

initial system:

$$\begin{array}{c}
\frac{}{\Vdash \cdot} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U} : \mathbb{U}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}}{\Vdash \Gamma, x : A} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : A}{\Gamma, x : A \vdash x : A} \text{ (var)} \\
\\
\frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{E}_k : \mathbb{U}} \text{ (E)} \quad \frac{\Vdash \Gamma \quad c \in \mathbb{E}_k}{\Gamma \vdash c : \mathbb{E}_k} \text{ (E}_i\text{)} \quad \frac{\Gamma \vdash e : \mathbb{E}_k \quad (\Gamma, x : \mathbb{E}_k \vdash e_c : A)_{c \in \mathbb{E}_k}}{\Gamma \vdash \text{let } x := e \text{ in } (c \Rightarrow e_c) : A\{x := e\}} \text{ (E}_e\text{)} \\
\\
\frac{\Gamma \vdash A_0 : \mathbb{U} \quad \cdots \quad \Gamma, x_0 : A_0, \dots, x_{n-1} : A_{n-1} \vdash A_n : \mathbb{U}}{\Gamma \vdash \Pi(x_0 : A_0, \dots, x_{n-1} : A_{n-1}). A_n : \mathbb{U}} \text{ (}\Pi\text{)} \\
\\
\frac{\Gamma, x_0 : A_0, \dots, x_{n-1} : A_{n-1} \vdash e : A_n}{\Gamma \vdash \lambda(x_0 : A_0, \dots, x_{n-1} : A_{n-1}). e : \Pi(x_0 : A_0, \dots, x_{n-1} : A_{n-1}). A_n} \text{ (}\Pi_i\text{)} \\
\\
\frac{\Gamma \vdash e : \Pi(x_0 : A_0, \dots, x_{n-1} : A_{n-1}). A_n \quad \Gamma \vdash e_0 : A_0 \quad \cdots \quad \Gamma \vdash e_{n-1} : A_{n-1}\{x_k := e_k\}}{\Gamma \vdash e @ (e_0, \dots, e_{n-1}) : A_n\{x_k := e_k\}} \text{ (}\Pi_e\text{)} \\
\\
\frac{\Gamma \vdash A_0 : \mathbb{U} \quad \cdots \quad \Gamma, x_0 : A_0, \dots, x_{n-1} : A_{n-1} \vdash A_n : \mathbb{U}}{\Gamma \vdash \Sigma(x_0 : A_0, \dots, x_{n-1} : A_{n-1}). A_n : \mathbb{U}} \text{ (}\Sigma\text{)} \\
\\
\frac{\Gamma \vdash e_0 : A_0 \quad \cdots \quad \Gamma \vdash e_n : A_n\{x_k := e_k\}}{\Gamma \vdash (e_0, \dots, e_n) : \Sigma(x_0 : A_0, \dots, x_{n-1} : A_{n-1}). A_n} \text{ (}\Sigma_i\text{)} \\
\\
\frac{\Gamma \vdash e_1 : \Sigma(x_0 : A_0, \dots, x_{n-1} : A_{n-1}). A_n \quad \Gamma, x_0 : A_0, \dots, x_n : A_n \vdash e_2 : B\{z := (x_0, \dots, x_n)\}}{\Gamma \vdash \text{let } (x_0, \dots, x_n) := e_1 \text{ in } e_2 : B\{z := e_1\}} \text{ (}\Sigma_e\text{)} \\
\\
\frac{\Gamma, x : A \vdash e : A}{\Gamma \vdash \text{fix } x. e : A} \text{ (fix)} \quad \frac{}{\Gamma \vdash c : A_c} \text{ (constant)}
\end{array}$$

polarize:

$$\begin{array}{c}
\frac{}{\Vdash \cdot} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U} : \mathbb{U}} \text{ (univ)} \quad \frac{\Gamma \vdash P : \mathbb{U}}{\Vdash \Gamma, x : P} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P} \text{ (var)} \\
\\
\frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{E}_k : \mathbb{U}} \text{ (E)} \quad \frac{\Vdash \Gamma \quad c \in \mathbb{E}_k}{\Gamma \vdash c : \mathbb{E}_k} \text{ (E}_i\text{)} \quad \frac{\Gamma \vdash e : \mathbb{E}_k \quad (\Gamma, x : \mathbb{E}_k \vdash e_c : N)_{c \in \mathbb{E}_k}}{\Gamma \vdash \text{let } x := e \text{ in } (c \Rightarrow e_c) : N\{x := e\}} \text{ (E}_e\text{)} \\
\\
\frac{\Gamma \vdash P_0 : \mathbb{U} \quad \cdots \quad \Gamma, x_0 : P_0, \dots, x_{n-1} : P_{n-1} \vdash N : \mathbb{U}}{\Gamma \vdash \Pi(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). N : \mathbb{U}} \text{ (}\Pi\text{)} \\
\\
\frac{\Gamma, x_0 : P_0, \dots, x_{n-1} : P_{n-1} \vdash e : N}{\Gamma \vdash \lambda(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). e : \Pi(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). N} \text{ (}\Pi_i\text{)} \\
\\
\frac{\Gamma \vdash e : \Pi(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). N \quad \Gamma \vdash e_0 : P_0 \quad \cdots \quad \Gamma \vdash e_{n-1} : P_{n-1}\{x_k := e_k\}}{\Gamma \vdash e @ (e_0, \dots, e_{n-1}) : N\{x_k := e_k\}} \text{ (}\Pi_e\text{)} \\
\\
\frac{\Gamma \vdash P_0 : \mathbb{U} \quad \cdots \quad \Gamma, x_0 : P_0, \dots, x_{n-1} : P_{n-1} \vdash P_n : \mathbb{U}}{\Gamma \vdash \Sigma(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). P_n : \mathbb{U}} \text{ (}\Sigma\text{)} \\
\\
\frac{\Gamma \vdash e_0 : P_0 \quad \cdots \quad \Gamma \vdash e_n : P_n\{x_k := e_k\}}{\Gamma \vdash (e_0, \dots, e_n) : \Sigma(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). P_n} \text{ (}\Sigma_i\text{)} \\
\\
\frac{\Gamma \vdash e_1 : \Sigma(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). P_n \quad \Gamma, x_0 : P_0, \dots, x_n : P_n \vdash e_2 : N\{z := (x_0, \dots, x_n)\}}{\Gamma \vdash \text{let } (x_0, \dots, x_n) := e_1 \text{ in } e_2 : N\{z := e_1\}} \text{ (}\Sigma_e\text{)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}}{\Gamma \vdash \uparrow P : \mathbb{U}} \text{ (}\uparrow\text{)} \quad \frac{\Gamma \vdash e : P}{\Gamma \vdash \text{return } e : \uparrow P} \text{ (}\uparrow_i\text{)} \quad \frac{\Gamma \vdash e_1 : \uparrow P \quad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \triangleright_x e_2 : N} \text{ (}\uparrow_e\text{)} \\
\\
\frac{\Gamma \vdash N : \mathbb{U}}{\Gamma \vdash \downarrow N : \mathbb{U}} \text{ (}\downarrow\text{)} \quad \frac{\Gamma \vdash e : N}{\Gamma \vdash \text{thunk } e : \downarrow N} \text{ (}\downarrow_i\text{)} \quad \frac{\Gamma \vdash e : \downarrow N}{\Gamma \vdash \text{force } e : N} \text{ (}\downarrow_e\text{)} \\
\\
\frac{\Gamma, x : \downarrow N \vdash e : N}{\Gamma \vdash \text{fix } x. e : N} \text{ (fix)} \quad \frac{}{\Gamma \vdash c : P_c} \text{ (constant)}
\end{array}$$

after closure conversion:

$$\begin{array}{c}
\frac{}{\Vdash \cdot} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U} : \mathbb{U}} \text{ (univ)} \quad \frac{\Gamma \vdash P : \mathbb{U}}{\Vdash \Gamma, x : P} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P} \text{ (var)} \\
\\
\frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{E}_k : \mathbb{U}} \text{ (E)} \quad \frac{\Vdash \Gamma \quad c \in \mathbb{E}_k}{\Gamma \vdash c : \mathbb{E}_k} \text{ (E}_i\text{)} \quad \frac{\Gamma \vdash e : \mathbb{E}_k \quad (\Gamma, x : \mathbb{E}_k \vdash e_c : N)_{c \in \mathbb{E}_k}}{\Gamma \vdash \text{let } x := e \text{ in } (c \Rightarrow e_c) : N\{x := e\}} \text{ (E}_e\text{)} \\
\\
\frac{\Gamma \vdash P_0 : \mathbb{U} \quad \dots \quad \Gamma, x_0 : P_0, \dots, x_{n-1} : P_{n-1} \vdash N : \mathbb{U}}{\Gamma \vdash \downarrow \Pi(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). N : \mathbb{U}} \text{ (}\Pi\text{)} \\
\\
\frac{x_0 : P_0, \dots, x_{n-1} : P_{n-1} \vdash e : N}{\Gamma \vdash \text{thunk}(\lambda(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). e) : \downarrow \Pi(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). N} \text{ (}\Pi_i\text{)} \\
\\
\frac{\Gamma \vdash e : \downarrow \Pi(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). N \quad \Gamma \vdash e_0 : P_0 \quad \dots \quad \Gamma \vdash e_{n-1} : P_{n-1}\{x_k := e_k\}}{\Gamma \vdash (\text{force } e) @ (e_0, \dots, e_{n-1}) : N\{x_k := e_k\}} \text{ (}\Pi_e\text{)} \\
\\
\frac{\Gamma \vdash P_0 : \mathbb{U} \quad \dots \quad \Gamma, x_0 : P_0, \dots, x_{n-1} : P_{n-1} \vdash P_n : \mathbb{U}}{\Gamma \vdash \Sigma(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). P_n : \mathbb{U}} \text{ (}\Sigma\text{)} \\
\\
\frac{\Gamma \vdash e_0 : P_0 \quad \dots \quad \Gamma \vdash e_n : P_n\{x_k := e_k\}}{\Gamma \vdash (e_0, \dots, e_n) : \Sigma(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). P_n} \text{ (}\Sigma_i\text{)} \\
\\
\frac{\Gamma \vdash e_1 : \Sigma(x_0 : P_0, \dots, x_{n-1} : P_{n-1}). P_n \quad \Gamma, x_0 : P_0, \dots, x_n : P_n \vdash e_2 : N\{z := (x_0, \dots, x_n)\}}{\Gamma \vdash \text{let } (x_0, \dots, x_n) := e_1 \text{ in } e_2 : N\{z := e_1\}} \text{ (}\Sigma_e\text{)} \\
\\
\frac{\Gamma \vdash P : \mathbb{U}}{\Gamma \vdash \uparrow P : \mathbb{U}} \text{ (}\uparrow\text{)} \quad \frac{\Gamma \vdash e : P}{\Gamma \vdash \text{return } e : \uparrow P} \text{ (}\uparrow_i\text{)} \quad \frac{\Gamma \vdash e_1 : \uparrow P \quad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \triangleright_x e_2 : N} \text{ (}\uparrow_e\text{)} \\
\\
\frac{\Gamma, x : \downarrow N \vdash e : N}{\Gamma \vdash \text{fix } x. e : N} \text{ (fix)} \quad \frac{}{\Gamma \vdash c : P_c} \text{ (constant)}
\end{array}$$