# 4.1

## Exercise 28

a)a=-111,m=99

a=-111=-197+87=(-2)*99+87=(-2)m+87

the quotient m:q=-2

the remainder r=87(with $0 \leq 87 \leq 99$) a div m=-2

a mod m=87

b)a=-9999,m=101

a=-9999=(-99)*101=(-99)*101+0=(-99)m+0

the quotient m:q=-99

the remainder r=0 (with $0 \leq 0 \leq 101$)

a div m=-99

a mod m=0

c)a=10299,m=999

a=10299=9990+309=10*999+309=10m+309

the quotient m:q=10

the remainder r=309 (with $0 \leq 309 \leq 999$)

a div m=10

a mod m=309

d)a=123456,m=1001

a=123456=123123+333=123*1001+333=10m+333

the quotient m:q=123

the remainder r=333(with $0 \leq 333 \leq 1001$)

a div m=123

a mod m=333

## Exercise 30

a)$a \equiv 43 (\text{mod } 23)$ and $-22 \leq a \leq 0$

$$\begin{aligned} a &\equiv 43 (mod\, 23) \\ &\equiv 43 - 23 (mod\, 23) \\ &\equiv 20 (mod\, 23) \\ &\equiv 20 - 23 (mod\, 23) \\ &\equiv -3 (mod\, 23) \end{aligned}$$

$$a = -3$$

b)$a \equiv 17 (\text{mod } 29)$ and $-14 \leq a \leq 14$

$$\begin{aligned} a &\equiv 17 (mod\, 29) \\ &\equiv 17 - 29 (mod\, 29) \\ &\equiv -12 (mod\, 29) \end{aligned}$$

$$a = -12$$

c)$a \equiv -11 (\text{mod } 21)$ and $90 \leq a \leq 110$

$$\begin{aligned}
a &\equiv -11(mod\, 21) \\
&\equiv -11 + 21(mod\, 21) \\
&\equiv 10(mod\, 21) \\
&\equiv 10 + 21(mod\, 21) \\
&\equiv 31(mod\, 21) \\
&\equiv 31 + 21(mod\, 21) \\
&\equiv 52(mod\, 21) \\
&\equiv 52 + 21(mod\, 21) \\
&\equiv 73(mod\, 21) \\
&\equiv 73 + 21(mod\, 21) \\
&\equiv 94(mod\, 21)
\end{aligned}$$

$$a = 94$$

## Exercise 36

a)(177 mod 31 + 270 mod 31)mod 31

177 mod 31

a=177=155+22=5*31+22=5d+22

177 mod 31=22

270 mod 31

a=270=248+22=8*31+22=8d+22

270 mod 31=22

$$
\begin{aligned}
(177\,mod\,31 + 270\,mod\,31)\,mod\,31 &= (22+22)\,mod\,31 \\
&= 44\,mod\,31 \\
&= 44-31\,mod\,31 \\
&= 13\,mod\,31 \\
&= 13
\end{aligned}
$$

b)(177 mod 31 * 270 mod 31)mod 31
177 mod 31
a=177=155+22=5*31+22=5d+22
177 mod 31=22
270 mod 31
a=270=248+22=8*31+22=8d+22
270 mod 31=22

$$
\begin{aligned}
(177\,mod\,31 * 270\,mod\,31)\,mod\,31 &= (22*22)\,mod\,31 \\
&= 484\,mod\,31
\end{aligned}
$$

484 mod 31
a=484=465+19=15*31+19=15d+19
484 mod 31=19

## Exercise 42

a,b,c and m are integers $m \geq 2, c > 0$ and $a \equiv b$(mod m) then $ac \equiv bc$(mod mc)
$a \equiv b$(mod m)
m divides a-b
m divides c-d
a-b=mf

c(a-b)=c(mf)

ac-bc=(mc)f

$ac - bc \equiv (mc)f$

# 5.1

## Exercise 4

a) What is the statement P (1)?

$P(1) : 1^3 = (\frac{1(1+2)}{2})^2$

b) Show that P (1) is true, completing the basis step of the proof of P(n) for all positive integers n.

P(1) is true.

when$a = 11 = 1^2 = 1$

c)What is the inductive hypothesis of a proof that P(n) is true fpr all positive integers n?

$1^3 + 2^3 + ... + k^3 = (\frac{k(k+1)}{2})^2$

d) What do you need to prove in the inductive step of a proof that P(n) is true for all positive integers n?

P(k+1)

e) Complete the inductive step of a proof that P(n) is true for all positive integers n, identifying where you use the inductive hypothesis.

$1^3 + 2^3 + ... + k^3 + (k+1)^3 = (\frac{(k+1)(k+1)+1}{2})^2$

f)Explain why these steps show that this formula is true whenever n is a positive integer.
Induction

## Exercise 32

prove that 3 divides $n^3 + 2n$ whenever n is positive integer.
n=1
$n^3 + 2n = 1^3 + 2(1) = 1 + 2 = 3$
3 divides $k^3 + 2k$
P(k+1) also true

$$\begin{aligned}(k+1)^3 + 2(k+1) &= k^3 + 3k^2 + 3k + 1 + 2k + 2 \\ &= k^3 + 3k^2 + 5k + 3 \\ &= (k^3 + 2k) + (3k^2 + 3k + 3) \\ &= (k^3 + 2k) + 3(k^2 + k + 1)\end{aligned}$$

$k^3+2k$is divisible by 3 and $3(k^2+k+)$ is divisible by 3, thus $(k+1)^3+2(k+1)$ is then also divisible by 3
thus P(k+1) is true
3 divides $n^3 + 2n$ for every positive integer n.

## Exercise 40

prove that if $A_1, A_2, ..., A_n$ and B are sets, then
$(A_1 \cap A_2 \cap \cdots \cap A_n) \cup B$
$\equiv (A_1 \cup B) \cap (A_2 \cup B) \cap \cdots \cap (A_n \cup B)$
n=P(k+1)

$$
\begin{aligned}
(A_1 \cap A_2 \cap ... \cap A_n) \cup B &= (A_1 \cup B) \cap (A_2 \cup B) \cap ... \cap (A_n) \cup B) \\
&= [(A_1 \cap A_2 \cap ... \cap A_k) \cap A_{k+1}] \cup B \\
&= [(A_1 \cap A_2 \cap ... \cap A_k) \cup B] \cap (A_{k+1} \cup B) \\
& distributive\, property\, of\, union\, of\, intersection \\
&= (A_1 \cup B) \cap (A_2 \cup B) \cap ... \cap (A_k \cup B) \cap (A_{k+1} \cup B) \\
& since\, the\, result\, is\, true\, for\, k \\
P(k+1) : (A_1 \cap A_2 \cap ... \cap A_{k+1}) \cup B &= (A_1 \cup B) \cap (A_2 \cup B) \cap ... \cap (A_{k+1} \cup B) \\
(A_1 \cap A_2 \cap ... \cap A_n) \cup B &= (A_1 \cup B) \cap (A_2 \cup B) \cap ... \cap (A_n) \cup B)
\end{aligned}
$$