

initial system:

$$\begin{array}{c}
\frac{}{\Vdash \cdot} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U} : \mathbb{U}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}}{\Vdash \Gamma, x : A} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : A}{\Gamma, x : A \vdash x : A} \text{ (var)} \\
\\
\frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{E}_k : \mathbb{U}} \text{ (E)} \quad \frac{\Vdash \Gamma \quad c \in \mathbb{E}_k}{\Gamma \vdash c : \mathbb{E}_k} \text{ (E}_i\text{)} \quad \frac{\Gamma \vdash e : \mathbb{E}_k \quad (\Gamma, x : \mathbb{E}_k \vdash e_c : A)_{c \in \mathbb{E}_k}}{\Gamma \vdash \text{let } x := e \text{ in } (c \Rightarrow e_c) : A\{x := e\}} \text{ (E}_e\text{)} \\
\\
\frac{\Gamma \vdash A_0 : \mathbb{U} \quad \cdots \quad \Gamma, x_0 : A_0, \dots, x_{n-1} : A_{n-1} \vdash A_n : \mathbb{U}}{\Gamma \vdash \Pi(x_0 : A_0, \dots, x_{n-1} : A_{n-1}). A_n : \mathbb{U}} \text{ (}\Pi\text{)} \\
\\
\frac{\Gamma, x_0 : A_0, \dots, x_{n-1} : A_{n-1} \vdash e : A_n}{\Gamma \vdash \lambda x. e : \Pi(x_0 : A_0, \dots, x_{n-1} : A_{n-1}). A_n} \text{ (}\Pi_i\text{)} \\
\\
\frac{\Gamma \vdash e : \Pi(x_0 : A_0, \dots, x_{n-1} : A_{n-1}). A_n \quad \Gamma \vdash e_0 : A_0 \quad \cdots \quad \Gamma \vdash e_{n-1} : A_{n-1}\{x_k := e_k\}}{\Gamma \vdash e @ (e_0, \dots, e_{n-1}) : A_n\{x_k := e_k\}} \text{ (}\Pi_e\text{)} \\
\\
\frac{\Gamma \vdash A_0 : \mathbb{U} \quad \cdots \quad \Gamma, x_0 : A_0, \dots, x_{n-1} : A_{n-1} \vdash A_n : \mathbb{U}}{\Gamma \vdash \Sigma(x_0 : A_0, \dots, x_{n-1} : A_{n-1}). A_n : \mathbb{U}} \text{ (}\Sigma\text{)} \\
\\
\frac{\Gamma \vdash e_0 : A_0 \quad \cdots \quad \Gamma \vdash e_n : A_n\{x_k := e_k\}}{\Gamma \vdash (e_1, \dots, e_n) : \Sigma(x_1 : A_0, \dots, x_{n-1} : A_{n-1}). A_n} \text{ (}\Sigma_i\text{)} \\
\\
\frac{\Gamma \vdash e_1 : \Sigma(x_1 : A_0, \dots, x_{n-1} : A_{n-1}). A_n \quad \Gamma, x_0 : A_0, \dots, x_n : A_n \vdash e_2 : B\{z := (x_0, \dots, x_n)\}}{\Gamma \vdash \text{let } (x_0, \dots, x_n) := e_1 \text{ in } e_2 : B\{z := e_1\}} \text{ (}\Sigma_e\text{)} \\
\\
\frac{\Gamma, x : A \vdash e : A}{\Gamma \vdash \text{fix } x. e : A} \text{ (fix)} \quad \frac{}{\Gamma \vdash c : A_c} \text{ (constant)}
\end{array}$$

after closure conversion:

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i}{\Vdash \Gamma, x : A} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : A}{\Gamma, x : A \vdash x : A} \text{ (var)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i \quad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Pi(x : A). B : \mathbb{U}_i} \text{ (\Pi)} \\
\\
\frac{x_1 : A_1, \dots, x_n : A_n \vdash e : B}{\Gamma \vdash \lambda(x_1, \dots, x_n). e : \Pi(x_1 : A_1, \dots, x_n : A_n). B} \text{ (\Pi}_i\text{)} \\
\\
\frac{\Gamma \vdash e : \Pi(x_1 : A_1, \dots, x_n : A_n). B \quad \Gamma \vdash e_1 : A_1 \quad \dots \quad \Gamma \vdash e_n : A_n}{\Gamma \vdash e @ (e_1, \dots, e_n) : B\{x_i := e_i\}} \text{ (\Pi}_e\text{)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i \quad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : A). B : \mathbb{U}_i} \text{ (\Sigma)} \\
\\
\frac{\Gamma \vdash e_1 : A_1 \quad \dots \quad \Gamma \vdash e_n : A_n\{x_1 := e_1\} \dots \{x_{n-1} := e_{n-1}\}}{\Gamma \vdash (e_1, \dots, e_n) : \Sigma(x_1 : A_1, \dots, x_{n-1} : A_{n-1}). A_n} \text{ (\Sigma}_i\text{)} \\
\\
\frac{\Gamma \vdash e_1 : \Sigma(x_1 : A_1, \dots, x_{n-1} : A_{n-1}). A_n \quad \Gamma, x_1 : A_1, \dots, x_n : A_n \vdash e_2 : B\{z := (x_1, \dots, x_n)\}}{\Gamma \vdash \text{let } (x_1, \dots, x_n) := e_1 \text{ in } e_2 : B\{z := e_1\}} \text{ (\Sigma}_e\text{)} \\
\\
\frac{x : A \vdash e : A}{\Gamma \vdash \text{rec } x. e : A} \text{ (rec)} \quad \frac{}{\Gamma \vdash c : A_c} \text{ (constant)}
\end{array}$$

polarize:

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i}{\Vdash \Gamma, x : P} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P} \text{ (var)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i \quad \Gamma, x : P \vdash N : \mathbb{U}_i}{\Gamma \vdash \Pi(x : P). N : \mathbb{U}_i} (\Pi) \\
\\
\frac{\Gamma, x : P \vdash e : N}{\Gamma \vdash \lambda x. e : \Pi(x : P). N} (\Pi_i) \\
\\
\frac{\Gamma \vdash e_1 : \Pi(x : P). N \quad \Gamma \vdash e_2 : P}{\Gamma \vdash e_1 @ e_2 : N\{x := e_1\}} (\Pi_e) \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i \quad \Gamma, x : P \vdash Q : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : P). Q : \mathbb{U}_i} (\Sigma) \\
\\
\frac{\Gamma \vdash e_1 : P_1 \quad \dots \quad \Gamma \vdash e_n : P_n\{x_1 := e_1\} \dots \{x_{n-1} := e_{n-1}\}}{\Gamma \vdash (e_1, \dots, e_n) : \Sigma(x_1 : P_1, \dots, x_{n-1} : P_{n-1}). P_n} (\Sigma_i) \\
\\
\frac{\Gamma \vdash e_1 : \Sigma(x_1 : P_1, \dots, x_{n-1} : P_{n-1}). P_n \quad \Gamma, x_1 : P_1, \dots, x_n : P_n \vdash e_2 : N\{z := (x_1, \dots, x_n)\}}{\Gamma \vdash \text{let } (x_1, \dots, x_n) := e_1 \text{ in } e_2 : N\{z := e_1\}} (\Sigma_e) \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i}{\Gamma \vdash \uparrow P : \mathbb{U}_i} (\uparrow) \quad \frac{\Gamma \vdash e : P}{\Gamma \vdash \text{return } e : \uparrow P} (\uparrow_i) \quad \frac{\Gamma \vdash e_1 : \uparrow P \quad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \triangleright_x e_2 : N} (\uparrow_e) \\
\\
\frac{\Gamma \vdash N : \mathbb{U}_i}{\Gamma \vdash \downarrow N : \mathbb{U}_i} (\downarrow) \quad \frac{\Gamma \vdash e : N}{\Gamma \vdash \text{thunk } e : \downarrow N} (\downarrow_i) \quad \frac{\Gamma \vdash e : \downarrow N}{\Gamma \vdash \text{force } e : N} (\downarrow_e) \\
\\
\frac{\Gamma, x : \downarrow N \vdash e : N}{\Gamma \vdash \text{rec } x. e : N} (\text{rec}) \quad \frac{}{\Gamma \vdash c : P_c} (\text{constant})
\end{array}$$

after closure conversion:

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i}{\Vdash \Gamma, x : P} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P} \text{ (var)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i \quad \Gamma, x : P \vdash N : \mathbb{U}_i}{\Gamma \vdash \Pi(x : P). N : \mathbb{U}_i} \text{ (\Pi)} \\
\\
\frac{x_1 : P_1, \dots, x_n : P_n \vdash e : N}{\Gamma \vdash \text{thunk}(\lambda(x_1, \dots, x_n). e) : \downarrow \Pi(x_1 : P_1, \dots, x_n : P_n). N} \text{ (\Pi}_i\text{)} \\
\\
\frac{\Gamma \vdash e : \downarrow \Pi(x_1 : P_1, \dots, x_n : P_n). N \quad \Gamma \vdash e_1 : P_1 \quad \dots \quad \Gamma \vdash e_n : P_n}{\Gamma \vdash (\text{force } e_1) @ (e_1, \dots, e_n) : N\{x_i := e_i\}} \text{ (\Pi}_e\text{)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i \quad \Gamma, x : P \vdash Q : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : P). Q : \mathbb{U}_i} \text{ (\Sigma)} \\
\\
\frac{\Gamma \vdash e_1 : P_1 \quad \dots \quad \Gamma \vdash e_n : P_n\{x_1 := e_1\} \dots \{x_{n-1} := e_{n-1}\}}{\Gamma \vdash \text{return}(e_1, \dots, e_n) : \Sigma(x_1 : P_1, \dots, x_{n-1} : P_{n-1}). P_n} \text{ (\Sigma}_i\text{)} \\
\\
\frac{\Gamma \vdash e : \Sigma(x_1 : P_1, \dots, x_{n-1} : P_{n-1}). P_n \quad \Gamma, x_1 : P_1, \dots, x_n : P_n \vdash e' : N\{z := (x_1, \dots, x_n)\}}{\Gamma \vdash \text{let}(x_1, \dots, x_n) := e \text{ in } e' : N\{z := e\}} \text{ (\Sigma}_e\text{)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i}{\Gamma \vdash \uparrow P : \mathbb{U}_i} \text{ (\uparrow)} \quad \frac{\Gamma \vdash e : P}{\Gamma \vdash \text{return } e : \uparrow P} \text{ (\uparrow}_i\text{)} \quad \frac{\Gamma \vdash e_1 : \uparrow P \quad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \triangleright_x e_2 : N} \text{ (\uparrow}_e\text{)} \\
\\
\frac{x : \downarrow N \vdash e : N}{\Gamma \vdash \text{rec } x. e : N} \text{ (rec)} \quad \frac{}{\Gamma \vdash c : \downarrow N_c} \text{ (constant)}
\end{array}$$