

7.4

Exercise 4

A coin is biased so that the probability a head comes up when it is flipped is 0.6. What is the expected number of heads that come up when it is flipped 10 times?

$$10 * 0.6 = 6$$

the answer is 6 times

Exercise 8

What is the expected sum of the numbers that appear when three fair dice are rolled?

$$3 * \frac{1+2+3+4+5+6}{6} = 10.5$$

so we can get 10.5

Exercise 10

Suppose that we flip a fair coin until either it comes up tails twice or we have flipped it six times. What is the expected number of times we flip the coin?

$$\text{if times is 2 } (0.5)^2 = 0.25$$

$$\text{if times is 3 } 2 * (0.5)^3 = 0.25$$

$$\text{if times is 4 } 3 * (0.5)^4 = 0.1875$$

$$\text{if times is 5 } 4 * (0.5)^5 = 0.125$$

$$\text{if times is 6 } 6 * (0.5)^5 = 0.1875$$

$$2 * 0.25 + 3 * 0.25 + 4 * 0.1875 + 5 * 0.125 + 6 * 0.1875 = 3.75$$

Exercise 36

$$p = \frac{3}{5}$$

$$q = 1 - \frac{3}{5} = \frac{2}{5}$$

$$E(X) = np$$

$$= \frac{3n}{5}$$

$$V(X) = npq$$

$$= \frac{3n}{5} * \frac{2}{5}$$

$$= \frac{6n}{25}$$

$$p(|X(s) - \frac{3n}{5}| \geq \sqrt{n}) \leq \frac{6n}{25}/(\sqrt{n})^2$$

$$= \frac{6n}{25} * \frac{1}{n}$$

$$= \frac{6}{25}$$

9.1

Exercise 4

a) a is taller than b

Reflexive: No

Symmetric: No

Antisymmetric:Yes

Transitive:Yes

b) a and b were born on the same day

Reflexive: Yes

Symmetric: Yes

Antisymmetric:No

Transitive:Yes

c) a has the same first name as b

Reflexive: Yes

Symmetric: Yes

Antisymmetric:No

Transitive:Yes

d) a and b have a common grandparent

Reflexive: Yes

Symmetric: Yes

Antisymmetric:No

Transitive:No

Exercise 6

a) $x+y=0$

Reflexive: No

Symmetric: Yes

Antisymmetric:No

Transitive:No

b) $x=\pm y$

Reflexive: Yes

Symmetric: Yes

Antisymmetric:No

Transitive:Yes

c) $x-y$ is a rational number.

Reflexive: Yes

Symmetric: Yes

Antisymmetric:No

Transitive:Yes

d) $x=2y$

Reflexive: No

Symmetric: No

Antisymmetric:Yes

Transitive:No

e) $xy \geq 0$

Reflexive: Yes

Symmetric: Yes

Antisymmetric:No

Transitive:No

f) $xy=0$

Reflexive: No

Symmetric: Yes

Antisymmetric:No

Transitive:No

g) $x=1$

Reflexive: No

Symmetric: No

Antisymmetric:Yes

Transitive:Yes

h) $x=1$ or $y=1$

Reflexive: No

Symmetric: Yes

Antisymmetric:No

Transitive:No

Exercise 32

$R=(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)$

$S=(2, 1), (3, 1), (3, 2), (4, 2)$

$S \circ R = (1, 1), (1, 2), (2, 1), (2, 2)$

Exercise 56

Show that the relation R on a set A is reflexive if and only if the complementary relation \overline{R} is irreflexive.

$$a \in A(a, a) \in R(a, a) \notin \overline{R}a \in A(a, a) \notin \overline{R}(a, a) \in R$$

so R is reflexive if and only if \overline{R} is irreflexive

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Exercise 2

$$R = (a, b)/a \neq b = (a, b)/a \neq b \cup (a, a)/a \in Z = (a, b)/a, b \in Z = Z * Z$$

reflexive closure of R is the set.

Exercise 20

a) R^2

$$R^2 = R \circ R = (a, b) = (a, b)$$

the airline will be have one stop in some intermediate city

b) R^3

$$R^3 = R \circ R = (a, b) = (a, b)$$

the airline will be have two stop in some intermediate city

c) R^*

$$R^* = R \cup R^2 \cup R^3 \cup \dots \cup R^n \text{ will have } n - 1 \text{ stop}$$

it is possible to fly from a to b

9.5

Exercise 2

Which of these relations on the set of all people are equivalence relations?

Determine the properties of an equivalence relation that the others lack.

a) $(a, b) \text{ — } a \text{ and } b \text{ are the same age}$

equivalence relation

b) $(a, b) \text{ — } a \text{ and } b \text{ have the same parents}$

equivalence relation

c) $(a, b) \text{ — } a \text{ and } b \text{ share a common parent}$

not Transitive

d) $(a, b) \text{ — } a \text{ and } b \text{ have met}$

not Transitive

e) $(a, b) \text{ — } a \text{ and } b \text{ speak a common language}$

not Transitive

Exercise 10

$$\begin{aligned}
 (x, y) \in R &\rightarrow x, y \in [X]R \rightarrow f(x) = x_0 = f(y) \\
 f(x) = f(y) &\rightarrow x_0 = y_0 \rightarrow [X]R = [y]R \rightarrow \in [x]R \rightarrow (x, y) \in R \\
 (x, y) \in R &\quad x, y \in A \\
 (x, y) \in R &\quad f(X) = f(y) = c_1 \\
 (x, y) \in R &\quad f(x) = f(y)
 \end{aligned}$$

the function f defined above satisfies the required condition.