initial system:

$$\frac{}{\Vdash \emptyset}\ (\mathsf{empty}) \qquad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}}\ (\mathsf{univ}) \qquad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}}\ (\mathsf{hier})$$

$$\frac{\Gamma \vdash A : \mathbb{U}_i}{\Vdash \Gamma, x : A}\ (\mathsf{ext}) \qquad \frac{\Vdash \Gamma, x : A}{\Gamma, x : A \vdash x : A}\ (\mathsf{var})$$

$$\frac{\Gamma \vdash A : \mathbb{U}_i \qquad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Pi(x : A).\, B : \mathbb{U}_i}\ (\Pi)$$

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x.\, e : \Pi(x : A).\, B}\ (\Pi_{\mathsf{i}}) \qquad \frac{\Gamma \vdash e_1 : \Pi(x : A).\, B \qquad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1 \mathbin{@} e_2 : B[e_1/x]}\ (\Pi_{\mathsf{e}})$$

$$\frac{\Gamma \vdash A : \mathbb{U}_i \qquad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : A).\, B : \mathbb{U}_i}\ (\Sigma)$$

$$\frac{\Gamma \vdash e_1 : A_1 \qquad \dots \qquad \Gamma \vdash e_n : A_n[e_1/x_1] \dots [e_{n-1}/x_{n-1}]}{\Gamma \vdash (e_1, \dots, e_n) : \Sigma(x_1 : A_1, \dots, x_{n-1} : A_{n-1}).\, A_n}\ (\Sigma_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x_1 : A_1, \dots, x_{n-1} : A_{n-1}).\, A_n \qquad \Gamma, x_1 : A_1, \dots, x_n : A_n \vdash e_2 : B[(x_1, \dots, x_n)/z]}{\Gamma \vdash \mathsf{let}\ (x_1, \dots, x_n) = e_1\ \mathsf{in}\ e_2 : B[e_1/z]}\ (\Sigma_{\mathsf{e}})$$

$$\frac{\Gamma, x : A \vdash e : A}{\Gamma \vdash \mathsf{rec}\, x.\, e : A}\ (\mathsf{rec}) \qquad \frac{}{\Gamma \vdash c : A_c}\ (\mathsf{constant})$$

modal:

$$\frac{}{\Vdash \emptyset}\ (\mathsf{empty}) \qquad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}}\ (\mathsf{univ}) \qquad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}}\ (\mathsf{hier})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i}{\Vdash \Gamma, x : N}\ (\mathsf{ext}) \qquad \frac{\Vdash \Gamma, x : N}{\Gamma, x : N \vdash x : \downarrow N}\ (\mathsf{var})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i \qquad \Gamma, x : N \vdash M : \mathbb{U}_i}{\Gamma \vdash \Pi(x : N).\, M : \mathbb{U}_i}\ (\Pi)$$

$$\frac{\Gamma, x : N \vdash e : M}{\Gamma \vdash \mathsf{return}\ (\mathsf{thunk}\ (\lambda x.\, e)) : \uparrow\downarrow \Pi(x : N).\, M}\ (\Pi_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \uparrow\downarrow \Pi(x : N).\, M \qquad \Gamma \vdash e_2 : \downarrow N}{\Gamma \vdash e_1 \rhd_f (\mathsf{force}\, f)\, @\, e_2 : M[e_1/x]}\ (\Pi_{\mathsf{e}})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i \qquad \Gamma, x : N \vdash M : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : N).\, M : \mathbb{U}_i}\ (\Sigma)$$

$$\frac{\Gamma \vdash e_1 : \downarrow N_1 \qquad \dots \qquad \Gamma \vdash e_n : \downarrow N_n[e_1/x_1] \dots [e_{n-1}/x_{n-1}]}{\Gamma \vdash (e_1, \dots, e_n) : \Sigma(x_1 : N_1, \dots, x_{n-1} : N_{n-1}).\, N_n}\ (\Sigma_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x_1 : N_1, \dots, x_{n-1} : N_{n-1}).\, N_n \qquad \Gamma, x_1 : N_1, \dots, x_n : N_n \vdash e_2 : B[(x_1, \dots, x_n)/z]}{\Gamma \vdash \mathsf{let}\ (x_1, \dots, x_n) = e_1\ \mathsf{in}\ e_2 : B[e_1/z]}\ (\Sigma_{\mathsf{e}})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i}{\Gamma \vdash \uparrow P : \mathbb{U}_i}\ (\uparrow) \qquad \frac{\Gamma \vdash e : P}{\Gamma \vdash \mathsf{return}\, e : \uparrow P}\ (\uparrow_{\mathsf{i}}) \qquad \frac{\Gamma \vdash e_1 : \uparrow\downarrow N \qquad \Gamma, x : N \vdash e_2 : M}{\Gamma \vdash e_1 \rhd_x e_2 : M}\ (\uparrow_{\mathsf{e}})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i}{\Gamma \vdash \downarrow N : \mathbb{U}_i}\ (\downarrow) \qquad \frac{\Gamma \vdash e : N}{\Gamma \vdash \mathsf{thunk}\, e : \downarrow N}\ (\downarrow_{\mathsf{i}}) \qquad \frac{\Gamma \vdash e : \downarrow N}{\Gamma \vdash \mathsf{force}\, e : N}\ (\downarrow_{\mathsf{e}})$$

$$\frac{\Gamma, x : N \vdash e : \downarrow N}{\Gamma \vdash \mathsf{rec}\, x.\, e : \downarrow N}\ (\mathsf{rec}) \qquad \frac{}{\Gamma \vdash c : \downarrow N_c}\ (\mathsf{constant})$$

$$P ::= x \mid \Sigma(x_1 : N_1, \dots, x_n : N_n).\, M \mid \downarrow N$$
$$N ::= \Pi(x : N).\, M \mid \uparrow P$$

after closure conversion:

$$\frac{}{\Vdash \emptyset}\ (\mathsf{empty}) \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}}\ (\mathsf{univ}) \quad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}}\ (\mathsf{hier})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i}{\Vdash \Gamma, x : N}\ (\mathsf{ext}) \quad \frac{\Vdash \Gamma, x : N}{\Gamma, x : N \vdash x : \downarrow N}\ (\mathsf{var})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i \qquad \Gamma, x : N \vdash M : \mathbb{U}_i}{\Gamma \vdash \Pi(x : N).\, M : \mathbb{U}_i}\ (\Pi)$$

$$\frac{x : N \vdash e : M}{\Gamma \vdash \mathsf{return}\,(\mathsf{thunk}\,(\lambda x.\, e)) : \uparrow\downarrow \Pi(x : N).\, M}\ (\Pi_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \uparrow\downarrow \Pi(x : N).\, M \qquad \Gamma \vdash e_2 : \downarrow N}{\Gamma \vdash e_1 \rhd_f (\mathsf{force}\, f)\, @\, e_2 : M[e_1/x]}\ (\Pi_{\mathsf{e}})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i \qquad \Gamma, x : N \vdash M : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : N).\, M : \mathbb{U}_i}\ (\Sigma)$$

$$\frac{\Gamma \vdash e_1 : \downarrow N_1 \qquad \dots \qquad \Gamma \vdash e_n : \downarrow N_n[e_1/x_1] \dots [e_{n-1}/x_{n-1}]}{\Gamma \vdash (e_1, \dots, e_n) : \Sigma(x_1 : N_1, \dots, x_{n-1} : N_{n-1}).\, N_n}\ (\Sigma_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x_1 : N_1, \dots, x_{n-1} : N_{n-1}).\, N_n \qquad \Gamma, x_1 : N_1, \dots, x_n : N_n \vdash e_2 : B[(x_1, \dots, x_n)/z]}{\Gamma \vdash \mathsf{let}\ (x_1, \dots, x_n) = e_1\ \mathsf{in}\ e_2 : B[e_1/z]}\ (\Sigma_{\mathsf{e}})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i}{\Gamma \vdash \uparrow P : \mathbb{U}_i}\ (\uparrow) \quad \frac{\Gamma \vdash e : P}{\Gamma \vdash \mathsf{return}\, e : \uparrow P}\ (\uparrow_{\mathsf{i}}) \quad \frac{\Gamma \vdash e_1 : \uparrow\downarrow N \qquad \Gamma, x : N \vdash e_2 : M}{\Gamma \vdash e_1 \rhd_x e_2 : M}\ (\uparrow_{\mathsf{e}})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i}{\Gamma \vdash \downarrow N : \mathbb{U}_i}\ (\downarrow) \quad \frac{\vdash e : N}{\Gamma \vdash \mathsf{thunk}\, e : \downarrow N}\ (\downarrow_{\mathsf{i}}) \quad \frac{\Gamma \vdash e : \downarrow N}{\Gamma \vdash \mathsf{force}\, e : N}\ (\downarrow_{\mathsf{e}})$$

$$\frac{\Gamma, x : N \vdash e : \downarrow N}{\Gamma \vdash \mathsf{rec}\, x.\, e : \downarrow N}\ (\mathsf{rec}) \quad \frac{}{\Gamma \vdash c : \downarrow N_c}\ (\mathsf{constant})$$

$$P ::= x \mid \Sigma(x_1 : N_1, \dots, x_n : N_n).\, M \mid \downarrow N$$
$$N ::= \Pi(x : N).\, M \mid \uparrow P$$