initial system:

$$\frac{}{\Vdash \emptyset}\ (\mathsf{empty}) \qquad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}}\ (\mathsf{univ}) \qquad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}}\ (\mathsf{hier})$$

$$\frac{\Gamma \vdash A : \mathbb{U}_i}{\Vdash \Gamma, x : A}\ (\mathsf{ext}) \qquad \frac{\Vdash \Gamma, x : A}{\Gamma, x : A \vdash x : A}\ (\mathsf{var})$$

$$\frac{\Gamma \vdash A : \mathbb{U}_i \qquad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Pi(x : A).\, B : \mathbb{U}_i}\ (\Pi)$$

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x.\, e : \Pi(x : A).\, B}\ (\Pi_{\mathsf{i}}) \qquad \frac{\Gamma \vdash e_1 : \Pi(x : A).\, B \qquad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1 @ e_2 : B[e_1/x]}\ (\Pi_{\mathsf{e}})$$

$$\frac{\Gamma \vdash A : \mathbb{U}_i \qquad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : A).\, B : \mathbb{U}_i}\ (\Sigma)$$

$$\frac{\Gamma \vdash e_1 : A_1 \qquad \ldots \qquad \Gamma \vdash e_n : A_n[e_1/x_1]\ldots[e_{n-1}/x_{n-1}]}{\Gamma \vdash (e_1, \ldots, e_n) : \Sigma(x_1 : A_1, \ldots, x_{n-1} : A_{n-1}).\, A_n}\ (\Sigma_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x_1 : A_1, \ldots, x_{n-1} : A_{n-1}).\, A_n \qquad \Gamma, x_1 : A_1, \ldots, x_n : A_n \vdash e_2 : B[(x_1, \ldots, x_n)/z]}{\Gamma \vdash \mathsf{let}\ (x_1, \ldots, x_n) = e_1\ \mathsf{in}\ e_2 : B[e_1/z]}\ (\Sigma_{\mathsf{e}})$$

$$\frac{\Gamma, x : A \vdash e : A}{\Gamma \vdash \mathsf{rec}\, x.\, e : A}\ (\mathsf{rec}) \qquad \frac{}{\Gamma \vdash c : A_c}\ (\mathsf{constant})$$

polarize:

$$\frac{}{\Vdash \varnothing}\ (\mathsf{empty}) \qquad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}}\ (\mathsf{univ}) \qquad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}}\ (\mathsf{hier})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i}{\Vdash \Gamma, x : P}\ (\mathsf{ext}) \qquad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P}\ (\mathsf{var})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i \qquad \Gamma, x : P \vdash N : \mathbb{U}_i}{\Gamma \vdash \Pi(x : P).\, N : \mathbb{U}_i}\ (\Pi)$$

$$\frac{\Gamma, x : P \vdash e : N}{\Gamma \vdash \lambda x.\, e : \Pi(x : P).\, N}\ (\Pi_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \Pi(x : P).\, N \qquad \Gamma \vdash e_2 : P}{\Gamma \vdash e_1 \mathbin{@} e_2 : N[e_1/x]}\ (\Pi_{\mathsf{e}})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i \qquad \Gamma, x : P \vdash Q : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : P).\, Q : \mathbb{U}_i}\ (\Sigma)$$

$$\frac{\Gamma \vdash e_1 : P_1 \qquad \ldots \qquad \Gamma \vdash e_n : P_n[e_1/x_1]\ldots[e_{n-1}/x_{n-1}]}{\Gamma \vdash (e_1, \ldots, e_n) : \Sigma(x_1 : P_1, \ldots, x_{n-1} : P_{n-1}).\, P_n}\ (\Sigma_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x_1 : P_1, \ldots, x_{n-1} : P_{n-1}).\, P_n \qquad \Gamma, x_1 : P_1, \ldots, x_n : P_n \vdash e_2 : N[(x_1, \ldots, x_n)/z]}{\Gamma \vdash \mathsf{let}\ (x_1, \ldots, x_n) = e_1\ \mathsf{in}\ e_2 : N[e_1/z]}\ (\Sigma_{\mathsf{e}})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i}{\Gamma \vdash {\uparrow}P : \mathbb{U}_i}\ ({\uparrow}) \qquad \frac{\Gamma \vdash e : P}{\Gamma \vdash \mathsf{return}\, e : {\uparrow}P}\ ({\uparrow}_{\mathsf{i}}) \qquad \frac{\Gamma \vdash e_1 : {\uparrow}P \qquad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \rhd_x e_2 : N}\ ({\uparrow}_{\mathsf{e}})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i}{\Gamma \vdash {\downarrow}N : \mathbb{U}_i}\ ({\downarrow}) \qquad \frac{\Gamma \vdash e : N}{\Gamma \vdash \mathsf{thunk}\, e : {\downarrow}N}\ ({\downarrow}_{\mathsf{i}}) \qquad \frac{\Gamma \vdash e : {\downarrow}N}{\Gamma \vdash \mathsf{force}\, e : N}\ ({\downarrow}_{\mathsf{e}})$$

$$\frac{\Gamma, x : {\downarrow}N \vdash e : {\downarrow}N}{\Gamma \vdash \mathsf{rec}\, x.\, e : {\downarrow}N}\ (\mathsf{rec}) \qquad \frac{}{\Gamma \vdash c : P_c}\ (\mathsf{constant})$$

$$\begin{aligned} P &::= x \mid \Sigma(x_1 : N_1, \ldots, x_n : N_n).\, M \mid {\downarrow}N \\ N &::= \Pi(x : N).\, M \mid {\uparrow}P \end{aligned}$$

after closure conversion:

$$\frac{}{\Vdash \varnothing}\ (\mathsf{empty}) \qquad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}}\ (\mathsf{univ}) \qquad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}}\ (\mathsf{hier})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i}{\Vdash \Gamma, x : N}\ (\mathsf{ext}) \qquad \frac{\Vdash \Gamma, x : N}{\Gamma, x : N \vdash x : \downarrow N}\ (\mathsf{var})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i \qquad \Gamma, x : N \vdash M : \mathbb{U}_i}{\Gamma \vdash \Pi(x : N).\, M : \mathbb{U}_i}\ (\Pi)$$

$$\frac{x : N \vdash e : M}{\Gamma \vdash \lambda x.\, e : \Pi(x : N).\, M}\ (\Pi_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \Pi(x : N).\, M \qquad \Gamma \vdash e_2 : \downarrow N}{\Gamma \vdash e_1 \mathbin{@} e_2 : M[e_1/x]}\ (\Pi_{\mathsf{e}})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i \qquad \Gamma, x : N \vdash M : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : N).\, M : \mathbb{U}_i}\ (\Sigma)$$

$$\frac{\Gamma \vdash e_1 : \downarrow N_1 \qquad \ldots \qquad \Gamma \vdash e_n : \downarrow N_n[e_1/x_1] \ldots [e_{n-1}/x_{n-1}]}{\Gamma \vdash \mathsf{return}\ (e_1, \ldots, e_n) : \uparrow\Sigma(x_1 : N_1, \ldots, x_{n-1} : N_{n-1}).\, N_n}\ (\Sigma_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \uparrow\Sigma(x_1 : N_1, \ldots, x_{n-1} : N_{n-1}).\, N_n \qquad \Gamma, x_1 : N_1, \ldots, x_n : N_n \vdash e_2 : B[(x_1, \ldots, x_n)/z]}{\Gamma \vdash e_1 \rhd_p \mathsf{let}\ (x_1, \ldots, x_n) = p\ \mathsf{in}\ e_2 : B[e_1/z]}\ (\Sigma_{\mathsf{e}})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i}{\Gamma \vdash \uparrow P : \mathbb{U}_i}\ (\uparrow) \qquad \frac{\Gamma \vdash e : P}{\Gamma \vdash \mathsf{return}\, e : \uparrow P}\ (\uparrow_{\mathsf{i}}) \qquad \frac{\Gamma \vdash e_1 : \uparrow\downarrow N \qquad \Gamma, x : N \vdash e_2 : M}{\Gamma \vdash e_1 \rhd_x e_2 : M}\ (\uparrow_{\mathsf{e}})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i}{\Gamma \vdash \downarrow N : \mathbb{U}_i}\ (\downarrow) \qquad \frac{\vdash e : N}{\Gamma \vdash \mathsf{thunk}\, e : \downarrow N}\ (\downarrow_{\mathsf{i}}) \qquad \frac{\Gamma \vdash e : \downarrow N}{\Gamma \vdash \mathsf{force}\, e : N}\ (\downarrow_{\mathsf{e}})$$

$$\frac{\Gamma, x : N \vdash e : \downarrow N}{\Gamma \vdash \mathsf{rec}\, x.\, e : \downarrow N}\ (\mathsf{rec}) \qquad \frac{}{\Gamma \vdash c : \downarrow N_c}\ (\mathsf{constant})$$

$$\begin{aligned} P &::= x \mid \Sigma(x_1 : N_1, \ldots, x_n : N_n).\, M \mid \downarrow N \\ N &::= \Pi(x : N).\, M \mid \uparrow P \end{aligned}$$