As for the part: (N, ∞),

Let $x = \frac{1}{t}$,

$$u'' + \frac{2}{t}u' = \frac{f(\frac{1}{t})}{t^4} \quad (1.1)$$

- Central finite difference discretization for all derivatives:

$$p_i \frac{U_{i-1} - 2U_i + U_{i+1}}{h^2} + r_i \frac{U_{i+1} - U_{i-1}}{2h} - q_i U_i = f_i,$$

Total local truncation error:

$$\min_{a<N<\infty} \{h_1^2 \max_{a<x<N} \| \frac{1}{12} f''(x) \| + h_2^2 \max_{a<x<N} \| x^6 f(x) + \frac{2}{3} x^7 f'(x) + \frac{1}{12} x^8 f''(x) \| \} \quad (1.2)$$

To find the best N, we need to solve the min-max problem.

Appendix:

$$p_i \frac{U_{i-1} - 2U_i + U_{i+1}}{h^2} + r_i \frac{U_{i+1} - U_{i-1}}{2h} - q_i U_i = f_i$$

$$T_i = \frac{p_i}{h^2}(U_i - hU_i' + \frac{h^2}{2}U_i'' - \frac{h^3}{6}U_i''' + \frac{h^4}{24}U_i'''' - 2U_i + U_i + hU_i' + \frac{h^2}{2}U_i'' + \frac{h^3}{6}U_i''' + \frac{h^4}{24}U_i'''')$$

$$\frac{r_i}{2h}(U_i + hU_i' + \frac{h^2}{2}U_i'' + \frac{h^3}{6}U_i''' + \frac{h^4}{24}U_i'''' - U_i + hU_i' - \frac{h^2}{2}U_i'' + \frac{h^3}{6}U_i''' - \frac{h^4}{24}U_i'''') - q_i U_i - f_i$$

$$= \frac{p_i}{h^2}(h^2 U_i'' + \frac{h^4}{12}U_i'''') + \frac{r_i}{2h}(2hU_i' + \frac{h^3}{3}U_i''') - q_i U_i - f_i$$

$$= \frac{p_i}{h^2}(h^2 U_i'' + \frac{h^4}{12}U_i'''') + \frac{r_i}{2h}(2hU_i' + \frac{h^3}{3}U_i''') - q_i U_i - f_i$$

$$= \frac{p_i}{h^2}\frac{h^4}{12}U_i'''' + \frac{r_i}{2h}\frac{h^3}{3}U_i'''$$

$$= \frac{h^2}{12} p_i U_i'''' + \frac{h^2}{6} r_i U_i'''$$

$$= h^2(\frac{1}{12} p_i U_i'''' + \frac{1}{6} r_i U_i''')$$

$$= h^2(x^6 f(x) + \frac{2}{3} x^7 f'(x) + \frac{1}{12} x^8 f''(x))$$