

数字世界精彩无限

# Fundamentals of Logic Design

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# Unit 2

## —— Boolean Algebra

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## 2.5 代数化简法

### ■ 一个逻辑函数有多种不同的表达式

$$F=AB+A\bar{C} \quad \dots\dots \text{与-或}$$

$$\overline{\overline{AB+A\bar{C}}}$$

$$=\overline{\overline{AB}} \cdot \overline{\overline{A\bar{C}}} \quad \dots\dots \text{与非-与非}$$

$$= \overline{(\bar{A}+\bar{B})} \cdot \overline{(\bar{A}+C)} \quad \dots\dots \text{或非-与非}$$

$$= \overline{(\bar{A}+\bar{B})} + \overline{(\bar{A}+C)} \quad \dots\dots \text{或非-或}$$

$$F=(A+B) \cdot (A+\bar{C}) \quad \dots\dots \text{或-与}$$

$$\overline{\overline{(A+B) \cdot (A+\bar{C})}}$$

$$= \overline{(\bar{A}+\bar{B})} + \overline{(\bar{A}+\bar{C})} \quad \dots\dots \text{或非-或非}$$

$$= \overline{\bar{A}} \cdot \overline{\bar{B}} + \overline{\bar{A}} \cdot \overline{C} \quad \dots\dots \text{与-或非}$$

$$= \overline{\bar{A}} \overline{\bar{B}} \cdot \overline{\bar{A}C} \quad \dots\dots \text{与非-与}$$



# ■ 同一类型的表达式也不是唯一的

$$F=AB+\bar{A}C$$

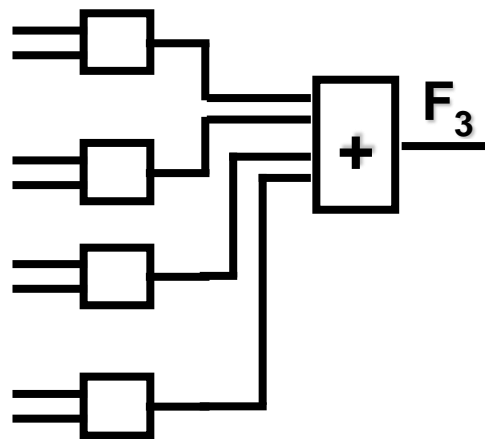
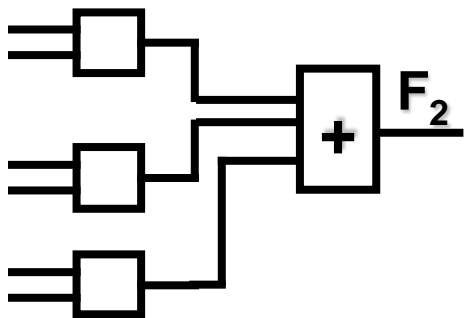
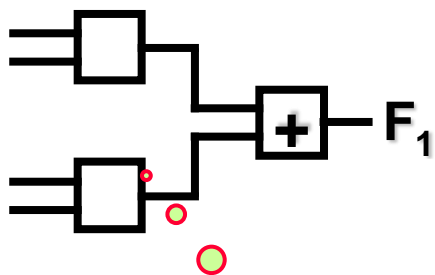
.....①  $F_1$

$$=AB+\bar{A}C+BC$$

.....②  $F_2$

$$=ABC+AB\bar{C}+\bar{A}BC+\bar{A}\bar{B}C$$

.....③  $F_3$



最简,元件少,可靠



## 2.5 代数化简法



**最简 (Minimum Expressions) ?**

- ① 与项 (和项) 的个数最少
- ② 每个与项 (和项) 中变量的个数最少



minimum cost

- ① 逻辑门的数量最少
- ② 逻辑门的输入个数最少

目的:

- ① 降低成本
- ② 提高可靠性

化简方法 {

- 代数法
- 卡诺图法



## 2.5 代数化简法

例:

$$\begin{aligned} F &= A + \underline{A\bar{B}\bar{C}} + \bar{A}CD + \bar{C}E + \bar{D}E \\ &= A + \underline{\bar{A}CD} + \bar{C}E + \bar{D}E \\ &= A + CD + \underline{\bar{C}E + \bar{D}E} \\ &= A + CD + E(\bar{C} + \bar{D}) \\ &= A + CD + \underline{E\bar{C}D} \\ &= A + CD + E \end{aligned}$$



例

$$\begin{aligned} F &= \underline{AB} + \underline{A\bar{C}} + \underline{\bar{B}C} + \underline{B\bar{C}} + \underline{\bar{B}D} + \underline{B\bar{D}} + \underline{ADE(F+G)} \\ &= \underline{A(\bar{B}C)} + \underline{\bar{B}C} + \underline{B\bar{C}} + \underline{\bar{B}D} + \underline{B\bar{D}} + \underline{ADE(F+G)} \\ &= \underline{A} + \underline{\bar{B}C} + \underline{B\bar{C}} + \underline{\bar{B}D} + \underline{B\bar{D}} + \underline{ADE(F+G)} \\ &= \underline{A} + \underline{\bar{B}C} + \underline{B\bar{C}} + \underline{\bar{B}D} + \underline{B\bar{D}} + \underline{C\bar{D}} \\ &= \underline{A} + \underline{\bar{B}C} + \underline{B\bar{C}} + \underline{\bar{B}D} + \underline{B\bar{D}} + \underline{C\bar{D}} \\ &= \underline{A} + \underline{\bar{B}C} + \underline{B\bar{C}} + \underline{\bar{B}D} + \underline{C\bar{D}} \\ &= \underline{A} + \underline{B\bar{C}} + \underline{\bar{B}D} + \underline{C\bar{D}} \end{aligned}$$





## 2.5 代数化简法

例:  $F = (\bar{B}+D)(\bar{B}+D+A+G)(C+E)(\bar{C}+G)(A+E+G)$

取对偶

  $J = \bar{B}D + \bar{B}DAG + CE + \bar{C}G + AEG$

$$= \bar{B}D + CE + \bar{C}G + AEG$$

$$= \bar{B}D + CE + \bar{C}G$$

取对偶



$$F = (\bar{B}+D)(C+E)(\bar{C}+G)$$

例

$$F = A + AB + \bar{A}C + BD + ACEF + \bar{B}E + DEF$$

$$= A + C + BD + \bar{B}E$$



## 2.5 代数化简法

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优点——

- 不受变量数目的约束；
- 对公理、定理和规则十分熟练时，化简较方便；

缺点——

- 技巧性强；
- 在很多情况下难以判断化简结果是否最简；