initial system:

$$\frac{}{\; \Vdash \cdot \;} \text{ (empty)} \qquad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U} : \mathbb{U}} \text{ (univ)} \qquad \frac{\Gamma \vdash A : \mathbb{U}}{\Vdash \Gamma, x : A} \text{ (ext)} \qquad \frac{\Vdash \Gamma, x : A}{\Gamma, x : A \vdash x : A} \text{ (var)}$$

$$\frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{E}_k : \mathbb{U}} \; (\mathbb{E}) \qquad \frac{\Vdash \Gamma \qquad c \in \mathbb{E}_k}{\Gamma \vdash c : \mathbb{E}_k} \; (\mathbb{E}_i) \qquad \frac{\Gamma \vdash e : \mathbb{E}_k \qquad (\Gamma, x : \mathbb{E}_k \vdash e_c : A)_{c \in \mathbb{E}_k}}{\Gamma \vdash \mathsf{let}\ x := e\ \mathsf{in}\ (c \Rightarrow e_c) : A\{x := e\}} \; (\mathbb{E}_e)$$

$$\frac{\Gamma \vdash A_0 : \mathbb{U} \qquad \cdots \qquad \Gamma, x_0 : A_0, \ldots, x_{n-1} : A_{n-1} \vdash A_n : \mathbb{U}}{\Gamma \vdash \Pi(x_0 : A_0, \ldots, x_{n-1} : A_{n-1}).\, A_n : \mathbb{U}} \; (\Pi)$$

$$\frac{\Gamma, x_0 : A_0, \ldots, x_{n-1} : A_{n-1} \vdash e : A_n}{\Gamma \vdash \lambda x.\, e : \Pi(x_0 : A_0, \ldots, x_{n-1} : A_{n-1}).\, A_n} \; (\Pi_i)$$

$$\frac{\Gamma \vdash e : \Pi(x_0 : A_0, \ldots, x_{n-1} : A_{n-1}).\, A_n \qquad \Gamma \vdash e_0 : A_0 \qquad \cdots \qquad \Gamma \vdash e_{n-1} : A_{n-1}\{x_k := e_k\}}{\Gamma \vdash e \, @ \, (e_0, \ldots, e_{n-1}) : A_n\{x_k := e_k\}} \; (\Pi_e)$$

$$\frac{\Gamma \vdash A_0 : \mathbb{U} \qquad \cdots \qquad \Gamma, x_0 : A_0, \ldots, x_{n-1} : A_{n-1} \vdash A_n : \mathbb{U}}{\Gamma \vdash \Sigma(x_0 : A_0, \ldots, x_{n-1} : A_{n-1}).\, A_n : \mathbb{U}} \; (\Sigma)$$

$$\frac{\Gamma \vdash e_0 : A_0 \qquad \ldots \qquad \Gamma \vdash e_n : A_n\{x_k := e_k\}}{\Gamma \vdash (e_1, \ldots, e_n) : \Sigma(x_1 : A_0, \ldots, x_{n-1} : A_{n-1}).\, A_n} \; (\Sigma_i)$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x_1 : A_0, \ldots, x_{n-1} : A_{n-1}).\, A_n \qquad \Gamma, x_0 : A_0, \ldots, x_n : A_n \vdash e_2 : B\{z := (x_0, \ldots, x_n)\}}{\Gamma \vdash \mathsf{let}\ (x_0, \ldots, x_n) := e_1\ \mathsf{in}\ e_2 : B\{z := e_1\}} \; (\Sigma_e)$$

$$\frac{\Gamma, x : A \vdash e : A}{\Gamma \vdash \mathsf{fix}\, x.\, e : A} \; (\mathsf{fix}) \qquad \frac{}{\Gamma \vdash c : A_c} \; (\mathsf{constant})$$

polarize:

$$\frac{}{\Vdash \varnothing} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U} : \mathbb{U}} \text{ (univ)} \quad \frac{\Gamma \vdash P : \mathbb{U}}{\Vdash \Gamma, x : P} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P} \text{ (var)}$$

$$\frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{E}_k : \mathbb{U}} \text{ (}\mathbb{E}\text{)} \quad \frac{\Vdash \Gamma \quad c \in \mathbb{E}_k}{\Gamma \vdash c : \mathbb{E}_k} \text{ (}\mathbb{E}_i\text{)} \quad \frac{\Gamma \vdash e : \mathbb{E}_k \quad (\Gamma, x : \mathbb{E}_k \vdash e_c : N)_{c \in \mathbb{E}_k}}{\Gamma \vdash \mathsf{let}\ x := e\ \mathsf{in}\ (c \Rightarrow e_c) : N\{x := e\}} \text{ (}\mathbb{E}_e\text{)}$$

$$\frac{\Gamma \vdash P_0 : \mathbb{U} \quad \cdots \quad \Gamma, x_0 : P_0, \ldots, x_{n-1} : P_{n-1} \vdash N : \mathbb{U}}{\Gamma \vdash \Pi(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}).N : \mathbb{U}} \text{ (}\Pi\text{)}$$

$$\frac{\Gamma, x_0 : P_0, \ldots, x_{n-1} : P_{n-1} \vdash e : N}{\Gamma \vdash \lambda x.e : \Pi(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}).N} \text{ (}\Pi_i\text{)}$$

$$\frac{\Gamma \vdash e : \Pi(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}).N \quad \Gamma \vdash e_0 : P_0 \quad \cdots \quad \Gamma \vdash e_{n-1} : P_{n-1}\{x_k := e_k\}}{\Gamma \vdash e @ (e_0, \ldots, e_{n-1}) : N\{x_k := e_k\}} \text{ (}\Pi_e\text{)}$$

$$\frac{\Gamma \vdash P_0 : \mathbb{U} \quad \cdots \quad \Gamma, x_0 : P_0, \ldots, x_{n-1} : P_{n-1} \vdash P_n : \mathbb{U}}{\Gamma \vdash \Sigma(x_0 : P_0, \ldots, x_{n-1} : P_{n-1}).P_n : \mathbb{U}} \text{ (}\Sigma\text{)}$$

$$\frac{\Gamma \vdash e_0 : P_0 \quad \ldots \quad \Gamma \vdash e_n : P_n\{x_k := e_k\}}{\Gamma \vdash (e_1, \ldots, e_n) : \Sigma(x_1 : P_0, \ldots, x_{n-1} : P_{n-1}).P_n} \text{ (}\Sigma_i\text{)}$$

$$\frac{\Gamma \vdash e_1 : \Sigma(x_1 : P_0, \ldots, x_{n-1} : P_{n-1}).P_n \quad \Gamma, x_0 : P_0, \ldots, x_n : P_n \vdash e_2 : N\{z := (x_0, \ldots, x_n)\}}{\Gamma \vdash \mathsf{let}\ (x_0, \ldots, x_n) := e_1\ \mathsf{in}\ e_2 : N\{z := e_1\}} \text{ (}\Sigma_e\text{)}$$

$$\frac{\Gamma \vdash P : \mathbb{U}}{\Gamma \vdash \uparrow P : \mathbb{U}} \text{ (}\uparrow\text{)} \quad \frac{\Gamma \vdash e : P}{\Gamma \vdash \mathsf{return}\ e : \uparrow P} \text{ (}\uparrow_i\text{)} \quad \frac{\Gamma \vdash e_1 : \uparrow P \quad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \rhd_x e_2 : N} \text{ (}\uparrow_e\text{)}$$

$$\frac{\Gamma \vdash N : \mathbb{U}}{\Gamma \vdash \downarrow N : \mathbb{U}} \text{ (}\downarrow\text{)} \quad \frac{\Gamma \vdash e : N}{\Gamma \vdash \mathsf{thunk}\ e : \downarrow N} \text{ (}\downarrow_i\text{)} \quad \frac{\Gamma \vdash e : \downarrow N}{\Gamma \vdash \mathsf{force}\ e : N} \text{ (}\downarrow_e\text{)}$$

$$\frac{\Gamma, x : \downarrow N \vdash e : N}{\Gamma \vdash \mathsf{fix}\ x.e : N} \text{ (fix)} \quad \frac{}{\Gamma \vdash c : P_c} \text{ (constant)}$$

after closure conversion:

$$\frac{}{\Vdash \varnothing} \; (\text{empty}) \qquad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U} : \mathbb{U}} \; (\text{univ}) \qquad \frac{\Gamma \vdash P : \mathbb{U}}{\Vdash \Gamma, x : P} \; (\text{ext}) \qquad \frac{\Vdash \Gamma, x : P}{\Gamma, x : P \vdash x : P} \; (\text{var})$$

$$\frac{\Gamma \vdash P : \mathbb{U} \qquad \Gamma, x : P \vdash N : \mathbb{U}}{\Gamma \vdash \Pi(x : P).\,N : \mathbb{U}} \; (\Pi)$$

$$\frac{x_1 : P_1, \ldots, x_n : P_n \vdash e : N}{\Gamma \vdash \mathsf{thunk}\,(\lambda(x_1, \ldots, x_n).\,e) : {\downarrow}\Pi(x_1 : P_1, \ldots, x_n : P_n).\,N} \; (\Pi_\mathsf{i})$$

$$\frac{\Gamma \vdash e : {\downarrow}\Pi(x_1 : P_1, \ldots, x_n : P_n).\,N \qquad \Gamma \vdash e_1 : P_1 \quad \ldots \quad \Gamma \vdash e_n : P_n}{\Gamma \vdash (\mathsf{force}\,e_1) \, @ \, (e_1, \ldots, e_n) : N\{x_i := e_i\}} \; (\Pi_\mathsf{e})$$

$$\frac{\Gamma \vdash P : \mathbb{U} \qquad \Gamma, x : P \vdash Q : \mathbb{U}}{\Gamma \vdash \Sigma(x : P).\,Q : \mathbb{U}} \; (\Sigma)$$

$$\frac{\Gamma \vdash e_1 : P_1 \qquad \ldots \qquad \Gamma \vdash e_n : P_n\{x_1 := e_1\} \ldots \{x_{n-1} := e_{n-1}\}}{\Gamma \vdash (e_1, \ldots, e_n) : \Sigma(x_1 : P_1, \ldots, x_{n-1} : P_{n-1}).\,P_n} \; (\Sigma_\mathsf{i})$$

$$\frac{\Gamma \vdash e : \Sigma(x_1 : P_1, \ldots, x_{n-1} : P_{n-1}).\,P_n \qquad \Gamma, x_1 : P_1, \ldots, x_n : P_n \vdash e' : N\{z := (x_1, \ldots, x_n)\}}{\Gamma \vdash \mathsf{let}\,(x_1, \ldots, x_n) := e \; \mathsf{in}\; e' : N\{z := e\}} \; (\Sigma_\mathsf{e})$$

$$\frac{\Gamma \vdash P : \mathbb{U}}{\Gamma \vdash {\uparrow}P : \mathbb{U}} \; (\uparrow) \qquad \frac{\Gamma \vdash e : P}{\Gamma \vdash \mathsf{return}\,e : {\uparrow}P} \; (\uparrow_\mathsf{i}) \qquad \frac{\Gamma \vdash e_1 : {\uparrow}P \qquad \Gamma, x : P \vdash e_2 : N}{\Gamma \vdash e_1 \rhd_x e_2 : N} \; (\uparrow_\mathsf{e})$$

$$\frac{x : {\downarrow}N \vdash e : N}{\Gamma \vdash \mathsf{fix}\,x.\,e : N} \; (\mathsf{fix}) \qquad \frac{}{\Gamma \vdash c : {\downarrow}N_c} \; (\mathsf{constant})$$