Midterm-II **Comp Sci 2C03** - *Data structures and Algorithms* *Winter 2021*

Name _____ Student No._____

- For your exam you are ONLY allowed to refer to the lecture notes, text book, tutorials and assignments.
- Surfing the internet for ideas and answers is not allowed.
- Clearly write your full name and student number on each sheet.

Time allotted: 2 hours **Total Marks: 33**
(for the test and uploading it on Avenue)

Multiple choice questions [6 marks]

Circle your answer. Each question has just one correct answer. Therefore multiple selections will not get a mark.

1. Suppose you have the following sorted list [1, 5, 8, 11, 19, 21, 25, 30, 33] and are using the binary search algorithm given on slide #11 in C3P1.pdf. Give the sequences of elements examined to find the key 5.

   (a) 19, 8, 5
   (b) 19, 5
   (c) 19, 25, 5
   (d) 19, 1, 5

2. Given the keys C O M P U T E, which of the below sequence of keys would produce the best-case binary search tree, when inserted into an initially empty BST.

   (a) C E M O P T U
   (b) U T P O M E C
   (c) O E T C M U P
   (d) O E P T M C U

3. Which of the trees given in Figure 1 is NOT a left leaning red-black tree (LLRBT) generated by the insert operation for LLRBT?

   (a) a
   (b) b

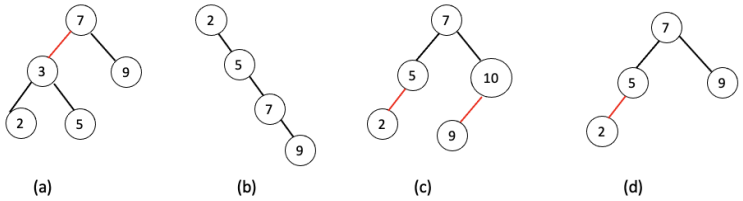


Figure 1: Trees for multiple choice Question 3.

(c) c

(d) d

4. A search or an insertion in a B-tree of order 10 with 81 keys requires at least 2 probes.

   (a) True

   (b) False

5. Which of the below scenario leads to expected constant running time for a random search hit in a hash table?

   a. All keys hash to the same index.

   b. All keys hash to different indices.

   c. All keys hash to an even-numbered index.

   d. None of the above.

6. How many strongly connected components does a directed acyclic graph (DAG) over V vertices have?

   a. $V - 1$

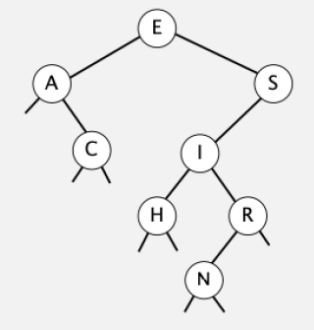
   b. 0

   c. V

   d. $V/2$

Provide detailed answers to the 6 questions below

1. Consider the below BST.



   a. Give the paths followed by the minimum and maximum operations, respectively. [2 marks]
   Answer: Minimum: E A; Maximum: E, S

   b. Give the path followed by the Floor(J) and Ceiling(B) operations, respectively. [2 marks]
   Answer: Floor(J): E S I R N or (E S I R N I); Ceiling(B): E A C

   c. Draw the resulting tree after deleting the key E from the tree. [2 marks]
   Answer: Answer given in Figure 2

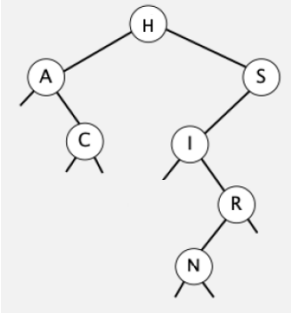


Figure 2: Solution for Q2c

2. Draw the (i) 2-3 tree and (ii) left leaning red-black tree that results when you insert the keys A L G O R I T in that order into an initially empty tree. Your solution must contain the trees obtained after every insertion. [6 marks]
Answer: (i) 2-3 Tree (See Figure 3)

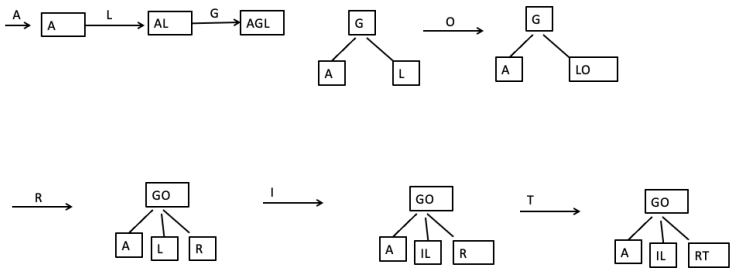


Figure 3: Solution for Q2c

Answer: (i) LLRBT Tree (See Figure 4)

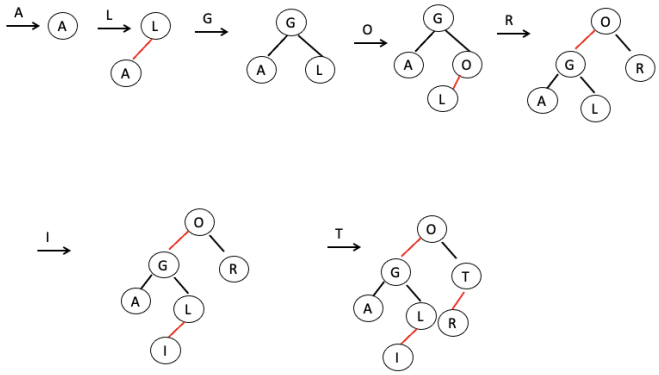


Figure 4: Solution for Q2c

3. Illustrate the sequence of probing and filling a Hash Table (an array) of size $m = 13$ with the following keys `<26, 52, 61, 68, 91>` using double hashing with $h_1(k) = k$ and $h_2(k) = 1 + (k \mod (m-1))$. [5 marks]
Answer: See Table 1 for the answer. Here is the explanation for it. The hash function

26			68		52			91	61			

Table 1: Solution for Q3 Double hashing

for double hashing discussed in class was $h(k,i) = (h_1(k) + ih_2(k)) \mod m = (k + i(1 + k \mod 12)) \mod 13$, for $i = 0, 1, \ldots, 12$. Based on it the keys in the given sequence would be inserted as follows:

- Key 26 - First slot probed is $h(26, 0) = (26 + 0) \mod 13 = 0$. Since it is empty, key 26 is inserted in it.

- Key 52 - First slot probed is $h(52, 0) = (52 + 0) \mod 13 = 0$. However since slot 0 is already filled, we fill the key 52 in the next available slot; that is,

$$h(52, 1) = (52 + 1 * (1 + (52 \mod 12))) \mod 13 = 5.$$

- Key 61 - First slot probed is $h(61, 0) = (61 + 0) \mod 13 = 9$. Since it is empty, key 61 is inserted in it.

- Key 68 - First slot probed is $h(68, 0) = (68 + 0) \mod 13 = 3$. Since it is empty, key 68 is inserted in it.

- Key 91 - First slot probed is $h(91, 0) = (91 + 0) \mod 13 = 0$. However since slot 0 is already filled, we fill the key 91 in the next available slot; that is,

$$h(91, 1) = (91 + 1 * (1 + (91 \mod 12))) \mod 13 = 8.$$

4. Draw the output tree for the undirected graph given in Figure 5 when

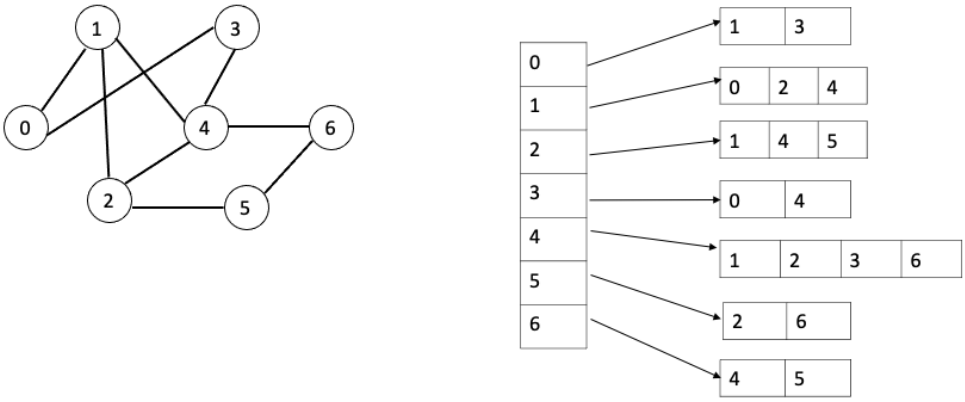
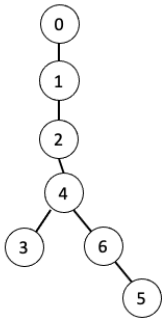


Figure 5: Graph for and the adjacency list for Question 4

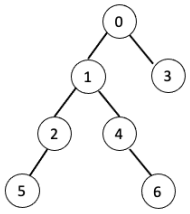
a. DFS is called on the source vertex 0. [3 marks]

b. BFS is called on the source vertex 0. [3 marks]

Answer:



Output tree for DFS



Output tree for BFS

5. Let G be the graph shown in Figure 6. Answer the below questions

   a. Give the topological sort of G. [3 marks]
   Answer: 4-3-2-1-0-7-9-5-8-6

   b. What is the reverse postorder vertex ordering of G. [1 mark]
   Answer: Same as above

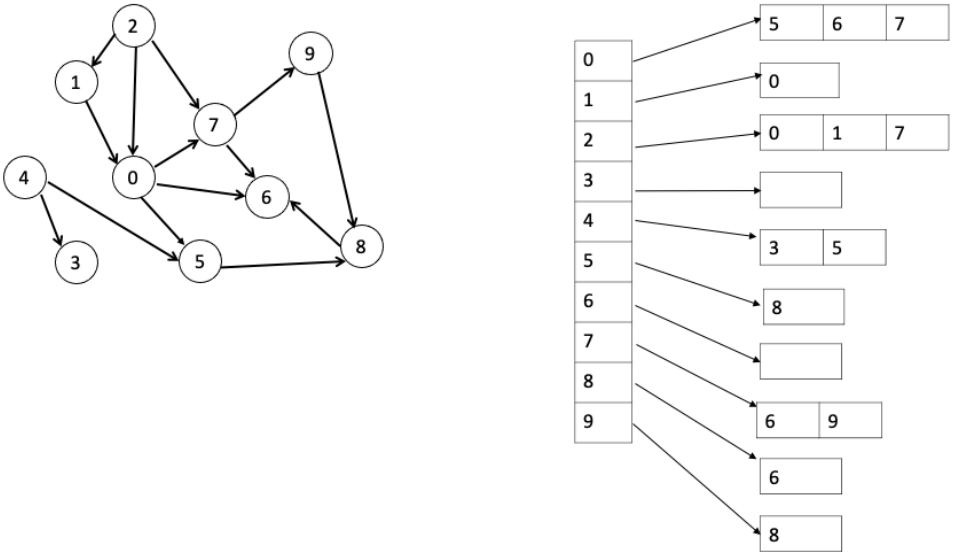


Figure 6: Digraph and its adjacency list for Question 5