

initial system:

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i}{\Vdash \Gamma, x : A} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : A}{\Gamma, x : A \vdash x : A} \text{ (var)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i \quad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Pi(x : A). B : \mathbb{U}_i} \text{ (\Pi)} \\
\\
\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : \Pi(x : A). B} \text{ (\Pi}_i\text{)} \quad \frac{\Gamma \vdash e_1 : \Pi(x : A). B \quad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1 @ e_2 : B[e_1/x]} \text{ (\Pi}_e\text{)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i \quad \Gamma, x : A \vdash B : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : A). B : \mathbb{U}_i} \text{ (\Sigma)} \\
\\
\frac{\Gamma \vdash e_1 : A_1 \quad \dots \quad \Gamma \vdash e_n : A_n[e_1/x_1] \dots [e_{n-1}/x_{n-1}]}{\Gamma \vdash (e_1, \dots, e_n) : \Sigma(x_1 : A_1, \dots, x_{n-1} : A_{n-1}). A_n} \text{ (\Sigma}_i\text{)} \\
\\
\frac{\Gamma \vdash e_1 : \Sigma(x_1 : A_1, \dots, x_{n-1} : A_{n-1}). A_n \quad \Gamma, x_1 : A_1, \dots, x_n : A_n \vdash e_2 : B[(x_1, \dots, x_n)/z]}{\Gamma \vdash \text{let } (x_1, \dots, x_n) = e_1 \text{ in } e_2 : B[e_1/z]} \text{ (\Sigma}_e\text{)} \\
\\
\frac{\Gamma, x : A \vdash e : A}{\Gamma \vdash \text{rec } x. e : A} \text{ (rec)} \quad \frac{}{\Gamma \vdash c : A_c} \text{ (constant)}
\end{array}$$

modal:

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash N : \mathbb{U}_i}{\Vdash \Gamma, x : N} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : N}{\Gamma, x : N \vdash x : \downarrow N} \text{ (var)} \\
\\
\frac{\Gamma \vdash N : \mathbb{U}_i \quad \Gamma, x : N \vdash M : \mathbb{U}_i}{\Gamma \vdash \Pi(x : N). M : \mathbb{U}_i} (\Pi) \\
\\
\frac{\Gamma, x : N \vdash e : M}{\Gamma \vdash \lambda x. e : \Pi(x : N). M} (\Pi_i) \\
\\
\frac{\Gamma \vdash e_1 : \Pi(x : N). M \quad \Gamma \vdash e_2 : \downarrow N}{\Gamma \vdash e_1 @ e_2 : M[e_1/x]} (\Pi_e) \\
\\
\frac{\Gamma \vdash N : \mathbb{U}_i \quad \Gamma, x : N \vdash M : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : N). M : \mathbb{U}_i} (\Sigma) \\
\\
\frac{\Gamma \vdash e_1 : \downarrow N_1 \quad \dots \quad \Gamma \vdash e_n : \downarrow N_n[e_1/x_1] \dots [e_{n-1}/x_{n-1}]}{\Gamma \vdash (e_1, \dots, e_n) : \Sigma(x_1 : N_1, \dots, x_{n-1} : N_{n-1}). N_n} (\Sigma_i) \\
\\
\frac{\Gamma \vdash e_1 : \Sigma(x_1 : N_1, \dots, x_{n-1} : N_{n-1}). N_n \quad \Gamma, x_1 : N_1, \dots, x_n : N_n \vdash e_2 : B[(x_1, \dots, x_n)/z]}{\Gamma \vdash \text{let } (x_1, \dots, x_n) = e_1 \text{ in } e_2 : B[e_1/z]} (\Sigma_e) \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i}{\Gamma \vdash \uparrow P : \mathbb{U}_i} (\uparrow) \quad \frac{\Gamma \vdash e : P}{\Gamma \vdash \text{return } e : \uparrow P} (\uparrow_i) \quad \frac{\Gamma \vdash e_1 : \uparrow \downarrow N \quad \Gamma, x : N \vdash e_2 : M}{\Gamma \vdash e_1 \triangleright_x e_2 : M} (\uparrow_e) \\
\\
\frac{\Gamma \vdash N : \mathbb{U}_i}{\Gamma \vdash \downarrow N : \mathbb{U}_i} (\downarrow) \quad \frac{\Gamma \vdash e : N}{\Gamma \vdash \text{thunk } e : \downarrow N} (\downarrow_i) \quad \frac{\Gamma \vdash e : \downarrow N}{\Gamma \vdash \text{force } e : N} (\downarrow_e) \\
\\
\frac{\Gamma, x : N \vdash e : \downarrow N}{\Gamma \vdash \text{rec } x. e : \downarrow N} (\text{rec}) \quad \frac{}{\Gamma \vdash c : \downarrow N_c} (\text{constant})
\end{array}$$

$$\begin{array}{l}
P ::= x \mid \Sigma(x_1 : N_1, \dots, x_n : N_n). M \mid \downarrow N \\
N ::= \Pi(x : N). M \mid \uparrow P
\end{array}$$

after closure conversion:

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash N : \mathbb{U}_i}{\Vdash \Gamma, x : N} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : N}{\Gamma, x : N \vdash x : \downarrow N} \text{ (var)} \\
\\
\frac{\Gamma \vdash N : \mathbb{U}_i \quad \Gamma, x : N \vdash M : \mathbb{U}_i}{\Gamma \vdash \Pi(x : N). M : \mathbb{U}_i} (\Pi) \\
\\
\frac{x : N \vdash e : M}{\Gamma \vdash \lambda x. e : \Pi(x : N). M} (\Pi_i) \\
\\
\frac{\Gamma \vdash e_1 : \Pi(x : N). M \quad \Gamma \vdash e_2 : \downarrow N}{\Gamma \vdash e_1 @ e_2 : M[e_1/x]} (\Pi_e) \\
\\
\frac{\Gamma \vdash N : \mathbb{U}_i \quad \Gamma, x : N \vdash M : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : N). M : \mathbb{U}_i} (\Sigma) \\
\\
\frac{\Gamma \vdash e_1 : \downarrow N_1 \quad \dots \quad \Gamma \vdash e_n : \downarrow N_n[e_1/x_1] \dots [e_{n-1}/x_{n-1}]}{\Gamma \vdash \text{return}(e_1, \dots, e_n) : \uparrow \Sigma(x_1 : N_1, \dots, x_{n-1} : N_{n-1}). N_n} (\Sigma_i) \\
\\
\frac{\Gamma \vdash e_1 : \uparrow \Sigma(x_1 : N_1, \dots, x_{n-1} : N_{n-1}). N_n \quad \Gamma, x_1 : N_1, \dots, x_n : N_n \vdash e_2 : B[(x_1, \dots, x_n)/z]}{\Gamma \vdash e_1 \triangleright_p \text{let } (x_1, \dots, x_n) = p \text{ in } e_2 : B[e_1/z]} (\Sigma_e) \\
\\
\frac{\Gamma \vdash P : \mathbb{U}_i}{\Gamma \vdash \uparrow P : \mathbb{U}_i} (\uparrow) \quad \frac{\Gamma \vdash e : P}{\Gamma \vdash \text{return } e : \uparrow P} (\uparrow_i) \quad \frac{\Gamma \vdash e_1 : \uparrow \downarrow N \quad \Gamma, x : N \vdash e_2 : M}{\Gamma \vdash e_1 \triangleright_x e_2 : M} (\uparrow_e) \\
\\
\frac{\Gamma \vdash N : \mathbb{U}_i}{\Gamma \vdash \downarrow N : \mathbb{U}_i} (\downarrow) \quad \frac{\vdash e : N}{\Gamma \vdash \text{thunk } e : \downarrow N} (\downarrow_i) \quad \frac{\Gamma \vdash e : \downarrow N}{\Gamma \vdash \text{force } e : N} (\downarrow_e) \\
\\
\frac{\Gamma, x : N \vdash e : \downarrow N}{\Gamma \vdash \text{rec } x. e : \downarrow N} (\text{rec}) \quad \frac{}{\Gamma \vdash c : \downarrow N_c} (\text{constant})
\end{array}$$

$$\begin{array}{l}
P ::= x \mid \Sigma(x_1 : N_1, \dots, x_n : N_n). M \mid \downarrow N \\
N ::= \Pi(x : N). M \mid \uparrow P
\end{array}$$