# Suppression of High PAPR for OFDM Signals

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***Abstract*— PAPR suppression has been a challenging problem in all OFDM systems as it imposes in band distortion to transmitter amplifiers. This phenomenon will be remained as an issue because newer methods inherit it from OFDM. Lowering the CL results in decrease of CR with in turn improves the high PAPR problem, but BER will act as a constraint. Clipping method, which is a straight forward road, is utilized in this research to lower the PAPR. Investigations having QPSK modulation, AWGN, and zero padding show 46 percent of PAPR reduction for simple clipping method. It is also proven that simple clipping can be even more efficient in comparison to SLM which is a probabilistic method. Comparing the results of simple clipping used in this research with previous efforts it can be inferred that simple clipping is almost optimized and those so-called novel or optimized techniques just increase the complexities of implementation.**

***Index Terms*—PAPR, OFDM, SLM, Clipping, QPSK**

## I. Introduction

PAPR is Peak to Average Power Ratio. Transmitter's power amplifiers (PAs) are peak-power limited and high PAPR results in in-band distortion and out-of-band radiation. Although it is possible to cancel the out-of-band radiations by filtering methods, in-band distortion continues to be a problem. Increasing the back-off, in which the output signal's amplitude is reduced by reducing the input signal amplitude, is probably the first solution coming to mind, but obviously it reduces PA efficiency. Moreover, high PAPR imposes negative impact on ADC[1] and DAC[2].

PAPR suppression has been a challenging problem in OFDM systems. It continues to be an issue because of its complexity, existing trade-offs and new areas that inherit high PAPR problem from OFDM. Numerous efforts have been done by different researchers to address high PAPR. In general. PAPR reduction methods can be classified into four groups of Probabilistic [1], [2], Coding [3], Companding [4], and Clipping/Filtering [5]. Complexity, data rate, and BER [3] performance are basics of comparison for PAPR reduction methods.

Probabilistic methods use mathematical rules to provide somehow unified data through which high PAPR is prevent before creeping. SLM[4] is probably the most renowned approach of this family. In coding techniques, the data in coded before transmission and in decoded after receiving. Companding comes from synthesis of compressing and expanding. In this method dynamic range input and variable-gain amplifier are utilized to effectively compress the data in transmitter before it is expanded at receiver. Clipping and filtration methods are simplest ones in which data is shaped in case of being far away from the mean.

## II. Methods

In order to understand the concept of PAPR there are prerequisites. Digital modulation is one of the fundamental topics needed in PAPR reduction. In electronics and telecommunications, modulation is the process of varying one or more properties of a periodic waveform, called the carrier signal, with a modulating signal that typically contains information to be transmitted. Most radio systems in the 20th century used frequency modulation (FM) or amplitude modulation (AM) to make the carrier carry the radio broadcast.

In general telecommunications, modulation is a process of conveying message signal, for example, a digital bit stream or an analog audio signal, inside another signal that can be physically transmitted. Modulation of a sine waveform transforms a narrow frequency range baseband message signal into a moderate to high frequency range passband signal, one that can pass through a filter.[5]

It can be concluded that modulation is the way data can be transferred in wireless world and the very matter is the goal of communication; hence, modulation is vital is wireless communications. Different techniques do this highly important process with different methods. The main goal of all those techniques is higher performance that can be imagined as higher data rates, when it comes to wireless world.

The aim of analog modulation is to transfer an analog baseband (or lowpass) signal, for example an audio signal or TV signal, over an analog bandpass channel at a different frequency, for example over a limited radio frequency band or a cable TV network channel.

The aim of digital modulation is to transfer a digital bit stream over an analog bandpass channel, for example over the public switched telephone network (where a bandpass filter limits the frequency range to 300–3400 Hz) or over a limited radio frequency band.[6]

Differences between the natural context of digital and analog world leads to analog and digital modulation diversity. In digital

[1] Analog-to-Digital Converter
[2] Digital-to-Analog Converter
[3] Bit Error Rate
[4] Selected Mapping
[5] Ref: https://en.wikipedia.org/wiki/Modulation
[6] Ref: https://en.wikipedia.org/wiki/Modulation

modulation there are three major families of techniques, each manipulating different factors of the carrier. Those are Frequency, amplitude, and phase shifting which are illustrated in Fig. 1 to 3, respectively.

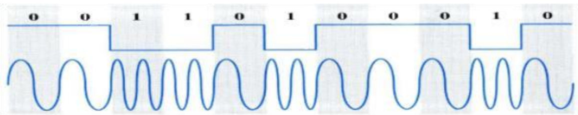


Fig. 1. Frequency Shift Keying (FSK)

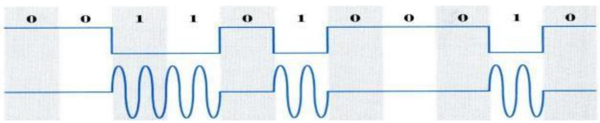


Fig. 2. Amplitude Shift Keying (ASK)

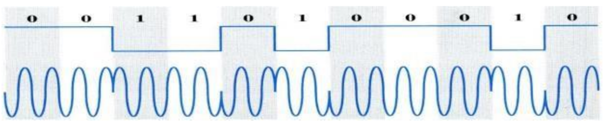


Fig. 3. Phase Shift Keying (PSK)

For PAPR reduction PSK is the underlying one. In fact, PSK is a group of techniques. Quadrature Phase-Shift Keying or QPSK is the one used in this project. It is sometimes called 4PSK. In the constellation diagram of QPKS, like Fig. 4, there are four symbols. Generally, symbols are packages of zeros and ones that are transmitted as a unit.

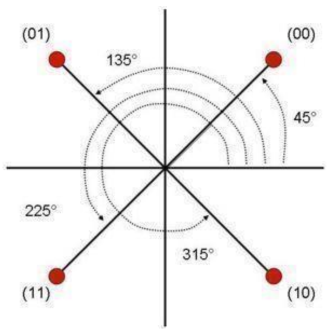


Fig. 4. QPSK Constellation

It can be intuitively determined what these four possible phase shifts should be like Fig. 5. It should be considered that modulation is only the beginning of the communication process; the receiver needs to be able to extract the original information from the modulated signal. Next, it makes sense to seek maximum separation between the four phase options, so that the receiver has less difficulty distinguishing one state from another. There is 360° of phase to work with and four phase states, and thus the separation should be 360°/4 = 90°. So, the four QPSK phase shifts are 45°, 135°, 225°, and 315°, as in Fig 5. It can be imagined that there are two carriers for QPSK, but they differ by phase not by frequency. Here is where I and Q are introduced in some papers. I and Q are the two components of each symbol. One for x part or real part, and the other for y or imaginary part.

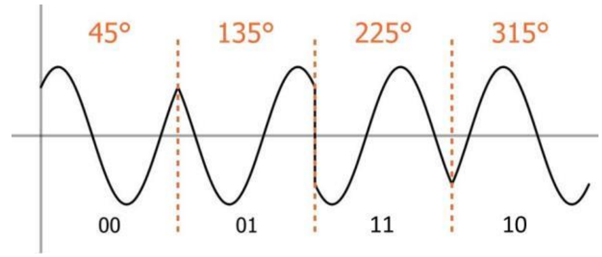


Fig. 5. QPSK shifting

The other requirement to dig into PAPR reduction is OFDM. In telecommunications, a carrier wave, carrier signal, or just carrier, is a waveform (usually sinusoidal) that is modulated (modified) with an input signal for the purpose of conveying information. OFDM or Orthogonal Frequency-Division Multiplexing is a multicarrier technique in which the concept of overlapping in frequency domain is used to achieve bandwidth conservation. The Idea is depicted in Fig. 6.

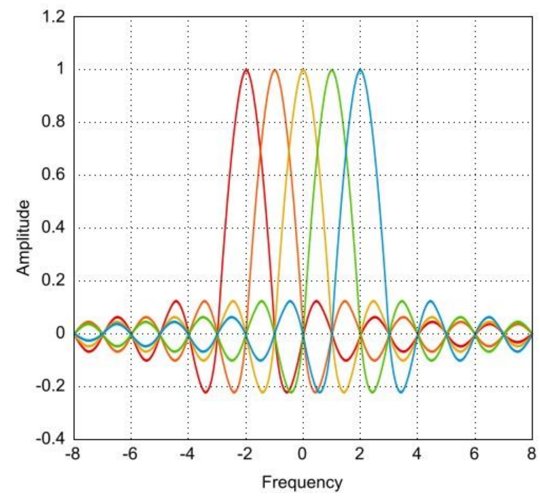


Fig. 6. Frequency domain illustration of OFDM

Channel equalization is simplified because OFDM may be viewed as using many slowly modulated narrowband signals rather than one rapidly modulated wideband signal. In OFDM, the sub-carrier frequencies are chosen so that the sub-carriers are orthogonal to each other, meaning that cross-talk between the sub-channels is eliminated and inter-carrier guard bands are not required. This greatly simplifies the design of both the transmitter and the receiver

The orthogonality allows for efficient modulator and demodulator implementation using the FFT algorithm on the receiver side, and inverse FFT on the sender side. The orthogonality requires that the sub-carrier spacing is in form of:

$$\Delta f = \frac{k}{T_U} \text{ (Hertz)} \tag{1}$$

Where $T_U$ (seconds) is the useful symbol duration[7], and k is a positive integer, typically equal to 1. Therefore, with N subcarriers, the total passband bandwidth will be:

$$B \approx N \cdot \Delta f \text{ (Hz)} \qquad (2)$$

Each subcarrier is modulated with a conventional digital modulation scheme (such as QPSK, 16QAM, etc.) at low symbol rate. However, the combination of many subcarriers enables data rates similar to conventional single-carrier modulation schemes within equivalent bandwidths.

This two-bits-per-symbol performance is possible because the carrier variations are not limited to two states. In ASK, for example, the carrier amplitude is either amplitude option A (representing a 1) or amplitude option B (representing a 0). As mentioned earlier, in QPSK, the carrier varies in terms of phase, not frequency, and there are four possible phase shifts.

PAPR is the last thing needs to be defined before getting into formulation and mathematics. PAPR or Peak-to-Average Power Ratio is defined as:

$$PAPR = \frac{Peak\ Power}{Average\ Power} = \frac{(Peak\ Amp)^2}{(RMS\ Amp)^2} \qquad (3)$$

PAPR elements can be shown as Fig. 7.

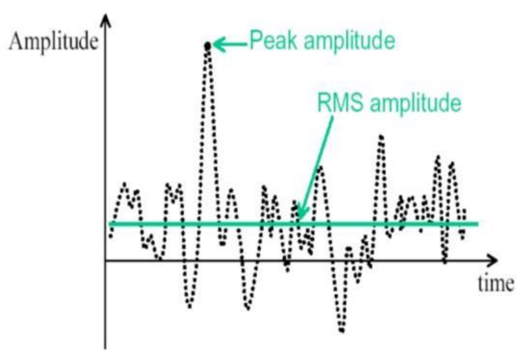
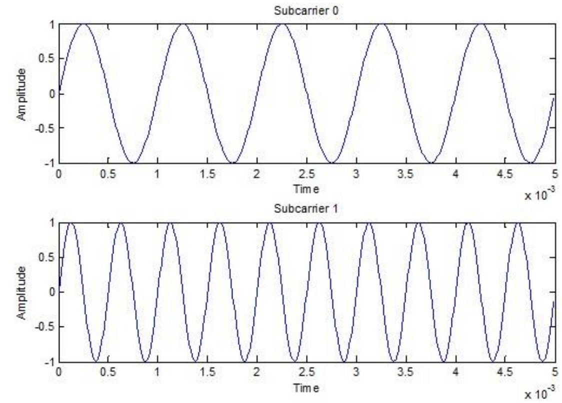
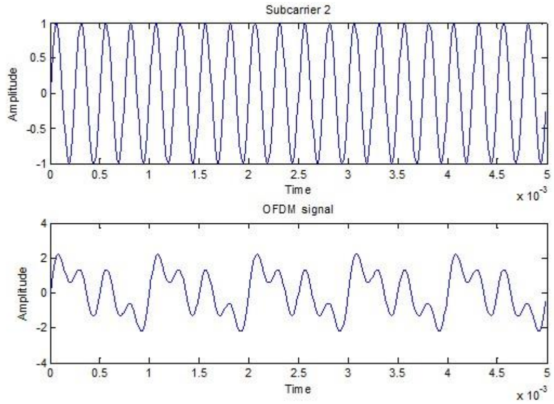



Fig. 7. PAPR elements

Because OFDM uses PSK modulation on each of its subcarriers and then all subcarriers are summed up together and sent, there are cases in which high PAPR occurs. Fig. 8 illustrates possible situations. It should be also regarded that all OFDM subcarriers are equally contributing in power domain and the very fact leads into equal amplitude waves of Fig. 8.




[7] the receiver-side window size



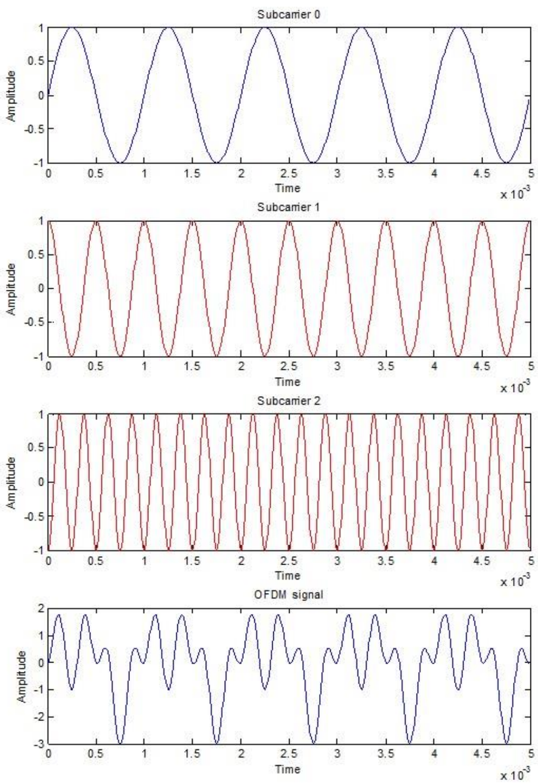




Fig. 8. How high PAPR occurs

The final transmitted OFDM signal (S) is stated as:

$$S(t)=\sum_{k=0}^{N-1} a_k e^{2\pi i (f_0+kf_s)t} \quad (4)$$

Where N is the number of carriers (or subcarriers). N symbols ($a_k$) are received at times 0, $\tau$, $2\tau$, … Obviously $a_k$ belongs to complex numbers (recall Fig. 4) and k has the range of 0,1, 2, …, N-1. The possible values [8] of $a_k$ is determined by modulation[9]. $f_0$ is Carrier Frequency and $f_s$ is Bandwidth (Refer to Fig. 6).

$$f_s=\frac{1}{\tau} \quad (5)$$

Regarding Eq. 5, it can be inferred that the transmitted signal is transferred during the interval of t=[0, $\frac{1}{f_s}$]. Also knowing that subcarriers have equal energy levels, it can be concluded that:

$$\sum_{k=0}^{N-1}|a_k|^2 = N\,|a_k|^2 \quad (6)$$

Supposing the normalized situation, Eq. 6 changes into Eq.7.

$$\sum_{k=0}^{N-1}|a_k|^2 = N \quad (7)$$

Normalization is achieved through using the new parameter of $\zeta$.

$$\zeta=\frac{f_0}{f_s} \quad (8)$$

For normalized waves the transmission interval will be t=[0,1). Now the Eq. 3 can be rewritten as:

$$\mathrm{PAPR}(a)=\frac{Max(|S(t)|^2)}{Avg(|S(t)|^2)}=\frac{Max(|\sum_{k=0}^{N-1} a_k e^{2\pi i(\zeta+k)t}|^2)}{N} \quad (9)$$

There is another concept, touched by different papers, called envelope power which is defines as $|S(t)|^2$. Using the new word of envelope, PAPR is studied trough PEMPR or Peak-to-Mean Envelope Ratio and defined as:

$$\mathrm{PAPR}(a)\leq \mathrm{PMEPR}(a)=\frac{Max(|\sum_{k=0}^{N-1} a_k e^{2\pi ikt}|^2)}{N} \quad (10)$$

A new function can be derived for simplifying the notations as:

$$F(t)=\sum_{k=0}^{N-1} a_k e^{2\pi ikt} \quad (11)$$

It can be seen that F(t) is a ceiling for S(t) which is feasible to deal with. Now the concept of CCDF or Complimentary Cumulative Distributive Function cab be understood. It is defined as:

$$CCDF[PAPR(x^n(t))]=\mathrm{prob}[PAPR(x^n(t))>\delta] \quad (12)$$

Where $\delta$ is just a threshold and $x^n(t)$ is the $N^{th}$ OFDM Symbol. CCDF is a ceiling for PAPR; hence, it is widely used in papers. The fundamental of clipping methods is based on the Eq. 13:

[8] Here it can also be considered as "state"

[9] four for QPKS: (1,1), (-1,1), (-1,-1), and (1,-1)

$$T(x[n]) = \begin{cases} x[n] & if\ |x[n]| \le CL \\ CLe^{j\angle x[n]} & if\ |x[n]| > CL \end{cases} \tag{13}$$

Where CL is Clipping Level and T(x) is trimmed signal. Clipping ratio (CR) which is relied on clipping level as well as mean value of input signal is defined as:

$$CR = 20log_{10}\left(\frac{CL}{E[x[n]]}\right) \tag{14}$$

Obviously, CR is stated in dB.

In some cases, PAPR or CCDF are stated in dB. Normal and logarithmic forms are related by:

$$PAPR(dB)=10*log(PAPR) \tag{15}$$

## III. RESULTS

A code as appendix A is developed to investigate the problem. For N=1024, using QPSK and assuming AWGN, The PAPR reduction is achieved as Fig. 9.

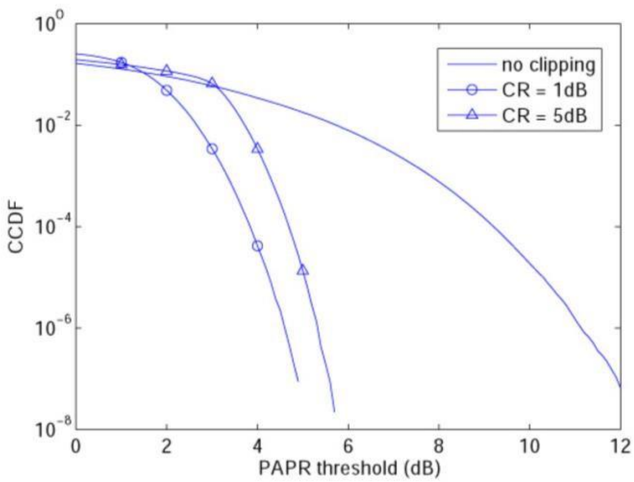



Fig. 9. PAPR reduction for N=1024, QPSK, and AWQN

For the same condition the BER performance is given by Fig. 10.

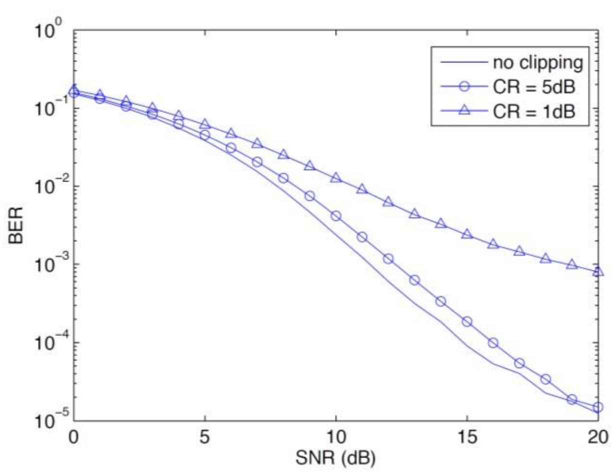



Fig. 10. PAPR reduction for N=1024, QPSK, and AWQN

Zero padding is a mathematical probabilistic technique and it can be used to inhibit the noise. For OFDM paradigm, zero padding is started with defining padding parameter or L. L is usually between 1 to 1.5. The subcarriers (Symbols) are extended by this factor in a way that N symbols are increased to LN symbols through adding LN-N zeros to the N signals.

For L=1, CL=0.7, using QPSK modulation and AWGN, PAPR of normal OFDM equal to 20.5339 and PAPR of clipped OFDM equal to 11.0855 are achieved in a normalized problem (Fig. 11).

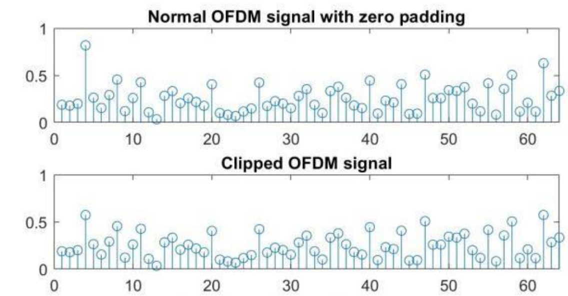



Fig. 11. Original vs Clipped OFDM signal L=1

Under the same condition, yet with L=1.5 PAPR of normal OFDM equal to 18.3249 and PAPR of clipped OFDM equal to 10.0510 were the results (Fig. 12).

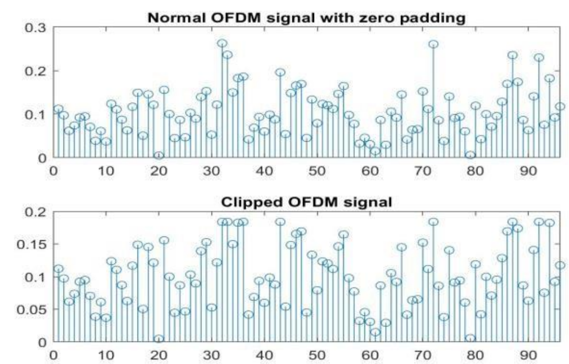



Fig. 12. Original vs Clipped OFDM signal L=1.5

In the code normalized Riemann Matrix is also used to provide SLM (Selected Mapping) technique. Comparison shows that clipping results in better results. However, SLM is beyond the borders of this research and here it is just a bonus.

## IV. DISCUSSION

Fig. 9 and Fig. 10 show that although lowering the CR leads to better PAPR, it brings poor BER. Therefore, in clipping methods always there is a trade-off between PAPR and BER which has to be optimized. Usually iterative methods are used to fulfill this need, but at the cost of more computational complexity.

It is also shown (Fig. 11 and 12) that zero padding is an effective technique resulting in both PAPR reduction and BER performance improvement.

Comparing the results of simple clipping used in this research with previous efforts [5] it can be inferred that simple clipping is almost optimized and those so-called novel or optimized techniques just increase the complexities of implementation.

For future works, the PAPR reduction in newer environments other than OFDM can be investigated. Also the compound methods in which synthesis of different approaches is used are candidates of optimized solutions.

## V. Summary

PAPR suppression has been a challenging problem in OFDM systems as it imposes in-band distortion as well as out of band radiation in transmitter amplifiers. It also impacts ADC and DAC poor performance. PAPR reduction methods can be classified into four groups of Probabilistic, Coding, Companding, and Clipping/Filtering. Complexity, data rate, and BER performance are basics of comparison for PAPR reduction methods.

Clipping method, which is a straight forward road, is utilized in this research to lower the PAPR. Despite of lower PAPR for lower values of CR, the BER will act as a constraint. Investigations having QPSK modulation, AWGN, and zero padding show 46 percent of PAPR reduction for simple clipping method.

Zero padding is declared to be effective to lower PAPR and improve BER at the same time. It is also proven that simple clipping can be even more efficient in comparison to SLM which is a probabilistic method.

To put in a nutshell clipping is able to inhibit PAPR problem completely, yet BER efficiency has to be checked.

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