2.4

Exercise 12

a)$a_n = 0$

replace n in $a_n = 0$ by n-1 and n-2:

$a_{n-1} = 0$

$a_{n-2} = 0$

$-3a_{n-1} + 4a_{n-2} = -3(0) + 4(0) = 0 + 0 = a_n$

b)$a_n = 1$

same with a just change $a_n = 1$

we will get$-3a_{n-1} + 4a_{n-2} = -3(1) + 4(1) = -3 + 4 = 1 = a_n$

c)$a_n = (-4)^n$

$$\begin{aligned}
-3a_{n-1} + 4a_{n-2} &= -3(-4)^{n-1} + 4(-4)^{n-2} \\
&= 3 * (-4) * (-4)^{n-2} + 4(-4)^{n-2} \\
&= (-4)^{n-2}(-3 * (-4) + 4) \\
&= (-4)^{n-2}(12 + 4) \\
&= (-4)^{n-2}(16) \\
&= (-4)^{n-2}(-4)^{-2} \\
&= (-4)^n \\
&= a_n
\end{aligned}$$

d)$a_n = 2(-4)^n + 3$

$$
\begin{aligned}
-3a_{n-1}+4a_{n-2} &= -3(2(-4)^{n-1}+3)+4(2(-4)^{n-2}+3) \\
&= -6(-4)^{n-1}-9+8(-4)^{n-2}+12 \\
&= (-6(-4)^{n-1}+8(-4)^{n-2}+12) \\
&= (-6*(-4)*(-4)^{n-2}+8(-4)^{n-2})+3 \\
&= (-4)^{n-2}(-6*(-4)+8)+3 \\
&= (-4)^{n-2}(24+8)+3 \\
&= (-4)^{n-2}*32+3 \\
&= (-4)^{n-2}*16*2+3 \\
&= (-4)^{n-2}*(-4)^2*2+3 \\
&= (-4)^{n}*2+3 \\
&= 2(-4)^{n}+3 \\
&= a_n
\end{aligned}
$$

Exercise 16

a)$a_n = -a_{n-1}, a_0 = 5$

$$
\begin{aligned}
a_n &= -a_{n-1} = (-1)^1 * a_{n-1} \\
&= -a_{n-2} = (-1)^2 * a_{n-2} \\
&= -a_{n-3} = (-1)^3 * a_{n-3} \\
&= -a_{n-4} = (-1)^4 * a_{n-4} \\
&= (-1)^n a_{n-n} \\
&= (-1)^n a_0 \\
&= 5(-1)^n
\end{aligned}
$$

b)$a_n = a_{n-1}+3, a_0 = 1$

$$\begin{aligned}
a_n &= a_{n-1} + 3 = a_{n-1} + 3 * 1 \\
&= a_{n-2} + 3 * 1 = a_{n-1} + 6 = a_{n-2} + 3 * 2 \\
&= a_{n-3} + 3 * 2 = a_{n-1} + 9 = a_{n-2} + 3 * 3 \\
&= a_{n-4} + 3 * 3 = a_{n-1} + 12 = a_{n-2} + 3 * 4 \\
&= a_{n-n} + 3 * n \\
&= a_0 + 3n \\
&= 1 + 3n \\
& 3n + 1
\end{aligned}$$

c)$a_n = a_{n-1} - n, a_0 = 4$

$$\begin{aligned}
a_n &= a_{n-1} - n = a_{n-1} - n + 0 \\
&= (a_{n-2} - (n-1)) - n = a_{n-2} - 2n + 0 + 1 \\
&= (a_{n-3} - (n-2)) - 2n + 1 = a_{n-3} - 3n + 0 + 1 + 2 \\
&= (a_{n-4} - (n-3)) - 3n + 3 = a_{n-4} - 4n + 0 + 1 + 2 + 3 \\
&= a_0 - n^2 + \frac{n-1}{n} \\
&= 4 - n^2 + \frac{n^2 - n}{2} \\
&= -\frac{1}{2}n^2 - \frac{1}{2}n + 4
\end{aligned}$$

d)$a_n = 2a_{n-1} - 3, a_0 = -1$

$$
\begin{aligned}
a_n &= 2a_{n-1} - 3 = 2^1 a_{n-1} - 3 \\
&= 2(2a_{n-2} - 3) - 3 = 2^2 a_{n-2} - (3 * 2^0 + 3 * 2^1) \\
&= 2^2(2a_{n-3} - 3) - (3 * 2^0 + 3 * 2^1) = 2^3 a_{n-2} - (3 * 2^0 + 3 * 2^1 + 3 * 2^2) \\
&= 2^3(2a_{n-3} - 3) - (3 * 2^0 + 3 * 2^1 + 3 * 2^2) = 2^4 a_{n-2} - (3 * 2^0 + 3 * 2^1 + 3 * 2^2 + 3 * 2^3) \\
&= 2^n * (-1) - 3 * \frac{2^n - 1}{2 - 1} \\
&= -2^n - 3(2^n - 1) \\
&= -2^n - 3 * 2^n + 3 \\
&= -4 * 2^n + 3
\end{aligned}
$$

e)$a_n = (n+1)a_{n-1}, a_0 = 2$

$$
\begin{aligned}
a_n &= (n+1)a_{n-1} \\
&= (n+1)(n)a_{n-2} \\
&= (n+1)(n)(n-1)a_{n-3} \\
&= (n+1)(n)(n-1)(n-2)a_{n-4} \\
&= (n+1)!a_0 \\
&= (n+1)! * 2 \\
&= 2 * (n+1)!
\end{aligned}
$$

f)$a_n = 2na_{n-1}, a_0 = 3$

$$\begin{aligned}
a_n &= 2na_{n-1} = 2^1 na_{n-1} \\
&= 2n(2(n-1)a_{n-2}) = 2^2 n(n-1)a_{n-2} \\
&= 2^2 n(n-1)(2(n-2)a_{n-2}) = 2^3 n(n-1)(n-2)a_{n-3} \\
&= 2^3 n(n-1)(n-2)(2(n-3)a_{n-2}) = 2^4 n(n-1)(n-2)(n-3)a_{n-4} \\
&= 2^n * n! * a_0 \\
&= 2^n * n! * 3 \\
&= 3n!2^n
\end{aligned}$$

g)$a_n = -a_{n-1} + n - 1, a_0 = 7$

$$\begin{aligned}
a_n = -a_{n-1} + n - 1 &= (-1)^1 a_{n-1} + (n-1) \\
&= (-1)^2 a_{n-2} + ((n-1) - (n-2)) \\
&= (-1)^3 a_{n-3} + ((n-1) - (n-2) + (n+3)) \\
&= (-1)^4 a_{n-4} + ((n-1) - (n-2) + (n+3) - (n-4)) \\
&= (-1)^n a_0 + \sum_{i=0}^{n-1} (-1)^{n-i+1} i \\
&= 7 * (-1)^n + \sum_{i=0}^{n-1} (-1)^{n-i+1} i
\end{aligned}$$

2.5

Exercise 2

a) the integers greater than 10

f(n)=n+10, Countably infinite

b) the odd negitive integers.

f(n)=1-2n, Countably infinite

c)the integers with absolute value less than 1,000,000 .

$C = \{x : \|x\| < 1,000,000\}$

$C = \{x : -1,000,000 < x < 1,000,000\}$

Finite

d) the real number between 0 and 2.

$D = \{x : x \in (0,2) \cap R\}$

uncountable

e) the set A*Z^+ where $A = \{2,3\}$.

Countably infinite

by f(2,n)=2n and f(3,n)

f) the intergers are multiples of 10.

Countably infinite

$F = \{10n : n \in Z\}$

Countably infinite

Exercise 10

a) finite.

A=all real number =0

B=all real number ¿0

$A - B = \varnothing$

b) countably infinite.

A=all real number

B=all real number that are not the positive intergers

$= A - ()^+$

A-B=A-($A - ()^+$)

=$()^+$

c) uncountable.

A=all real number

B=all positive real number

A-B=$\{0, R^-\}$