

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Box \Gamma}{\Box \Gamma \vdash \mathbb{L}_i : \Box \mathbb{L}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \Box \mathbb{L}_i}{\Gamma \vdash A : \Box \mathbb{L}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash A : \Box \mathbb{L}_i}{\Vdash \Gamma, x : A} \text{ (ext)} \quad \frac{\Vdash \Box \Gamma, x : A}{\Box \Gamma, x : A \vdash x : A} \text{ (var)} \\
\\
\frac{\Gamma \vdash A : \Box \mathbb{L}_i \quad \Delta, x : A \vdash B : \Box \mathbb{L}_i}{\Gamma, \Delta \vdash \Pi(x : A). B : \Box \mathbb{L}_i} (\Pi) \\
\\
\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : \Pi(x : A). B} (\Pi_i) \quad \frac{\Gamma \vdash e_1 : \Pi(x : A). B \quad \Delta \vdash e_2 : A}{\Gamma, \Delta \vdash e_1 @ e_2 : B[e_1/x]} (\Pi_e) \\
\\
\frac{\Gamma \vdash A : \Box \mathbb{L}_i \quad \Delta, x : A \vdash B : \Box \mathbb{L}_i}{\Gamma, \Delta \vdash \Sigma(x : A). B : \Box \mathbb{L}_i} (\Sigma) \\
\\
\frac{\Gamma \vdash e_1 : A \quad \Delta \vdash e_2 : B[e_1/x]}{\Gamma, \Delta \vdash (e_1, e_2) : \Sigma(x : A). B} (\Sigma_i) \quad \frac{\Gamma \vdash e_1 : \Sigma(x : A). B \quad \Delta, x : A, y : B \vdash e_2 : C[(x, y)/z]}{\Gamma, \Delta \vdash \text{let } (x, y) = e_1 \text{ in } e_2 : C[e_1/z]} (\Sigma_e) \\
\\
\frac{\Gamma \vdash A : \Box \mathbb{L}_i}{\Gamma \vdash \Box A : \Box \mathbb{L}_i} (\Box) \\
\\
\frac{\Box \Gamma \vdash e : A}{\Box \Gamma \vdash \text{box } e : \Box A} (\Box_i) \quad \frac{\Box \Gamma \vdash e : \Box A}{\Box \Gamma \vdash \text{unbox } e : A} (\Box_e) \\
\\
\frac{\Gamma \vdash A : \Box \mathbb{L}_i}{\Gamma \vdash [A; n] : \Box \mathbb{L}_i} \text{ (array)} \\
\\
\frac{\Gamma_1 \vdash e_1 : A \quad \dots \quad \Gamma_n \vdash e_n : A}{\Gamma_1, \dots, \Gamma_n \vdash [e_1, \dots, e_n] : [A; n]} \text{ (array}_i\text{)} \\
\\
\frac{\Gamma \vdash e_1 : [A; n] \quad \Delta, x_1 : A, \dots, x_n : A \vdash e_2 : B}{\Gamma, \Delta \vdash \text{let } [x_1, \dots, x_n] = e_1 \text{ in } e_2 : B} \text{ (array}_e\text{)} \\
\\
\frac{\Box \Gamma, x : \Box A \vdash e : \Box A}{\Box \Gamma \vdash \text{rec } x. e : \Box A} \text{ (rec)} \\
\\
\frac{\Gamma \vdash e : B}{\Gamma, x : \Box A \vdash e : B} \text{ (wkg)} \quad \frac{\Gamma, x : \Box A, y : \Box A \vdash e : B}{\Gamma, x : \Box A \vdash e[x/y] : B} \text{ (ctr)}
\end{array}$$

A type  $A$  is relevant if  $\vdash e : A \rightarrow A \otimes A$  is derivable for some  $e$ .

A type  $A$  is affine if  $\vdash e : A \rightarrow \top$  is derivable for some  $e$ .

A term  $e$  is a well-typed closed proper term if (1)  $\vdash e : A$  is derivable for some type  $A$ , and (2) for any variable  $x$  in  $e$ , if it occurs more than once, then the type of  $x$  is relevant. If it doesn't occur, then the type of  $x$  is affine. (3) All the free variables in a recursion-term ( $\text{rec } x. e$ ) must be relevant.