

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Box \Gamma}{\Box \Gamma \vdash \mathbb{L}_i : \Box \mathbb{L}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \Box \mathbb{L}_i}{\Gamma \vdash A : \Box \mathbb{L}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash A : \Box \mathbb{L}_i}{\Vdash \Gamma, x : A} \text{ (ext)} \quad \frac{\Vdash \Box \Gamma, x : A}{\Box \Gamma, x : A \vdash x : A} \text{ (var)} \\
\\
\frac{\Gamma \vdash A : \Box \mathbb{L}_i \quad \Delta, x : A \vdash B : \Box \mathbb{L}_i}{\Gamma, \Delta \vdash \Pi(x : A). B : \Box \mathbb{L}_i} \text{ (\Pi)} \\
\\
\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : \Pi(x : A). B} \text{ (\Pi}_i\text{)} \quad \frac{\Gamma \vdash e_1 : \Pi(x : A). B \quad \Delta \vdash e_2 : A}{\Gamma, \Delta \vdash e_1 @ e_2 : B[e_1/x]} \text{ (\Pi}_e\text{)} \\
\\
\frac{\Gamma \vdash A : \Box \mathbb{L}_i \quad \Delta, x : A \vdash B : \Box \mathbb{L}_i}{\Gamma, \Delta \vdash \Sigma(x : A). B : \Box \mathbb{L}_i} \text{ (\Sigma)} \\
\\
\frac{\Gamma \vdash e_1 : A \quad \Delta \vdash e_2 : B[e_1/x]}{\Gamma, \Delta \vdash (e_1, e_2) : \Sigma(x : A). B} \text{ (\Sigma}_i\text{)} \quad \frac{\Gamma \vdash e_1 : \Sigma(x : A). B \quad \Delta, x : A, y : B \vdash e_2 : C[(x, y)/z]}{\Gamma, \Delta \vdash \text{let } (x, y) = e_1 \text{ in } e_2 : C[e_1/z]} \text{ (\Sigma}_e\text{)} \\
\\
\frac{\Gamma \vdash A : \Box \mathbb{L}_i}{\Gamma \vdash \Box A : \Box \mathbb{L}_i} \text{ (\Box)} \\
\\
\frac{\Box \Gamma \vdash e : A}{\Box \Gamma \vdash \text{box } e : \Box A} \text{ (\Box}_i\text{)} \quad \frac{\Box \Gamma \vdash e : \Box A}{\Box \Gamma \vdash \text{unbox } e : A} \text{ (\Box}_e\text{)} \\
\\
\frac{\Box \Gamma, x : \Box A \vdash e : \Box A}{\Box \Gamma \vdash \text{rec } x. e : \Box A} \text{ (rec)} \\
\\
\frac{\Gamma \vdash e : B}{\Gamma, x : \Box A \vdash e : B} \text{ (wkg)} \quad \frac{\Gamma, x : \Box A, y : \Box A \vdash e : B}{\Gamma, x : \Box A \vdash e[x/y] : B} \text{ (ctr)}
\end{array}$$

A type A is relevant if $\vdash e : A \rightarrow A \otimes A$ is derivable for some e .

A type A is affine if $\vdash e : A \rightarrow \top$ is derivable for some e .

A term e is a well-typed closed proper term if (1) $\vdash e : A$ is derivable for some type A , and (2) for any variable x in e , if it occurs more than once, then the type of x is relevant. If it doesn't occur, then the type of x is affine. (3) All the free variables in a recursion-term ($\text{rec } x. e$) must be relevant.