

initial system:

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i}{\Vdash \Gamma, x : A} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : A}{\Gamma, x : A \vdash x : A} \text{ (var)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i \quad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Pi(x : A). B : \mathbb{U}_i} \text{ (\Pi)} \\
\\
\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : \Pi(x : A). B} \text{ (\Pi}_i\text{)} \quad \frac{\Gamma \vdash e_1 : \Pi(x : A). B \quad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1 @ e_2 : B[e_1/x]} \text{ (\Pi}_e\text{)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i \quad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : A). B : \mathbb{U}_i} \text{ (\Sigma)} \\
\\
\frac{\Gamma \vdash e_1 : A_1 \quad \dots \quad \Gamma \vdash e_n : A_n[e_1/x_1] \dots [e_{n-1}/x_{n-1}]}{\Gamma \vdash (e_1, \dots, e_n) : \Sigma(x_1 : A_1, \dots, x_{n-1} : A_{n-1}). A_n} \text{ (\Sigma}_i\text{)} \\
\\
\frac{\Gamma \vdash e_1 : \Sigma(x_1 : A_1, \dots, x_{n-1} : A_{n-1}). A_n \quad \Gamma, x_1 : A_1, \dots, x_n : A_n \vdash e_2 : B[(x_1, \dots, x_n)/z]}{\Gamma \vdash \text{let } (x_1, \dots, x_n) = e_1 \text{ in } e_2 : B[e_1/z]} \text{ (\Sigma}_e\text{)} \\
\\
\frac{\Gamma, x : A \vdash e : A}{\Gamma \vdash \text{rec } x. e : A} \text{ (rec)} \quad \frac{}{\Gamma \vdash c : A_c} \text{ (constant)}
\end{array}$$

polarize:

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i}{\Vdash \Gamma, x : P} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P} \text{ (var)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i \quad \Gamma, x : P \vdash N : \mathbb{U}_i}{\Gamma \vdash \Pi(x : P). N : \mathbb{U}_i} \text{ (\Pi)} \\
\\
\frac{\Gamma, x : P \vdash e : N}{\Gamma \vdash \lambda x. e : \Pi(x : P). N} \text{ (\Pi}_i\text{)} \\
\\
\frac{\Gamma \vdash e_1 : \Pi(x : P). N \quad \Gamma \vdash e_2 : P}{\Gamma \vdash e_1 @ e_2 : N[e_1/x]} \text{ (\Pi}_e\text{)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i \quad \Gamma, x : P \vdash Q : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : P). Q : \mathbb{U}_i} \text{ (\Sigma)} \\
\\
\frac{\Gamma \vdash e_1 : P_1 \quad \dots \quad \Gamma \vdash e_n : P_n[e_1/x_1] \dots [e_{n-1}/x_{n-1}]}{\Gamma \vdash (e_1, \dots, e_n) : \Sigma(x_1 : P_1, \dots, x_{n-1} : P_{n-1}). P_n} \text{ (\Sigma}_i\text{)} \\
\\
\frac{\Gamma \vdash e_1 : \Sigma(x_1 : P_1, \dots, x_{n-1} : P_{n-1}). P_n \quad \Gamma, x_1 : P_1, \dots, x_n : P_n \vdash e_2 : N[(x_1, \dots, x_n)/z]}{\Gamma \vdash \text{let } (x_1, \dots, x_n) = e_1 \text{ in } e_2 : N[e_1/z]} \text{ (\Sigma}_e\text{)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i}{\Gamma \vdash \uparrow P : \mathbb{U}_i} \text{ (\uparrow)} \quad \frac{\Gamma \vdash e : P}{\Gamma \vdash \text{return } e : \uparrow P} \text{ (\uparrow}_i\text{)} \quad \frac{\Gamma \vdash e_1 : \uparrow P \quad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \triangleright_x e_2 : N} \text{ (\uparrow}_e\text{)} \\
\\
\frac{\Gamma \vdash N : \mathbb{U}_i}{\Gamma \vdash \downarrow N : \mathbb{U}_i} \text{ (\downarrow)} \quad \frac{\Gamma \vdash e : N}{\Gamma \vdash \text{thunk } e : \downarrow N} \text{ (\downarrow}_i\text{)} \quad \frac{\Gamma \vdash e : \downarrow N}{\Gamma \vdash \text{force } e : N} \text{ (\downarrow}_e\text{)} \\
\\
\frac{\Gamma, x : \downarrow N \vdash e : N}{\Gamma \vdash \text{rec } x. e : N} \text{ (rec)} \quad \frac{}{\Gamma \vdash c : P_c} \text{ (constant)}
\end{array}$$

after closure conversion:

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i}{\Vdash \Gamma, x : P} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P} \text{ (var)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i \quad \Gamma, x : P \vdash N : \mathbb{U}_i}{\Gamma \vdash \Pi(x : P). N : \mathbb{U}_i} \text{ (\Pi)} \\
\\
\frac{x_1 : P_1, \dots, x_n : P_n \vdash e : N}{\Gamma \vdash \text{return}(\text{thunk}(\lambda(x_1, \dots, x_n). e)) : \uparrow \downarrow \Pi(x_1 : P_1, \dots, x_n : P_n). N} \text{ (\Pi}_i\text{)} \\
\\
\frac{\Gamma \vdash e : \uparrow \downarrow \Pi(x_1 : P_1, \dots, x_n : P_n). N \quad \Gamma \vdash e_1 : P_1 \quad \dots \quad \Gamma \vdash e_n : P_n}{\Gamma \vdash e_1 \triangleright_f (\text{force } f) @ (e_1, \dots, e_n) : N[e_i/x_i]} \text{ (\Pi}_e\text{)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i \quad \Gamma, x : P \vdash Q : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : P). Q : \mathbb{U}_i} \text{ (\Sigma)} \\
\\
\frac{\Gamma \vdash e_1 : P_1 \quad \dots \quad \Gamma \vdash e_n : P_n[e_1/x_1] \dots [e_{n-1}/x_{n-1}]}{\Gamma \vdash \text{return}(e_1, \dots, e_n) : \uparrow \Sigma(x_1 : P_1, \dots, x_{n-1} : P_{n-1}). P_n} \text{ (\Sigma}_i\text{)} \\
\\
\frac{\Gamma \vdash e : \uparrow \Sigma(x_1 : P_1, \dots, x_{n-1} : P_{n-1}). P_n \quad \Gamma, x_1 : P_1, \dots, x_n : P_n \vdash e' : N[(x_1, \dots, x_n)/z]}{\Gamma \vdash e \triangleright_x \text{let } (x_1, \dots, x_n) = x \text{ in } e' : N[e/z]} \text{ (\Sigma}_e\text{)} \\
\\
\frac{x : \downarrow N \vdash e : N}{\Gamma \vdash \text{rec } x. e : N} \text{ (rec)} \quad \frac{}{\Gamma \vdash c : \downarrow N_c} \text{ (constant)}
\end{array}$$