$$\frac{ \Vdash \Box \Gamma}{\Box \Gamma \vdash \bot \Box \bot} (\mathsf{univ}) \quad \frac{\Gamma \vdash A : \Box \bot}{\Gamma \vdash A : \Box \bot} (\mathsf{hier})$$

$$\frac{\Gamma \vdash A : \Box \bot}{\Box \vdash \Gamma, x : A} (\mathsf{ext}) \quad \frac{\Box \vdash \Box \Gamma, x : A}{\Box \Gamma, x : A \vdash x : A} (\mathsf{var})$$

$$\frac{\Gamma \vdash A : \Box \bot}{\Box \vdash \Gamma, x : A} (\mathsf{ext}) \quad \frac{\Box \vdash \Box \Gamma, x : A}{\Box \Gamma, x : A \vdash x : A} (\mathsf{var})$$

$$\frac{\Gamma \vdash A : \Box \bot}{\Gamma, \Delta \vdash \Pi(x : A), B : \Box \bot} (\mathsf{II})$$

$$\frac{\Gamma \vdash A : \Box \bot}{\Gamma, \Delta \vdash \Pi(x : A), B} \quad \frac{\Delta \vdash e_2 : A}{\Gamma, \Delta \vdash e_1 \otimes e_2 : B[e_1/x]} (\mathsf{II})$$

$$\frac{\Gamma \vdash A : \Box \bot}{\Gamma, \Delta \vdash \Sigma(x : A), B} \quad \frac{\Delta, x : A \vdash B : \Box \bot}{\Gamma, \Delta \vdash \exists} (\Sigma)$$

$$\frac{\Gamma \vdash e_1 : A \quad \Delta \vdash e_2 : B[e_1/x]}{\Gamma, \Delta \vdash (e_1, e_2) : \Sigma(x : A), B} (\Sigma_i) \quad \frac{\Gamma \vdash e_1 : \Sigma(x : A), B \quad \Delta, x : A, y : B \vdash e_2 : C[(x, y)/z]}{\Gamma, \Delta \vdash \exists} (\Sigma_i)$$

$$\frac{\Gamma \vdash A : \Box \bot}{\Gamma, \Delta \vdash (e_1, e_2) : \Sigma(x : A), B} (\Sigma_i) \quad \frac{\Gamma \vdash e_1 : \Delta \cup \bot}{\Gamma, \Delta \vdash \exists} (\Box)$$

$$\frac{\Box \Gamma \vdash e : A}{\Box \Gamma \vdash \exists} (\Box) \quad \frac{\Box \Gamma \vdash e : \Box A}{\Box \Gamma \vdash \exists} (\Box)$$

$$\frac{\Box \Gamma \vdash e : A}{\Box \Gamma \vdash \exists} (\Box) \quad \frac{\Box \Gamma \vdash e : \Box A}{\Box \Gamma \vdash \exists} (\exists x : n] (\exists x : n]$$

$$\frac{\Gamma \vdash A : \Box \bot}{\Gamma, \bot} (\exists x : n] : [A, x : n] : [A, x : n] (\exists x : n]$$

$$\frac{\Gamma \vdash e_1 : A : \Box \bot}{\Gamma, \Delta \vdash \exists} (\exists x : n] (\exists x : n] (\exists x : n]$$

$$\frac{\Gamma \vdash e_1 : A : \Box \bot}{\Gamma, \Delta \vdash \exists} (\exists x : n] (\exists x : n] (\exists x : n] (\exists x : n]$$

$$\frac{\Gamma \vdash e_1 : A : \Box \bot}{\Gamma, \Delta \vdash \exists} (\exists x : n] (\exists$$