2.1

Exercise 12

(a)

$\varnothing \in \{\varnothing\}$

True, for the reason that $\{\varnothing\}$ meaning a empty set set so that $\varnothing$ is a element of empty set.

(b)

$\varnothing \in \{\varnothing, \{\varnothing\}\}$

True, same with (a) the $\varnothing$ is a element of that set.

(c)

$\{\varnothing\} \in \{\varnothing\}$

False, they are same set that meaning they can not element in that set.

(d)

$\{\varnothing\} \in \{\{\varnothing\}\}$

True, $\{\varnothing\}$ is a element of $\{\{\varnothing\}\}$.

(e)

$\{\varnothing\} \subset \{\varnothing, \{\varnothing\}\}$ True, $\{\varnothing\}$ is a element of $\{\varnothing, \{\varnothing\}\}$.

(f)

$\{\{\varnothing\}\} \subset \{\varnothing, \{\varnothing\}\}$ True, $\{\{\varnothing\}\}$ can be undersatnd the set $\{\varnothing\}$ so that is subset of the big set.

(g)

$\{\{\varnothing\}\} \subset \{\{\varnothing\}, \{\varnothing\}\}$ False, this euqal can be write like this $\{\{\varnothing\}\} =$

$\{\{\varnothing\}, \{\varnothing\}\}$ so that is False.

2.2

Exercise 14

$$\begin{cases} A - B = 1, 5, 7, 8) \\ B - A = 2, 10 \\ A \cap B = 3, 6, 9 \end{cases}$$

$$\begin{aligned} A &= (A \cap B) \cup (A - B) \\ &= \{3, 6, 9\} \cup \{1, 5, 7, 8\} \\ &= \{1, 3, 5, 6, 7, 8, 9\} \\ B &= (A \cap B) \cup (B - A) \\ &= \{3, 6, 9\} \cup \{2, 10\} \\ &= \{2, 3, 6, 9, 10\} \end{aligned}$$

so $A = \{1, 3, 5, 6, 7, 8, 9\}$ and $B = \{2, 3, 6, 9, 10\}$

Exercise 20

(a)

$(A \cup B) \subseteq (A \cup B \cup C)$

$$x \in A \cup B$$
$$x \in A \vee x \in B$$
$$x \in A \vee x \in B \vee x \in C$$
$$x \in A \cup B \cup C$$
$$(A \cup B) \subseteq (A \cup B \cup C)$$

(b)

$(A \cap B \cap C) \subseteq (A \cap B)$

$$x \in A \cap B \cap C$$
$$x \in A \wedge x \in B \wedge x \in C$$
$$x \in A \wedge x \in B$$
$$x \in A \cap B$$
$$(A \cap B \cap C) \subseteq (A \cap B)$$

(c)

$(A - B) - C \subseteq A - C$

$$x \in (A - C) \cap (C - B)$$
$$x \in (A - C) \wedge \neg(x \in C)$$
$$x \in A \wedge \neg(x \in B) \wedge \neg(x \in C)$$
$$x \in A \wedge \neg(x \in C)$$
$$x \in A - C$$
$$(A - B) - C \subseteq A - C$$

(d)

$(A - C) \cap (C - B) = \varnothing$

$$x \in (A - C) \cap (C - B)$$

$$x \in (A - C) \land x \in (C - B)$$

$$x \in A \land \neg(x \in C) \land x \in C \land \neg(x \in B)$$

$$x \in A \land F \land \neg(x \in B)$$

$$x \in \varnothing$$

$$\varnothing \subseteq (A - C)$$

$$(A - C) \cap (C - B) = \varnothing$$

(e)

$(B - A) \cup (C - A) = (B \cup C) - A$

$$x \in (B - A) \cup (C - A)$$

$$x \in (B - A) \lor x \in (C - A)$$

$$(x \in B \land \neg(x \in A)) \lor (x \in C \land \neg(x \in A))$$

$$(x \in B \lor x \in C) \land \neg(x \in A)$$

$$(x \in B \lor C) \land \neg(x \in A)$$

$$x \in (B \cup C) - A$$

$$x \in B \cup C \land \neg(x \in A)$$

$$(x \in B \lor x \in C) \land \neg(x \in A)$$

$$(x \in B \land \neg(x \in A)) \lor (x \in C \land \neg(x \in A))$$

$$x \in B - A \lor x \in C - A$$

$$x \in (B - A) \cup (C - A)$$

$$(B - A) \cup (C - A) = (B \cup C) - A$$

Exercise 48

$$
\begin{aligned}
(A \oplus B) \oplus (C \oplus D) = (A \oplus c) \oplus (B \oplus D) &= A \oplus (B \oplus (C \oplus D)) \\
&= A \oplus (B \oplus (D \oplus C)) \\
&= A \oplus ((B \oplus D) \oplus C) \\
&= A \oplus (C \oplus (B \oplus D)) \\
&= (A \oplus C) \oplus (B \oplus D)
\end{aligned}
$$

2.3

Exercise 12

(a)

f(n)=n-1

one-to-one, each one number just cna get one result.

(b)

$f(n) = n^2 + 1$

not one-to-one, the negitive and positive can be get the same result.like 1 and -1.

(c)

$f(n) = n^3$

one-to-one, each one unumber just can get one result.

(d)

$f(n) = \lceil n/2 \rceil$

not one-to-one, when n=0.5 and 1 they can het the same result 1.

Exercise 14

(a)

f(m,n)=2m-n

Onto, each one can number can get at little one combo of m and n.

(b)

$f(m, n) = m^2 - n^2$

Not onto, for example you can not get 2 by this equation.

(c)

f(m,n)=m+n+1

Onto, any number you can get.

(d)

$f(m, n) = |m| - |n|$

Onto, you can get any number even negitive.

(e)

$f(m, n) = m^2 - 4$

Not onto, you can not get -5 in this equation. or any number less than -5.

Exercise 20

(a)one-to-one but not onto

$f(n) = n^2$

(b)onto nut not one-to-one

$f(n) = \lceil \frac{n}{2} \rceil$

(c)both onto and one-to-one(but different from the identity function)

$$f(n) = \begin{cases} n-1 & \text{if n is odd.} \\ n+1 & \text{if n is even.} \end{cases}$$

(d)neither one to one nor onto

f(n)=0

Exercise 48

$[x + \frac{1}{2}]$

when x is midway of two integer $[x + \frac{1}{2}]$ will euqal of larger than larger of two integer.

Bonus

Exercise 74

if $|A| = |B|$ is one-to-one that meaning $f(a_1) = b_1$

if onto B=f(A) B should have all match of A, the function is one-to-one so that will be onto.

so f is one-to-one if and only if f is onto when $|A| = |B|$.