Final Exam **Comp Sci 2C03** - *Data structures and Algorithms* *Fall 2021*

Name _____ Student No._____

For your exam you are ONLY allowed to refer to the lecture notes, text book, and tutorials and assignment solutions.

Time allotted: 3 hours (write and submit PDF) **Total Marks: 40**

Multiple choice questions [10 marks]

Circle your answer. Each question has just one correct answer. Therefore multiple selections will not get a mark.

1. The function $T(n) = \frac{n \log n + 2n + 1}{\log n}$ is in

   (a) $\Theta(n^3)$

   (b) $\Omega(n^2)$

   (c) $O(n)$

   (d) None of the above.

2. Let L be a doubly linked list consisting of n nodes. Suppose x is a node of L and not the head or tail of L. How many nodes remain in L after the execution of the following code fragment (assume that $x.next.next \neq null$): `x.next = x.next.next and x.next.next.prev = x`?

   (a) $n - 2$

   (b) $n - 1$

   (c) n

   (d) 0

3. Which of the following statements is FALSE?

   (a) Selection sort is not input sensitive

   (b) Insertion sort is not input sensitive

   (c) Insertion sort is the array of choice for small (< 10) sized subarrays

   (d) Insertion sort is used as a sub-module in Shellsort

4. What will be the order of elements of the array $A = 21, 37, 3, 1, 43, 15, 17, 11$ after first iteration of Bottom-up Mergesort is complete?

(a) 21, 37, 3, 1, 43, 15, 17, 11

(b) 1, 3, 21, 37, 11, 15, 17, 43

(c) 21, 37, 3, 1, 15, 43, 11, 17

(d) 21, 37, 1, 3, 15, 43, 11, 17

5. Which of the trees given in Figure 1 is NOT a left leaning red-black tree (LLRBT) generated by the insert operation for LLRBT?

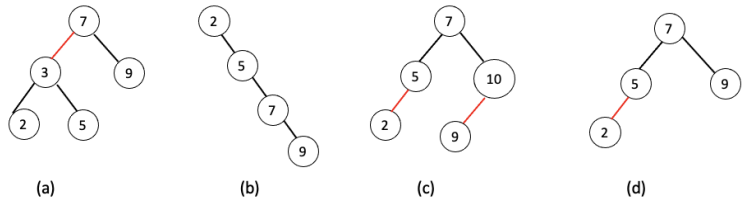

Figure 1: Trees for multiple choice Question 5.

(a) a

(b) b

(c) c

(d) d

6. A search or an insertion in a B-tree of order 4 with 128 keys requires at most _____ probes.

(a) 7

(b) 9

(c) 5

(d) 4

7. Adding a constant to every edge weight to an edge weight directed graph consisting of negative edge weights does not change the solution to the single-source shortest-paths problem.

(a) True

(b) False

8. Which of the following is NOT true about MSD radix sort?

(a) its processing starts from the most significant digit

(b) it is not a stable sort

(c) it is an in place sorting algorithm

(d) it is a non-comparison based sort

9. Consider the `text = a a a a a a a a`, and the pattern `pattern = a a b`. Compute the number of character comparisons done by KMP (KMPC) and the Boyer-Moore (with only bad character rule) (BMC) algorithms?

   (a) KMPC = 9 and BMC = 6

   (b) KMPC = 6 and BMC = 6

   (c) KMPC = 5 and BMC = 8

   (d) KMPC = 5 and BMC = 6

10. Consider the message A B A C A B A. The prefix trie generated by the Huffman coding for this message is given in Figure 2. What is the encoding of each character?

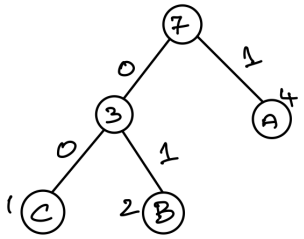

Figure 2: Prefix Trie for Q10.

   (a) A = 1, B = 10, C = 00

   (b) A = 1, B = 01, C = 00

   (c) A = 11, B = 01, C = 00

   (d) None of the above.

Provide detailed answers to the 6 questions below. [30 marks]

1. Draw the recursion tree for the recurrence $T(n) = T(n/4) + T(3n/4) + n$, and explain in few lines that $T(n) = O(n \log n)$. [4 marks]

2. Construct the Max. Binary Heap on the input array C O M P U T E R S. [5 marks]

3. (a) Find an insertion order for the keys A D J L M O R that leads to a binary search tree (BST) of minimum height and draw this tree. [3 marks]

   (b) Find an insertion order for the keys A D J L M O R X that leads to a 2-3 tree of minimum height and draw this tree. [3 marks]

4. (a) Consider Dijkstra's algorithm to compute the shortest path for an edge weighted digraph. In the algorithm if we replace the priority queue data structure with a FIFO queue, will the algorithm still produce the shortest path? Explain your answer. [2 marks]

   (b) Given an MST for an edge-weighted graph G and a new edge e, describe how to find an MST of the new graph in time proportional to V. [3 marks]

5. (a) Compute the border array of the string $w = (ab)^n$ for $n > 2$ and $n \in \mathbb{N}$. [2 marks]

   (b) Perform substring search using the Rabin-Karp algorithm on the following text = 6 8 9 4 7 9, and pattern = 4 7 9. The algorithm uses the following hash function: $h(x) = x \mod 53$. When performing the substring search compute the hash functions for x_3, x_4, x_5 using rolling hash. [4 marks]

6. What is the LZW encoding of the following input: A B A C A B A B A A C A A B A A? You may assume that the input contains 7-bit ASCII characters and the output is in 8 bit codewords in hexadecimal. Provide the trie representing the symbol table. What is the compression ratio achieved? The hexadecimal values for $A = 41, B = 42$ and $C = 43$. [4 marks]