
数字世界精彩无限

Unit 12

——Design Sequential Circuits with Flip Flops

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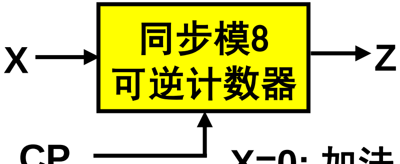
Some examples

- 模8可逆计数器
- 自动售卖机
- 时序锁
- 二进制串行加法器
- 串行输入的8421BCD码检测器
- 奇偶校验器
- 码制转换器
- 序列信号发生器

利用触发器设计同步时序逻辑_例1

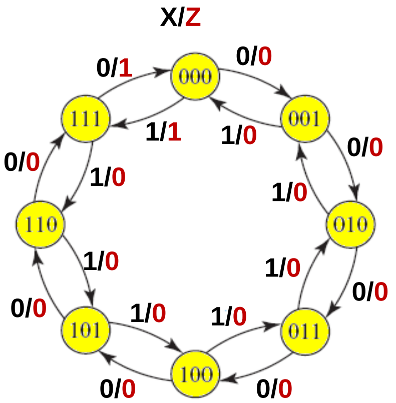
例1：利用T触发器设计一个同步模8可逆计数器



X=0: 加法；X=1: 减法
Z ：进位及借位

1. 原始状态图及状态表

需要3个T触发器



T触发器驱动表

输入端T	次态 Q_{n+1}
0	Q_n
1	$\bar{Q}_n$

确定T_3：看$Q_3^n \to Q_3^{n+1}$
确定T_2：看$Q_2^n \to Q_2^{n+1}$
确定T_1：看$Q_1^n \to Q_1^{n+1}$

2. 状态转换真值表

输入	现态			次态			输入			输出
X	Q_3^n	Q_2^n	Q_1^n	Q_3^{n+1}	Q_2^{n+1}	Q_1^{n+1}	T_3	T_2	T_1	Z
0	0	0	0	0	0	1	0	0	1	0
0	0	0	1	0	1	0	0	1	1	0
0	0	1	0	0	1	1	0	0	1	0
0	0	1	1	1	0	0	1	1	1	0
0	1	0	0	1	0	1	0	0	1	0
0	1	0	1	1	1	0	0	1	1	0
0	1	1	0	1	1	1	0	0	1	0
0	1	1	1	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	0	0	0	0	0	1	0
1	0	1	0	0	0	1	0	1	1	0
1	0	1	1	0	1	0	0	0	1	0
1	1	0	0	0	1	1	1	1	1	0
1	1	0	1	1	0	0	0	0	1	0
1	1	1	0	1	0	1	0	1	1	1
1	1	1	1	1	1	0	0	0	1	0

利用触发器设计同步时序逻辑_例1

3. 卡诺图化简

XQ_3^n \ $Q_2^nQ_1^n$	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	1	0	0	0
10	1	0	0	0

$$T_3 = \overline{X}Q_2^nQ_1^n + X\overline{Q_2^n}\,\overline{Q_1^n}$$

XQ_3^n \ $Q_2^nQ_1^n$	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	1

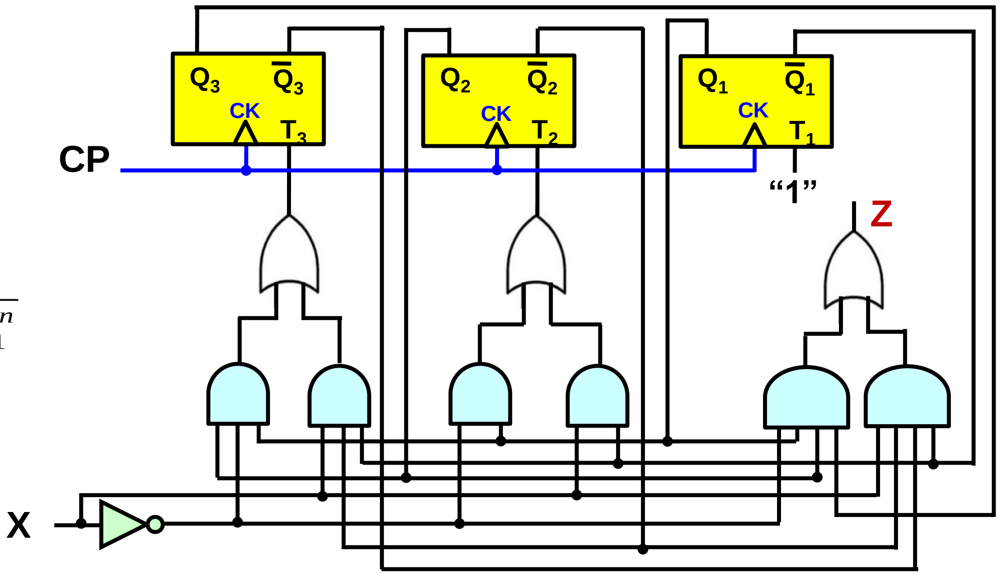
$$T_2 = \overline{X}Q_1^n + X\overline{Q_1^n}$$

XQ_3^n \ $Q_2^nQ_1^n$	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	0	0	0	0
10	1	0	0	0

$$T_1 = 1$$

$$Z = X\overline{Q_3^n}\,\overline{Q_2^n}\,\overline{Q_1^n} + \overline{X}Q_3^nQ_2^nQ_1^n$$

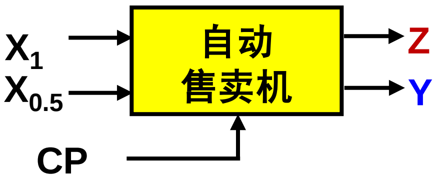
4. 电路实现



利用触发器设计同步时序逻辑_例2

例2：利用D触发器设计一个自动售卖机

- 只接收硬币： 0.5￥，1￥
- 每次投币只接收一枚硬币
- 机器收到1.5 ￥，给出一瓶饮料
- 机器收到2.0￥，给出一瓶饮料，找回0.5￥



$X_1 X_{0.5}$ ＝ 00: 0￥

$X_1 X_{0.5}$ ＝ 01: 0.5￥

$X_1 X_{0.5}$ ＝ 10: 1￥

Y=1/ 0: 给/不给 饮料

Z=1/ 0 : 找零/不找零

1. 原始状态图及状态表

① 状态设定

Solution 1:

Mealy circuit

S_0—初始状态，无投币

S_1—机器收到0.5￥

S_2—机器收到1.0￥ （2个 0.5￥, or 1个1.0￥ ）

if （机器又收到1个0.5￥）

then Y=1,且 Z=0, 回到 S_0

Else If （机器又收到1个1￥）

then Y=1, 且Z=1,回到S_0

利用触发器设计同步时序逻辑_例2

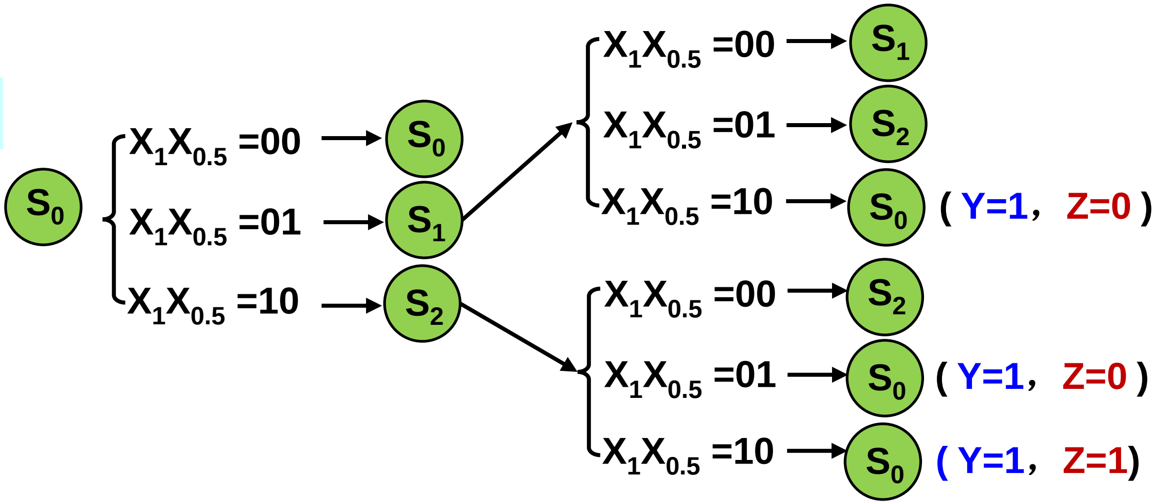
② 状态转换分析

Solution 1: Mealy circuit

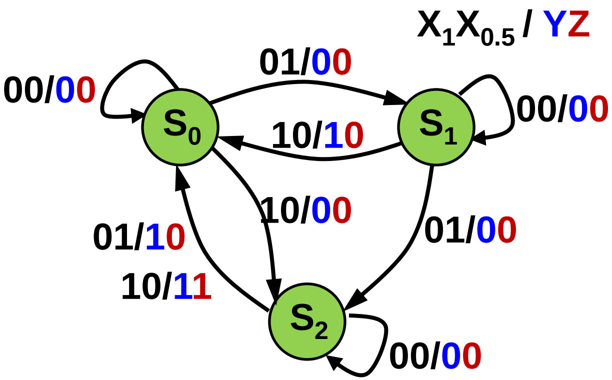
S_0—无投币

S_1—0.5￥

S_2—机器收到1.0￥



③ Mealy 状态图



④ 状态表

现态 S^n	S^{n+1}/ Z			
	$X_1X_{0.5}$=00	$X_1X_{0.5}$=01	$X_1X_{0.5}$=10	$X_1X_{0.5}$=11
S_0	S_0 / 00	S_1 / 00	S_2 / 00	X/ XX
S_1	S_1 / 00	S_2 / 00	S_0 / 10	X / XX
S_2	S_2 / 00	S_0 / 10	S_0 / 11	X / XX

利用触发器设计同步时序逻辑_例2

④ 状态表

现态 S^n	S^{n+1}/ Z			
	$X_1X_{0.5}$=00	$X_1X_{0.5}$=01	$X_1X_{0.5}$=10	$X_1X_{0.5}$=11
S_0	S_0 / 00	S_1 / 00	S_2 / 00	X/ XX
S_1	S_1 / 00	S_2 / 00	S_0 / 10	X / XX
S_2	S_2 / 00	S_0 / 10	S_0 / 11	X / XX

2. 状态化简

3. 状态分配

S_0 —— 00
S_1 —— 01
S_2 —— 10

	0	1
0	S_0	S_1
1	S_2	

需要2个D触发器

4. 状态转换真值

确定D_2：看Q_2^{n+1}
确定D_1：看Q_1^{n+1}

输入		现态		次态		输入		输出	
X_1	$X_{0.5}$	Q_2^n	Q_1^n	Q_2^{n+1}	Q_1^{n+1}	D_2	D_1	Y	Z
0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	1	0	0
0	0	1	0	1	0	1	0	0	0
0	0	1	1	X	X	X	X	X	X
0	1	0	0	0	1	0	1	0	0
0	1	0	1	1	0	1	0	0	0
0	1	1	0	0	0	0	0	1	0
0	1	1	1	X	X	X	X	X	X
1	0	0	0	1	0	1	0	0	0
1	0	0	1	0	0	0	0	1	0
1	0	1	0	0	0	0	0	1	1
1	0	1	1	X	X	X	X	X	X
1	1	0	0	X	X	X	X	X	X
1	1	0	1	X	X	X	X	X	X
1	1	1	0	X	X	X	X	X	X
1	1	1	1	X	X	X	X	X	X

利用触发器设计同步时序逻辑_例2

5. 卡诺图化简

$X_1X_{0.5}$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	0	X	1
01	0	1	X	0
11	X	X	X	X
10	1	0	X	0

$$D_2 = \overline{X_1}\,\overline{X_{0.5}}Q_2^n + Q_1^n X_{0.5} + X_1\overline{Q_1}^n\overline{Q_2}^n$$

$X_1X_{0.5}$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	0	X	0
01	0	0	X	1
11	X	X	X	X
10	0	1	X	1

$$Y = Q_2^n X_{0.5} + Q_2^n X_1 + X_1 Q_1^n$$

$X_1X_{0.5}$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	1	X	0
01	1	0	X	0
11	X	X	X	X
10	0	0	X	0

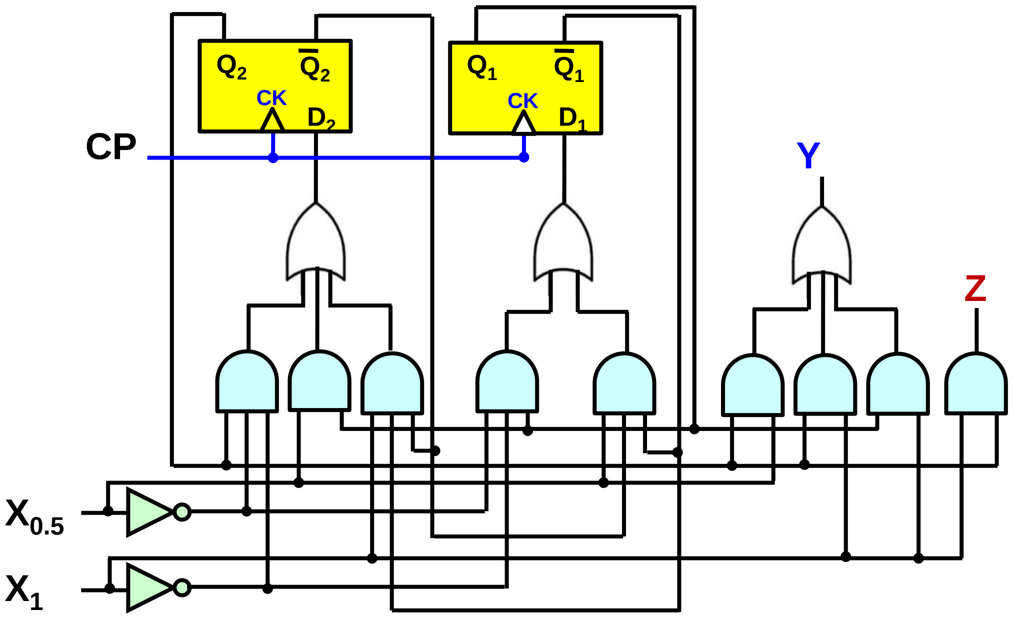
$$D_1 = \overline{X_1}\,\overline{X_{0.5}}Q_1^n + X_{0.5}\overline{Q_1}^n\overline{Q_2}^n$$

$X_1X_{0.5}$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	0	X	0
01	0	0	X	0
11	X	X	X	X
10	0	0	X	1

$$Z = X_1 Q_2^n$$

利用触发器设计同步时序逻辑_例2

6. 电路实现



! 电路需要预置

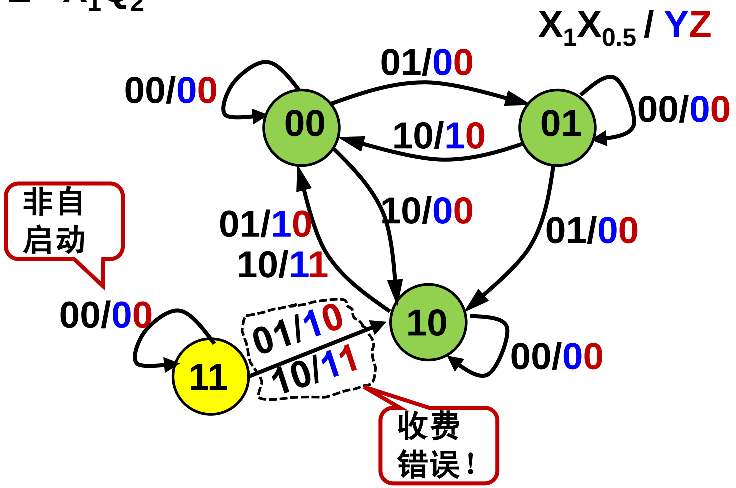
7. 检查无关项

无关状态：$Q_2^nQ_1^n=11$

$X_1X_{0.5}$ 分别为 00 ,01,10时，带入计算

$$\begin{cases} Q_2^{n+1}=D_2=\overline{X}_1\overline{X}_{0.5}Q_1^n+Q_1X_{0.5}+X_1\overline{Q}_1^n\overline{Q}_2^n \\ Q_1^{n+1}=D_1=\overline{X}_1\overline{X}_{0.5}Q_2^n+X_{0.5}\overline{Q}_1^n\overline{Q}_2^n \end{cases}$$

$$\begin{cases} Y=Q_2^nX_{0.5}+Q_2^nX_1+X_1Q_1^n \\ Z=X_1Q_2^n \end{cases}$$



利用触发器设计同步时序逻辑_例2

1. 原始状态图及状态表

① 状态设定（标记收到的钱数）

S_0—初始状态，机器收到0￥

S_1—机器收到0.5￥

S_2—机器收到1.0￥

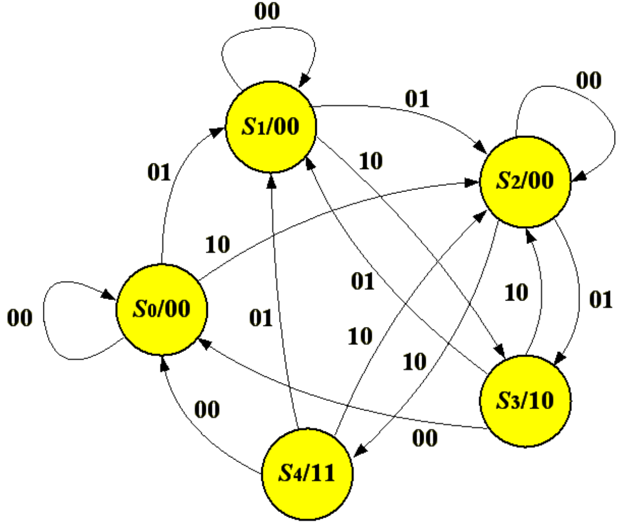
S_3—机器收到1.5￥

S_4—机器收到2.0￥

Solution 2:

Moor circuit

② Moor 状态图



③ Moor 状态表

现态 S_n	次态S_{n+1} $X_1X_2 = 00$	$X_1X_2 = 01$	$X_1X_2 = 10$	输出 YZ
S_0	S_0	S_1	S_2	00
S_1	S_1	S_2	S_3	00
S_2	S_2	S_3	S_4	00
S_3	S_0	S_1	S_2	10
S_4	S_0	S_1	S_2	11

2. 状态化简

3. 状态分配

需要3个D触发器

Q_3^n \ $Q_2^nQ_1^n$	00	01	11	10
0	S_0	S_3		S_1
1	S_4			S_2

S_0 —— 000

S_1 —— 010

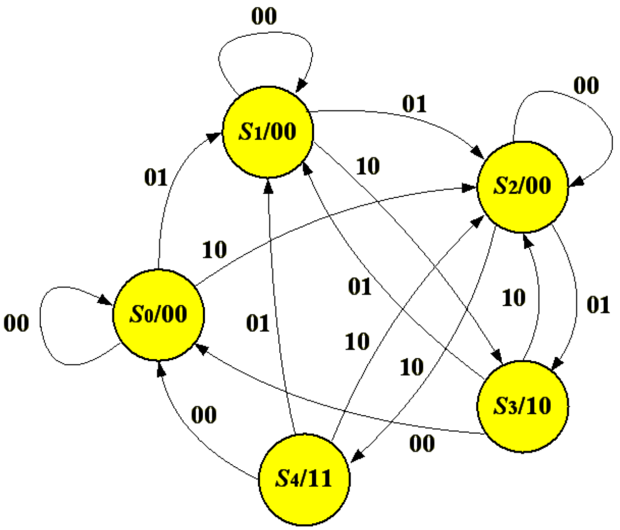
S_2 —— 110

S_3 —— 001

S_4 —— 100

利用触发器设计同步时序逻辑_例2

4. 状态转换真值表



S_0 —— 000

S_1 —— 010

S_2 —— 110

S_3 —— 001

S_4 —— 100

输入		现态			次态			输入			输出	
X_1	$X_{0.5}$	Q_3^n	Q_2^n	Q_1^n	Q_3^{n+1}	Q_2^{n+1}	Q_1^{n+1}	D_3	D_2	D_1	Y	Z
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	1	0	0	1	0	0	0
0	0	1	0	0	0	0	0	0	0	0	1	1
0	0	1	1	0	1	1	0	1	1	0	0	0
0	1	1	1	0	0	0	1	0	0	1	0	0
0	1	0	0	0	0	1	0	0	1	0	0	0
0	1	0	1	0	1	1	0	1	1	0	0	0
0	1	0	0	1	0	1	0	0	1	0	1	0
0	1	1	0	0	0	1	0	0	1	0	1	1
1	0	0	0	0	1	1	0	1	1	0	0	0
1	0	0	1	0	0	0	1	0	0	1	0	0
1	0	1	1	0	1	0	0	1	0	0	0	0
1	0	0	0	1	1	1	0	1	1	0	1	0
1	0	1	0	0	1	1	0	1	1	0	1	1
1	1	X	X	X	X	X	X	X	X	X	X	X

利用触发器设计同步时序逻辑_例2

5. 卡诺图化简

$X_1=0$

$X_{0.5}Q_3^n$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	0	X	0
01	0	X	X	1
11	0	X	X	0
10	0	0	X	1

$X_1=1$

$X_{0.5}Q_3^n$ \ $Q_2^nQ_1^n$	00	01	11	10
00	1	1	X	0
01	1	X	X	1
11	X	X	X	X
10	X	X	X	X

$$D_3=\overline{X}_{0.5}Q_3^nQ_2^n+\overline{Q}_3^n X_{0.5}Q_2^n+X_1\overline{Q}_2^n$$

$X_1=0$

$X_{0.5}Q_3^n$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	0	X	1
01	0	X	X	1
11	1	X	X	0
10	1	1	X	1

$X_1=1$

$X_{0.5}Q_3^n$ \ $Q_2^nQ_1^n$	00	01	11	10
00	1	1	X	0
01	1	X	X	0
11	X	X	X	X
10	X	X	X	X

$$D_2=\overline{X}_{0.5}Q_3^n+\overline{Q}_2^n X_{0.5}+X_1\overline{Q}_2^n+\overline{X}_1\overline{X}_{0.5}Q_2^n$$

$X_1=0$

$X_{0.5}Q_3^n$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	0	X	0
01	0	X	X	0
11	0	X	X	1
10	0	0	X	0

$X_1=1$

$X_{0.5}Q_3^n$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	0	X	1
01	0	X	X	0
11	X	X	X	X
10	X	X	X	X

$$D_1=X_{0.5}Q_3^nQ_2^n+\overline{Q}_3^n X_1Q_2^n$$

利用触发器设计同步时序逻辑_例2

$X_1=0$

$X_{0.5}Q_3^n$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	1	X	0
01	1	X	X	0
11	1	X	X	0
10	0	1	X	1

$X_1=1$

$X_{0.5}Q_3^n$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	1	X	0
01	1	X	X	0
11	X	X	X	X
10	X	X	X	X

$$Y = \overline{Q}_2^nQ_3^n + Q_1^n\overline{Q}_2^n + X_{0.5}\overline{Q}_3^nQ_2^n$$

$X_1=0$

$X_{0.5}Q_3^n$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	0	X	0
01	1	X	X	0
11	1	X	X	0
10	0	0	X	0

$X_1=1$

$X_{0.5}Q_3^n$ \ $Q_2^nQ_1^n$	00	01	11	10
00	0	0	X	0
01	1	X	X	0
11	X	X	X	X
10	X	X	X	X

$$Z = \overline{Q}_2^nQ_3^n$$

$$\begin{cases} D_3 = \overline{X}_{0.5}Q_3^nQ_2^n + \overline{Q}_3^n X_{0.5}Q_2^n + X_1\overline{Q}_2^n \\ D_2 = \overline{X}_{0.5}Q_3^n + \overline{Q}_2^n X_{0.5} + X_1\overline{Q}_2^n + \overline{X}_1\overline{X}_{0.5}Q_2^n \\ D_1 = X_{0.5}Q_3^nQ_2^n + \overline{Q}_3^n X_1Q_2^n \\ Y = \overline{Q}_2^nQ_3^n + Q_1^n\overline{Q}_2^n + X_{0.5}\overline{Q}_3^nQ_2^n \\ Z = \overline{Q}_2^nQ_3^n \end{cases}$$

6. 电路实现(略)

7. 检查无关项（略）

Moor型电路与Mealy型电路比较

- Moor型电路中的状态总数相对要多一些，需要使用较多的触发器资源。
- Moor型电路的输出只与状态有关，输出没有毛刺。