Numerical Solutions & Analysis for PDEs

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Abstract

❑ **Partial Differential Equations (PDEs)** can be applied broadly and effectively in our daily life such as temperature distribution in physical phenomena and option pricing in financial mathematics.

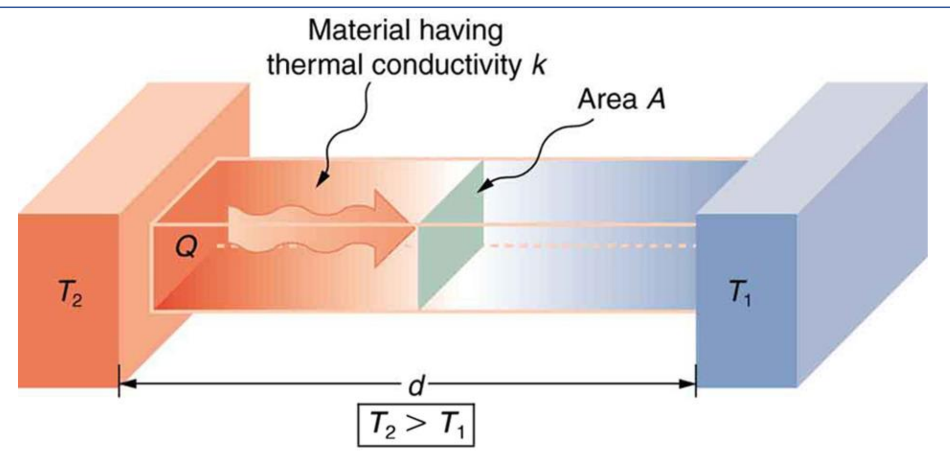
❑ **The heat equation is a parabolic PDE** that describes the distribution of heat (or variation in temperature) in a given region over time.

Objectives & Applications

❑ Develop and validate some finite difference methods to solve parabolic PDEs such as the heat equation numerically.

❑ Build a model to solve the special clothing design problem for high temperature work or heat conduction through a wall.

Models



❑ **Heat Equation in 1D**

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial t^2}, a < x < b, 0 < t$$

❑ **Boundary conditions**

$$u(a,t) = g_1(t), u(b,t) = g_2(t)$$

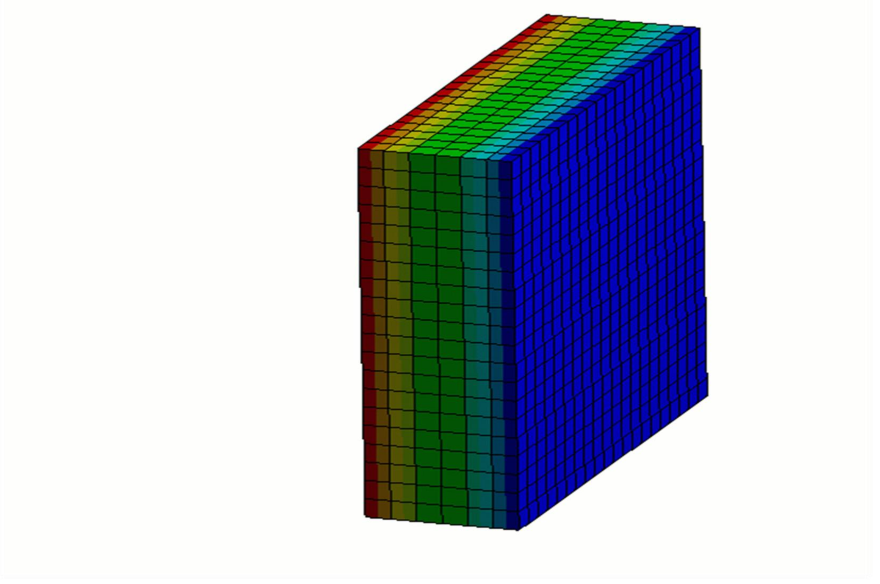
❑ **Initial Condition:**

$$u(x,0) = u_0(x)$$

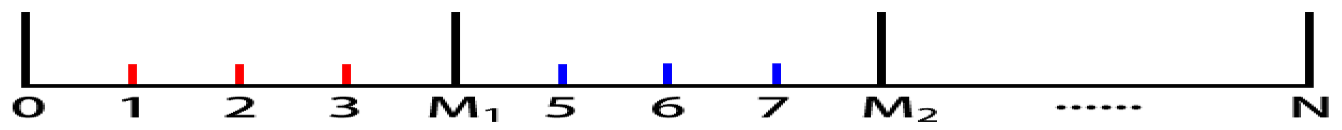
Methods

❑ **Finite-Difference Approximations to the Heat Conduction Equation**

The left figure is the 3D model of a wall of a house. In this case, the wall has three layers while different layers have different thermal properties.



❑ **Simplify the 3D model into a 1D model**



❑ **Use finite-difference approximations to the heat conduction equation**

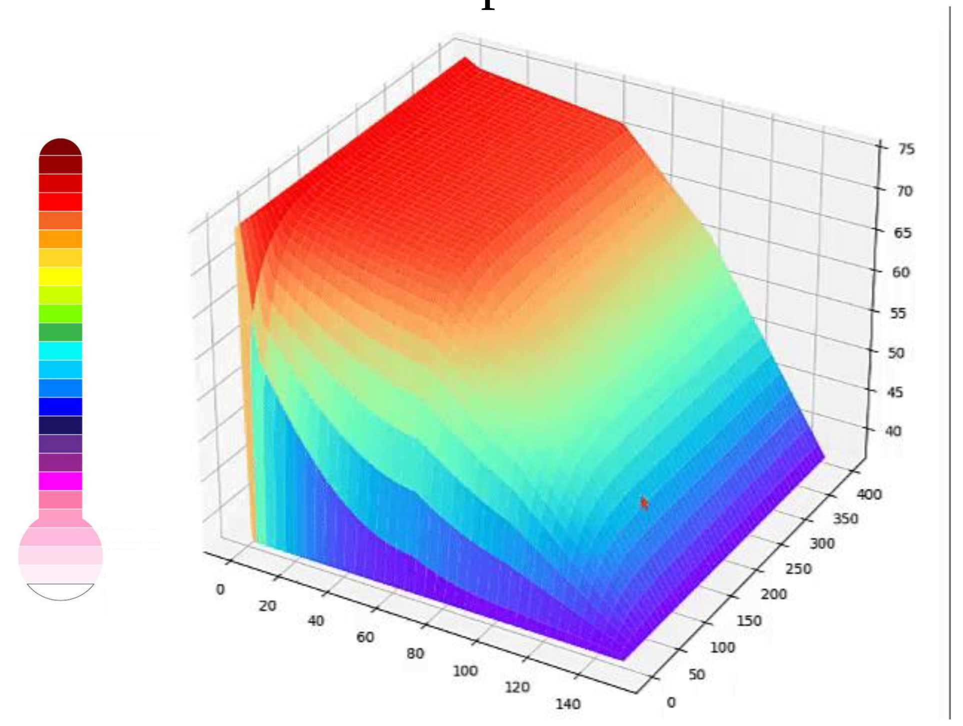
$$A_1 u_{0,j+1} + B_1 u_{1,j+1} + C_1 u_{2,j+1} = v'_{1,j}$$
$$\cdots = \cdots$$
$$A_{M_1-1} u_{M_1-2,j+1} + B_{M_1-1} u_{M_1-1,j+1} + C_{M_1-1} u_{M_1,j+1} = v_{M_1-1,j}$$
$$A_{M_1} u_{M_1-1,j+1} + B_{M_1} u_{M_1,j+1} + C_{M_1} u_{M_1+1,j+1} = v_{M_1,j}$$
$$A_{M_1+1} u_{M_1,j+1} + B_{M_1+1} u_{M_1+1,j+1} + C_{M_1+1} u_{M_1+2,j+1} = v_{M_1+1,j}$$
$$\cdots = \cdots$$
$$A_{M_2} u_{M_2-1,j+1} + B_{M_2} u_{M_2,j+1} + C_{M_2} u_{M_2+1,j+1} = v_{M_2,j}$$
$$\cdots = \cdots$$
$$A_N u_{N-1,j+1} + B_N u_{N,j+1} + C_N u_{N+1,j+1} = v'_{N,j}$$

$$\begin{bmatrix} B_1 & C_1 & & & & & & \\ A_2 & B_2 & C_2 & & & & & \\ & A_3 & B_3 & C_3 & & & & \\ & & A_{M_1} & B_{M_1} & C_{M_1} & & & \\ & & & \ddots & \ddots & \ddots & & \\ & & & & A_{M_2} & B_{M_2} & C_{M_2} & \\ & & & & & \ddots & \ddots & \ddots \\ & & & & & & A_N & B_N \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_{M_1} \\ \vdots \\ u_{M_2} \\ \vdots \\ u_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_M \\ \vdots \\ v_{M_2} \\ \vdots \\ v_N \end{bmatrix}$$

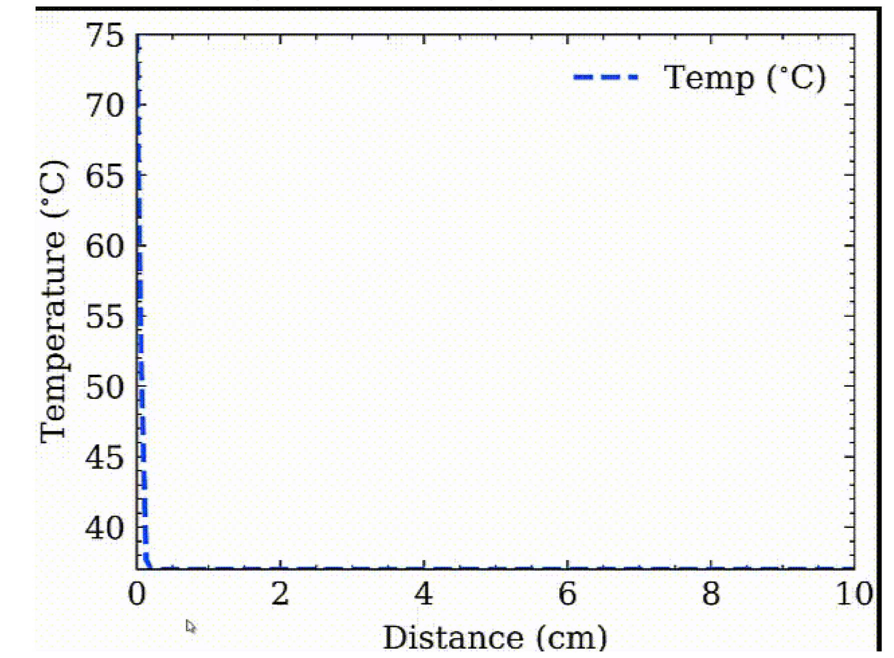
Results

❑ **The numerical solution of heat equation**

❑ Visualize the solution in a 3D picture



❑ Visualize the solution dynamically



Future Work

❑ Carry out furthur and deeper error analysis and work out a general code to generate the numerical solution for two or three dimensional heat conduction equations.

❑ Use the **immersed interface method** to deal with the discontinuous boundaries.

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References

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