- Depicts the variation of angle theta over time for a simple pendulum, computed using the Euler method with the program "pendulum."
- Lines connecting calculated values are solid.
- Despite error reduction with smaller time increments, all calculated behaviors are flawed.
- In Figure 3.2, both oscillation amplitude and total energy increase over time in the Euler solution.

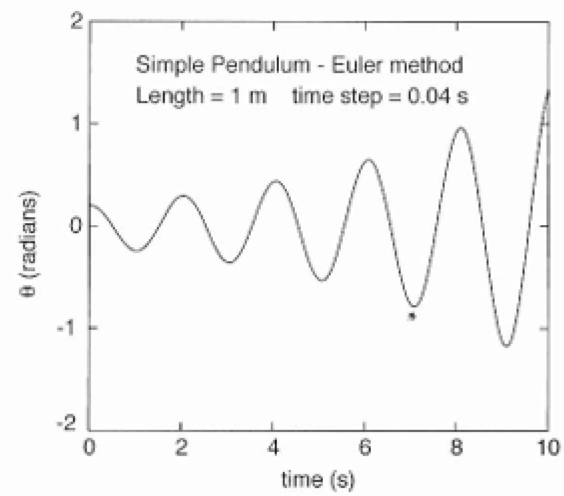

FIGURE 3.2: θ as a function of time for our simple pendulum, calculated using the Euler method with program `pendulum`. Here, and in most of the other figures in this chapter, we have connected the calculated values with solid lines.

- The Euler solution inherently displays instability, with energy increasing over time even with smaller time increments.
- Described by E = 1/2 m (omega l)^2 + mg l (1 - cos theta).
- First term represents kinetic energy, 1/2 m v^2, where v = omega l.
- Second term signifies gravitational potential energy, mgh, with h as the mass height above the lowest point on its trajectory.
- For small theta, energy simplifies to E = mg l (1 - cos theta).
- By substituting omega and theta from the Euler method into the energy equation, E(i+1) = E(i) + mg l (omega(i+1)^2 - omega(i)^2).

- Since the second term on the right is always positive, the energy of the Euler "solution" consistently increases with time, regardless of how small we make Δt
- However, despite this instability, the Euler method was still used for problems in Chapters 1 and 2.
- The suitability of a method depends on the specific problem context.
- In Chapters 1 and 2, although the Euler method didn't perfectly conserve energy, the errors were negligible for those particular problems.
- For problems involving oscillatory motion, like those in this chapter, we often need to consider behavior over many oscillation periods.
- A numerical method must conserve energy over the long term to be useful in such cases.
- The Euler method isn't suitable for these problems, leading to the consideration of other methods for solving ordinary differential equations.
- Appendix A discusses several different numerical approaches, including the Runge-Kutta and Verlet methods, which work well for dealing with oscillatory problems.
- A modified version of the Euler method, known as the Euler-Cromer method, is also suitable.
- The Euler-Cromer method involves a minor modification where the new value of one variable is used to calculate the new value of the other variable.
- This slight alteration can significantly impact the behavior of the algorithm, ensuring better conservation of energy over time.
- introduction of damping to the frictionless pendulum described earlier.
- Origin of damping varies, including factors like bearing friction and air resistance.

- Assumption: Damping force is directly proportional to velocity (rate of change of angle).
- Form of frictional force: Negative constant (rate of damping) multiplied by velocity (rate of change of angle).
- Equation of motion for damped pendulum: Second derivative of angle with respect to time plus damping coefficient times first derivative of angle with respect to time plus natural frequency squared times angle equals zero.
- Solution approach: Can be solved analytically due to linearity of the equation.
- Physical behavior regimes:
    - Underdamped regime: Occurs with small friction, characterized by oscillatory behavior with exponential decay.
- Variability in damping: Other functional forms possible, depending on factors like air resistance.
- Further exploration: Readers encouraged to investigate behaviors with different damping forms.
- Behavior of Damped Pendulum:
    - Underdamped regime exhibits oscillatory behavior with a frequency of
    - 53−974
    - 4
    - 53−97
    -
    - and amplitude decay over time.
    - Overdamped regime demonstrates monotonic, exponential decay of amplitude.
    - Critical damping occurs at the boundary between underdamped and overdamped regimes.
- Effects of Driving Force:
    - Addition of a driving force introduces excitement to the pendulum's behavior.

- Driving force assumed to be sinusoidal with time, characterized by amplitude "F sub d" and angular frequency "Omega sub d."
- Equation of motion includes external driving force term "F sub d times sine of Omega sub d times t," influencing energy exchange within the system.

- Nonlinear Pendulum:
    - Consideration of a nonlinear pendulum without friction or driving force.
    - Total mechanical energy conserved, leading to periodic motion with amplitude-dependent period.
    - Numerical approach used due to complexity of analytical demonstration.
- Observations:
    - Motion remains periodic but is no longer described by simple sine or cosine functions.
    - Period of motion now dependent on amplitude, with longer periods for larger amplitudes.
    - Results align with intuitive understanding of pendulum behavior.
- Next Steps:
    - Integration of damping, driving force, and nonlinearity to explore complex pendulum motion further in the next section.