initial system:

$$\frac{i \neq \infty}{\Vdash^i \cdot}\ (\mathsf{empty}) \qquad \frac{\Vdash^i \Gamma}{\Gamma \vdash^i \mathbb{U}_j : \mathbb{U}_\infty}\ (\mathsf{univ}) \qquad \frac{\Gamma \vdash^i A : \mathbb{U}_j}{\Vdash^i \Gamma, x : A}\ (\mathsf{ext}) \qquad \frac{\Vdash^i \Gamma, x : A}{\Gamma, x : A \vdash^i x : A}\ (\mathsf{var})$$

$$\frac{\Vdash^i \Gamma}{\Gamma \vdash^i \mathbb{E}_k : \mathbb{U}_\infty}\ (\mathbb{E}) \qquad \frac{\Vdash^i \Gamma \qquad c \in \mathbb{E}_k}{\Gamma \vdash^i c : \mathbb{E}_k}\ (\mathbb{E}_\mathsf{i}) \qquad \frac{\Gamma \vdash^i e : \mathbb{E}_k \qquad (\Gamma, x : \mathbb{E}_k \vdash^i e_c : A)_{c \in \mathbb{E}_k}}{\Gamma \vdash^i \mathsf{let}\ x := e\ \mathsf{in}\ (c \Rightarrow e_c) : A\{x := e\}}\ (\mathbb{E}_\mathsf{e})$$

$$\frac{\Gamma \vdash^i A_0 : \mathbb{U}_{j_0} \qquad \cdots \qquad \Gamma, x_0 : A_0, \ldots, x_{n-1} : A_{n-1} \vdash^i A_n : \mathbb{U}_{j_n} \qquad j \leq j_0 \sqcap \cdots \sqcap j_n}{\Gamma \vdash^i \Pi^j (x_0 : A_0, \ldots, x_{n-1} : A_{n-1}).\, A_n : \mathbb{U}_j}\ (\Pi)$$

$$\frac{\Gamma, x_0 : A_0, \ldots, x_{n-1} : A_{n-1} \vdash^i e : A_n}{\Gamma \vdash^i \lambda^j x.\, e : \Pi^j (x_0 : A_0, \ldots, x_{n-1} : A_{n-1}).\, A_n}\ (\Pi_\mathsf{i})$$

$$\frac{\Gamma \vdash^i e : \Pi^j (x_0 : A_0, \ldots, x_{n-1} : A_{n-1}).\, A_n \qquad \Gamma \vdash^i e_0 : A_0 \qquad \cdots \qquad \Gamma \vdash^i e_{n-1} : A_{n-1}\{x_k := e_k\}}{\Gamma \vdash^i e\,@^j\,(e_0, \ldots, e_{n-1}) : A_n\{x_k := e_k\}}\ (\Pi_\mathsf{e})$$

$$\frac{\Gamma \vdash^i A_0 : \mathbb{U}_{j_0} \qquad \cdots \qquad \Gamma, x_0 : A_0, \ldots, x_{n-1} : A_{n-1} \vdash^i A_n : \mathbb{U}_{j_n} \qquad j \leq j_0 \sqcap \cdots \sqcap j_n}{\Gamma \vdash^i \Sigma^j (x_0 : A_0, \ldots, x_{n-1} : A_{n-1}).\, A_n : \mathbb{U}_j}\ (\Sigma)$$

$$\frac{\Gamma \vdash^i e_0 : A_0 \qquad \ldots \qquad \Gamma \vdash^i e_n : A_n\{x_k := e_k\}}{\Gamma \vdash^i (e_1, \ldots, e_n)^j : \Sigma^j (x_1 : A_1, \ldots, x_{n-1} : A_{n-1}).\, A_n}\ (\Sigma_\mathsf{i})$$

$$\frac{\Gamma \vdash^i e_1 : \Sigma^j (x_1 : A_1, \ldots, x_{n-1} : A_{n-1}).\, A_n \qquad \Gamma, x_0 : A_0, \ldots, x_n : A_n \vdash^i e_2 : B\{z := (x_0, \ldots, x_n)^j\}}{\Gamma \vdash^i \mathsf{let}\ (x_0, \ldots, x_n)^j := e_1\ \mathsf{in}\ e_2 : B\{z := e_1\}}\ (\Sigma_\mathsf{e})$$

$$\frac{\Gamma \vdash^i A : \mathbb{U}_j}{\Gamma \vdash^i \Box A : \mathbb{U}_j}\ (\Box)$$

$$\frac{\Gamma \vdash^i e : A \qquad i \leq \lfloor \Gamma \rfloor \qquad \Vdash^{i-1} \Delta}{\Gamma, \Delta \vdash^{i-1} \mathsf{box}\, e : \Box A}\ (\Box_\mathsf{i}) \qquad \frac{\Gamma \vdash^{i-1} e : \Box A \qquad i \leq \lfloor \Gamma \rfloor \qquad \Vdash^i \Delta}{\Gamma, \Delta \vdash^i \mathsf{unbox}\, e : A}\ (\Box_\mathsf{e})$$

$$\frac{\Gamma \vdash^i A : \mathbb{U}_j}{\Gamma \vdash^i !A : \mathbb{U}_\infty}\ (!) \qquad \frac{!\Gamma \vdash^i e : A \qquad \Vdash^i \Delta}{!\Gamma, \Delta \vdash^{i-1} \mathsf{lift}\, e : !A^{-i}}\ (!_\mathsf{i}) \qquad \frac{!\Gamma \vdash^{i-1} e : !A \qquad \Vdash^i \Delta}{!\Gamma, \Delta \vdash^i \mathsf{unlift}\, e : A^{+i}}\ (!_\mathsf{e})$$

$$\frac{\Gamma, x : A \vdash e : A}{\Gamma \vdash \mathsf{fix}\, x.\, e : A}\ (\mathsf{fix}) \qquad \frac{}{\Gamma \vdash c : A_c}\ (\mathsf{constant})$$

after closure conversion:

$$\frac{}{\Vdash \emptyset}\ (\mathsf{empty}) \qquad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}}\ (\mathsf{univ}) \qquad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}}\ (\mathsf{hier})$$

$$\frac{\Gamma \vdash A : \mathbb{U}_i}{\Vdash \Gamma, x : A}\ (\mathsf{ext}) \qquad \frac{\Vdash \Gamma, x : A}{\Gamma, x : A \vdash x : A}\ (\mathsf{var})$$

$$\frac{\Gamma \vdash A : \mathbb{U}_i \qquad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Pi(x : A).\, B : \mathbb{U}_i}\ (\Pi)$$

$$\frac{x_1 : A_1, \ldots, x_n : A_n \vdash e : B}{\Gamma \vdash \lambda(x_1, \ldots, x_n).\, e : \Pi(x_1 : A_1, \ldots, x_n : A_n).\, B}\ (\Pi_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e : \Pi(x_1 : A_1, \ldots, x_n : A_n).\, B \qquad \Gamma \vdash e_1 : A_1 \quad \ldots \quad \Gamma \vdash e_n : A_n}{\Gamma \vdash e \,@\, (e_1, \ldots, e_n) : B\{x_i := e_i\}}\ (\Pi_{\mathsf{e}})$$

$$\frac{\Gamma \vdash A : \mathbb{U}_i \qquad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : A).\, B : \mathbb{U}_i}\ (\Sigma)$$

$$\frac{\Gamma \vdash e_1 : A_1 \qquad \ldots \qquad \Gamma \vdash e_n : A_n\{x_1 := e_1\} \ldots \{x_{n-1} := e_{n-1}\}}{\Gamma \vdash (e_1, \ldots, e_n) : \Sigma(x_1 : A_1, \ldots, x_{n-1} : A_{n-1}).\, A_n}\ (\Sigma_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x_1 : A_1, \ldots, x_{n-1} : A_{n-1}).\, A_n \qquad \Gamma, x_1 : A_1, \ldots, x_n : A_n \vdash e_2 : B\{z := (x_1, \ldots, x_n)\}}{\Gamma \vdash \mathsf{let}\ (x_1, \ldots, x_n) := e_1\ \mathsf{in}\ e_2 : B\{z := e_1\}}\ (\Sigma_{\mathsf{e}})$$

$$\frac{x : A \vdash e : A}{\Gamma \vdash \mathsf{rec}\, x.\, e : A}\ (\mathsf{rec}) \qquad \frac{}{\Gamma \vdash c : A_c}\ (\mathsf{constant})$$

polarize:

$$\frac{}{\Vdash \varnothing}\ (\mathsf{empty}) \qquad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}}\ (\mathsf{univ}) \qquad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}}\ (\mathsf{hier})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i}{\Vdash \Gamma, x : P}\ (\mathsf{ext}) \qquad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P}\ (\mathsf{var})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i \qquad \Gamma, x : P \vdash N : \mathbb{U}_i}{\Gamma \vdash \Pi(x : P).\, N : \mathbb{U}_i}\ (\Pi)$$

$$\frac{\Gamma, x : P \vdash e : N}{\Gamma \vdash \lambda x.\, e : \Pi(x : P).\, N}\ (\Pi_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \Pi(x : P).\, N \qquad \Gamma \vdash e_2 : P}{\Gamma \vdash e_1 @ e_2 : N\{x := e_1\}}\ (\Pi_{\mathsf{e}})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i \qquad \Gamma, x : P \vdash Q : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : P).\, Q : \mathbb{U}_i}\ (\Sigma)$$

$$\frac{\Gamma \vdash e_1 : P_1 \qquad \ldots \qquad \Gamma \vdash e_n : P_n\{x_1 := e_1\}\ldots\{x_{n-1} := e_{n-1}\}}{\Gamma \vdash (e_1, \ldots, e_n) : \Sigma(x_1 : P_1, \ldots, x_{n-1} : P_{n-1}).\, P_n}\ (\Sigma_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x_1 : P_1, \ldots, x_{n-1} : P_{n-1}).\, P_n \qquad \Gamma, x_1 : P_1, \ldots, x_n : P_n \vdash e_2 : N\{z := (x_1, \ldots, x_n)\}}{\Gamma \vdash \mathsf{let}\ (x_1, \ldots, x_n) := e_1\ \mathsf{in}\ e_2 : N\{z := e_1\}}\ (\Sigma_{\mathsf{e}})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i}{\Gamma \vdash {\uparrow}P : \mathbb{U}_i}\ (\uparrow) \qquad \frac{\Gamma \vdash e : P}{\Gamma \vdash \mathsf{return}\, e : {\uparrow}P}\ (\uparrow_{\mathsf{i}}) \qquad \frac{\Gamma \vdash e_1 : {\uparrow}P \qquad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \rhd_x e_2 : N}\ (\uparrow_{\mathsf{e}})$$

$$\frac{\Gamma \vdash N : \mathbb{U}_i}{\Gamma \vdash {\downarrow}N : \mathbb{U}_i}\ (\downarrow) \qquad \frac{\Gamma \vdash e : N}{\Gamma \vdash \mathsf{thunk}\, e : {\downarrow}N}\ (\downarrow_{\mathsf{i}}) \qquad \frac{\Gamma \vdash e : {\downarrow}N}{\Gamma \vdash \mathsf{force}\, e : N}\ (\downarrow_{\mathsf{e}})$$

$$\frac{\Gamma, x : {\downarrow}N \vdash e : N}{\Gamma \vdash \mathsf{rec}\, x.\, e : N}\ (\mathsf{rec}) \qquad \frac{}{\Gamma \vdash c : P_c}\ (\mathsf{constant})$$

after closure conversion:

$$\frac{}{\Vdash \emptyset}\ (\mathsf{empty}) \qquad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}}\ (\mathsf{univ}) \qquad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}}\ (\mathsf{hier})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i}{\Vdash \Gamma, x : P}\ (\mathsf{ext}) \qquad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P}\ (\mathsf{var})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i \qquad \Gamma, x : P \vdash N : \mathbb{U}_i}{\Gamma \vdash \Pi(x : P).\, N : \mathbb{U}_i}\ (\Pi)$$

$$\frac{x_1 : P_1, \ldots, x_n : P_n \vdash e : N}{\Gamma \vdash \mathsf{thunk}\,(\lambda(x_1, \ldots, x_n).\, e) : {\downarrow}\Pi(x_1 : P_1, \ldots, x_n : P_n).\, N}\ (\Pi_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e : {\downarrow}\Pi(x_1 : P_1, \ldots, x_n : P_n).\, N \qquad \Gamma \vdash e_1 : P_1 \quad \ldots \quad \Gamma \vdash e_n : P_n}{\Gamma \vdash (\mathsf{force}\, e_1)\ @\ (e_1, \ldots, e_n) : N\{x_i := e_i\}}\ (\Pi_{\mathsf{e}})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i \qquad \Gamma, x : P \vdash Q : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : P).\, Q : \mathbb{U}_i}\ (\Sigma)$$

$$\frac{\Gamma \vdash e_1 : P_1 \qquad \ldots \qquad \Gamma \vdash e_n : P_n\{x_1 := e_1\} \ldots \{x_{n-1} := e_{n-1}\}}{\Gamma \vdash \mathsf{return}\,(e_1, \ldots, e_n) : \Sigma(x_1 : P_1, \ldots, x_{n-1} : P_{n-1}).\, P_n}\ (\Sigma_{\mathsf{i}})$$

$$\frac{\Gamma \vdash e : \Sigma(x_1 : P_1, \ldots, x_{n-1} : P_{n-1}).\, P_n \qquad \Gamma, x_1 : P_1, \ldots, x_n : P_n \vdash e' : N\{z := (x_1, \ldots, x_n)\}}{\Gamma \vdash \mathsf{let}\ (x_1, \ldots, x_n) := e\ \mathsf{in}\ e' : N\{z := e\}}\ (\Sigma_{\mathsf{e}})$$

$$\frac{\Gamma \vdash P : \mathbb{U}_i}{\Gamma \vdash {\uparrow}P : \mathbb{U}_i}\ ({\uparrow}) \qquad \frac{\Gamma \vdash e : P}{\Gamma \vdash \mathsf{return}\, e : {\uparrow}P}\ ({\uparrow}_{\mathsf{i}}) \qquad \frac{\Gamma \vdash e_1 : {\uparrow}P \qquad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \rhd_x e_2 : N}\ ({\uparrow}_{\mathsf{e}})$$

$$\frac{x : {\downarrow}N \vdash e : N}{\Gamma \vdash \mathsf{rec}\, x.\, e : N}\ (\mathsf{rec}) \qquad \frac{}{\Gamma \vdash c : {\downarrow}N_c}\ (\mathsf{constant})$$