initial system:

$$\frac{}{\Vdash \cdot}\ (\text{empty}) \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U} : \mathbb{U}}\ (\text{univ}) \quad \frac{\Gamma \vdash A : \mathbb{U}}{\Vdash \Gamma, x : A}\ (\text{ext}) \quad \frac{\Vdash \Gamma, x : A}{\Gamma, x : A \vdash x : A}\ (\text{var})$$

$$\frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{E}_k : \mathbb{U}}\ (\mathbb{E}) \quad \frac{\Vdash \Gamma \qquad c \in \mathbb{E}_k}{\Gamma \vdash c : \mathbb{E}_k}\ (\mathbb{E}_\mathsf{i}) \quad \frac{\Gamma \vdash e : \mathbb{E}_k \qquad (\Gamma, x : \mathbb{E}_k \vdash e_c : A)_{c \in \mathbb{E}_k}}{\Gamma \vdash \mathsf{let}\ x := e\ \mathsf{in}\ (c \Rightarrow e_c) : A\{x := e\}}\ (\mathbb{E}_\mathsf{e})$$

$$\frac{\Gamma \vdash A_0 : \mathbb{U} \qquad \cdots \qquad \Gamma, x_0 : A_0, \ldots, x_{n-1} : A_{n-1} \vdash A_n : \mathbb{U}}{\Gamma \vdash \Pi(x_0 : A_0, \ldots, x_{n-1} : A_{n-1}). A_n : \mathbb{U}}\ (\Pi)$$

$$\frac{\Gamma, x_0 : A_0, \ldots, x_{n-1} : A_{n-1} \vdash e : A_n}{\Gamma \vdash \lambda(x_0 : A_0, \ldots, x_{n-1} : A_{n-1}). e : \Pi(x_0 : A_0, \ldots, x_{n-1} : A_{n-1}). A_n}\ (\Pi_\mathsf{i})$$

$$\frac{\Gamma \vdash e : \Pi(x_0 : A_0, \ldots, x_{n-1} : A_{n-1}). A_n \qquad \Gamma \vdash e_0 : A_0 \qquad \cdots \qquad \Gamma \vdash e_{n-1} : A_{n-1}\{x_k := e_k\}}{\Gamma \vdash e @ (e_0, \ldots, e_{n-1}) : A_n\{x_k := e_k\}}\ (\Pi_\mathsf{e})$$

$$\frac{\Gamma, x : A \vdash e : A}{\Gamma \vdash \mathsf{fix}\, x.\, e : A}\ (\mathsf{fix}) \qquad \frac{}{\Gamma \vdash c : A_c}\ (\mathsf{constant})$$

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polarize:

$$\frac{}{\Vdash \cdot} \text{ (empty)} \qquad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U} : \mathbb{U}} \text{ (univ)} \qquad \frac{\Gamma \vdash P : \mathbb{U}}{\Vdash \Gamma, x : P} \text{ (ext)} \qquad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P} \text{ (var)}$$

$$\frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{E}_k : \mathbb{U}} \text{ ($\mathbb{E}$)} \qquad \frac{\Vdash \Gamma \qquad c \in \mathbb{E}_k}{\Gamma \vdash c : \mathbb{E}_k} \text{ ($\mathbb{E}_i$)} \qquad \frac{\Gamma \vdash e : \mathbb{E}_k \qquad (\Gamma, x : \mathbb{E}_k \vdash e_c : N)_{c \in \mathbb{E}_k}}{\Gamma \vdash \text{let } x := e \text{ in } (c \Rightarrow e_c) : N\{x := e\}} \text{ ($\mathbb{E}_e$)}$$

$$\frac{\Gamma \vdash P_0 : \mathbb{U} \qquad \cdots \qquad \Gamma, x_0 : P_0, \ldots, x_{n-1} : P_{n-1} \vdash N : \mathbb{U}}{\Gamma \vdash \Pi(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}). N : \mathbb{U}} \text{ (Π)}$$

$$\frac{\Gamma, x_0 : P_0, \ldots, x_{n-1} : P_{n-1} \vdash e : N}{\Gamma \vdash \lambda(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}). e : \Pi(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}). N} \text{ (Π_i)}$$

$$\frac{\Gamma \vdash e : \Pi(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}). N \qquad \Gamma \vdash e_0 : P_0 \qquad \cdots \qquad \Gamma \vdash e_{n-1} : P_{n-1}\{x_k := e_k\}}{\Gamma \vdash e @ (e_0, \ldots, e_{n-1}) : N\{x_k := e_k\}} \text{ (Π_e)}$$

$$\frac{\Gamma \vdash P_0 : \mathbb{U} \qquad \cdots \qquad \Gamma, x_0 : P_0, \ldots, x_{n-1} : P_{n-1} \vdash P_n : \mathbb{U}}{\Gamma \vdash \Sigma(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}). P_n : \mathbb{U}} \text{ (Σ)}$$

$$\frac{\Gamma \vdash e_0 : P_0 \qquad \ldots \qquad \Gamma \vdash e_n : P_n\{x_k := e_k\}}{\Gamma \vdash (e_0, \ldots, e_n) : \Sigma(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}). P_n} \text{ (Σ_i)}$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}). P_n \qquad \Gamma, x_0 : P_0, \ldots, x_n : P_n \vdash e_2 : N\{z := (x_0, \ldots, x_n)\}}{\Gamma \vdash \text{let } (x_0, \ldots, x_n) := e_1 \text{ in } e_2 : N\{z := e_1\}} \text{ (Σ_e)}$$

$$\frac{\Gamma \vdash P : \mathbb{U}}{\Gamma \vdash {\uparrow}P : \mathbb{U}} \text{ ($\uparrow$)} \qquad \frac{\Gamma \vdash e : P}{\Gamma \vdash \text{return } e : {\uparrow}P} \text{ ($\uparrow_i$)} \qquad \frac{\Gamma \vdash e_1 : {\uparrow}P \qquad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \rhd_x e_2 : N} \text{ ($\uparrow_e$)}$$

$$\frac{\Gamma \vdash N : \mathbb{U}}{\Gamma \vdash {\downarrow}N : \mathbb{U}} \text{ ($\downarrow$)} \qquad \frac{\Gamma \vdash e : N}{\Gamma \vdash \text{thunk } e : {\downarrow}N} \text{ ($\downarrow_i$)} \qquad \frac{\Gamma \vdash e : {\downarrow}N}{\Gamma \vdash \text{force } e : N} \text{ ($\downarrow_e$)}$$

$$\frac{\Gamma, x : {\downarrow}N \vdash e : N}{\Gamma \vdash \text{fix } x. e : N} \text{ (fix)} \qquad \frac{}{\Gamma \vdash c : P_c} \text{ (constant)}$$

after closure conversion:

$$\frac{}{\Vdash \cdot} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U} : \mathbb{U}} \text{ (univ)} \quad \frac{\Gamma \vdash P : \mathbb{U}}{\Vdash \Gamma, x : P} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P} \text{ (var)}$$

$$\frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{E}_k : \mathbb{U}} \text{ (}\mathbb{E}\text{)} \quad \frac{\Vdash \Gamma \quad c \in \mathbb{E}_k}{\Gamma \vdash c : \mathbb{E}_k} \text{ (}\mathbb{E}_i\text{)} \quad \frac{\Gamma \vdash e : \mathbb{E}_k \quad (\Gamma, x : \mathbb{E}_k \vdash e_c : N)_{c \in \mathbb{E}_k}}{\Gamma \vdash \mathsf{let}\ x := e\ \mathsf{in}\ (c \Rightarrow e_c) : N\{x := e\}} \text{ (}\mathbb{E}_e\text{)}$$

$$\frac{\Gamma \vdash P_0 : \mathbb{U} \quad \cdots \quad \Gamma, x_0 : P_0, \ldots, x_{n-1} : P_{n-1} \vdash N : \mathbb{U}}{\Gamma \vdash \downarrow\Pi(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}).\, N : \mathbb{U}} \text{ (}\Pi\text{)}$$

$$\frac{x_0 : P_0, \ldots, x_{n-1} : P_{n-1} \vdash e : N}{\Gamma \vdash \mathsf{thunk}\ (\lambda(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}).\, e) : \downarrow\Pi(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}).\, N} \text{ (}\Pi_i\text{)}$$

$$\frac{\Gamma \vdash e : \downarrow\Pi(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}).\, N \quad \Gamma \vdash e_0 : P_0 \quad \cdots \quad \Gamma \vdash e_{n-1} : P_{n-1}\{x_k := e_k\}}{\Gamma \vdash (\mathsf{force}\ e) @ (e_0, \ldots, e_{n-1}) : N\{x_k := e_k\}} \text{ (}\Pi_e\text{)}$$

$$\frac{\Gamma \vdash P_0 : \mathbb{U} \quad \cdots \quad \Gamma, x_0 : P_0, \ldots, x_{n-1} : P_{n-1} \vdash P_n : \mathbb{U}}{\Gamma \vdash \Sigma(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}).\, P_n : \mathbb{U}} \text{ (}\Sigma\text{)}$$

$$\frac{\Gamma \vdash e_0 : P_0 \quad \ldots \quad \Gamma \vdash e_n : P_n\{x_k := e_k\}}{\Gamma \vdash (e_0, \ldots, e_n) : \Sigma(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}).\, P_n} \text{ (}\Sigma_i\text{)}$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}).\, P_n \quad \Gamma, x_0 : P_0, \ldots, x_n : P_n \vdash e_2 : N\{z := (x_0, \ldots, x_n)\}}{\Gamma \vdash \mathsf{let}\ (x_0, \ldots, x_n) := e_1\ \mathsf{in}\ e_2 : N\{z := e_1\}} \text{ (}\Sigma_e\text{)}$$

$$\frac{\Gamma \vdash P : \mathbb{U}}{\Gamma \vdash \uparrow P : \mathbb{U}} \text{ (}\uparrow\text{)} \quad \frac{\Gamma \vdash e : P}{\Gamma \vdash \mathsf{return}\ e : \uparrow P} \text{ (}\uparrow_i\text{)} \quad \frac{\Gamma \vdash e_1 : \uparrow P \quad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \rhd_x e_2 : N} \text{ (}\uparrow_e\text{)}$$

$$\frac{\Gamma, x : \downarrow N \vdash e : N}{\Gamma \vdash \mathsf{fix}\ x.\, e : N} \text{ (fix)} \quad \frac{}{\Gamma \vdash c : P_c} \text{ (constant)}$$