

$$\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Box \Gamma}{\Box \Gamma \vdash \Box \mathbb{L}_i : \Box \mathbb{L}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \Box \mathbb{L}_i}{\Gamma \vdash A : \Box \mathbb{L}_{i+1}} \text{ (hier)}$$

$$\frac{\Gamma \vdash A : \Box \mathbb{L}_i}{\Vdash \Gamma, x : A} \text{ (ext)} \quad \frac{\Vdash \Box \Gamma, x : A}{\Box \Gamma, x : A \vdash x : A} \text{ (var)}$$

$$\frac{\Gamma \vdash A : \Box \mathbb{L}_i \quad \Delta, x : A \vdash B : \Box \mathbb{L}_i}{\Gamma, \Delta \vdash \Pi(x : A). B : \Box \mathbb{L}_i} \text{ (\Pi)}$$

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : \Pi(x : A). B} \text{ (\Pi}_i\text{)} \quad \frac{\Gamma \vdash e_1 : \Pi(x : A). B \quad \Delta \vdash e_2 : A}{\Gamma, \Delta \vdash e_1 @ e_2 : B\{x := e_1\}} \text{ (\Pi}_e\text{)}$$

$$\frac{\Gamma \vdash A : \Box \mathbb{L}_i \quad \Delta \vdash B : \Box \mathbb{L}_i}{\Gamma, \Delta \vdash A \multimap B : \Box \mathbb{L}_i} \text{ (\multimap)}$$

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : A \multimap B} \text{ (\multimap}_i\text{)} \quad \frac{\Gamma \vdash e_1 : A \multimap B \quad \Delta \vdash e_2 : A}{\Gamma, \Delta \vdash e_1 @ e_2 : B} \text{ (\multimap}_e\text{)}$$

$$\frac{\Gamma \vdash A : \Box \mathbb{L}_i \quad \Delta, x : A \vdash B : \Box \mathbb{L}_i}{\Gamma, \Delta \vdash \Sigma(x : A). B : \Box \mathbb{L}_i} \text{ (\Sigma)}$$

$$\frac{\Gamma \vdash e_1 : A \quad \Delta \vdash e_2 : B\{x := e_1\}}{\Gamma, \Delta \vdash (e_1, e_2) : \Sigma(x : A). B} \text{ (\Sigma}_i\text{)}$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x : A). B \quad \Delta, x : A, y : B \vdash e_2 : C\{z := (x, y)\}}{\Gamma, \Delta \vdash \text{let } (x, y) = e_1 \text{ in } e_2 : C\{z := e_1\}} \text{ (\Sigma}_e\text{)}$$

$$\frac{\Gamma \vdash A : \Box \mathbb{L}_i \quad \Delta \vdash B : \Box \mathbb{L}_i}{\Gamma, \Delta \vdash A \otimes B : \Box \mathbb{L}_i} \text{ (\otimes)}$$

$$\frac{\Gamma \vdash e_1 : A \quad \Delta \vdash e_2 : B}{\Gamma, \Delta \vdash (e_1, e_2) : A \otimes B} \text{ (\otimes}_i\text{)} \quad \frac{\Gamma \vdash e_1 : A \otimes B \quad \Delta, x : A, y : B \vdash e_2 : C}{\Gamma, \Delta \vdash \text{let } (x, y) = e_1 \text{ in } e_2 : C} \text{ (\otimes}_e\text{)}$$

$$\frac{\Box \Gamma, x : \Box A \vdash e : A}{\Box \Gamma \vdash \text{rec } x. e : A} \text{ (rec)}$$

$$\frac{\Gamma \vdash A : \Box \mathbb{L}_i}{\Gamma \vdash \Box A : \Box \mathbb{L}_i} \text{ (\Box)} \quad \frac{\Box \Gamma \vdash e : A}{\Box \Gamma \vdash \text{box } e : \Box A} \text{ (\Box}_i\text{)} \quad \frac{\Box \Gamma \vdash e : \Box A}{\Box \Gamma \vdash \text{unbox } e : A} \text{ (\Box}_e\text{)}$$

$$\frac{\Gamma \vdash e : A}{\Gamma, x : \Box B \vdash e : A} \text{ (wkg)} \quad \frac{\Gamma, x : \Box B, y : \Box B \vdash e : A}{\Gamma, x : \Box B \vdash e\{y := x\} : A} \text{ (ctr)}$$