_{11} + x_{21} = (1-\rho)x_{21} \end{cases}$ 　解得：$x_{11} = -x_{21}$

则 $\lambda_2 = 1-\rho$ 相对应的单位特征向量 $\vec{x}_2 = (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})'$