

$$\begin{array}{c}
\frac{}{\Vdash \emptyset} \text{ (empty)} \quad \frac{\Vdash \Gamma}{\Gamma \vdash \mathbb{U}_i : \mathbb{U}_{i+1}} \text{ (univ)} \quad \frac{\Gamma \vdash A : \mathbb{U}_i}{\Gamma \vdash A : \mathbb{U}_{i+1}} \text{ (hier)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i}{\Vdash \Gamma, x : A} \text{ (ext)} \quad \frac{\Vdash \Gamma, x : A}{\Gamma, x : A \vdash x : A} \text{ (var)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i \quad \Gamma [x : A] \vdash B : \mathbb{U}_i}{\Gamma \vdash \Pi(x : A). B : \mathbb{U}_i} \text{ (\Pi)} \\
\\
\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : \Pi(x : A). B} \text{ (\Pi}_i\text{)} \quad \frac{\Gamma \vdash e_1 : \Pi(x : A). B \quad \Gamma \vdash e_2 : A}{\Gamma \vdash e_1 @ e_2 : B[e_1/x]} \text{ (\Pi}_e\text{)} \\
\\
\frac{\Gamma \vdash A : \mathbb{U}_i \quad \Gamma [x : A] \vdash B : \mathbb{U}_i}{\Gamma \vdash \Sigma(x : A). B : \mathbb{U}_i} \text{ (\Sigma)} \\
\\
\frac{\Gamma \vdash e_1 : A \quad \Gamma \vdash e_2 : B[e_1/x]}{\Gamma \vdash (e_1, e_2) : \Sigma(x : A). B} \text{ (\Sigma}_i\text{)} \quad \frac{\Gamma \vdash e_1 : \Sigma(x : A). B \quad \Gamma, x : A, y : B \vdash e_2 : C[(x, y)/z]}{\Gamma \vdash \text{let } (x, y) = e_1 \text{ in } e_2 : C[e_1/z]} \text{ (\Sigma}_e\text{)} \\
\\
\frac{\Vdash \Gamma}{\Gamma \vdash \top : \mathbb{U}_i} \text{ (\top)} \quad \frac{}{\vdash \text{unit} : \top} \text{ (\top}_i\text{)} \\
\\
\frac{\Gamma, x : A \vdash e : A}{\Gamma \vdash \text{rec } x. e : A} \text{ (rec)}
\end{array}$$

A type  $A$  is relevant if  $\vdash e : A \rightarrow A \otimes A$  is derivable for some  $e$ .

A type  $A$  is affine if  $\vdash e : A \rightarrow \top$  is derivable for some  $e$ .

A term  $e$  is a well-typed closed proper term if (1)  $\vdash e : A$  is derivable for some type  $A$ , and (2) for any variable  $x$  in  $e$ , if it occurs more than once, then the type of  $x$  is relevant. If it doesn't occur, then the type of  $x$  is affine. (3) All the free variables in a recursion-term ( $\text{rec } x. e$ ) must be relevant.