Homework 3

学号 姓名

1. Design context-free grammars for the following languages:

   a) L = $\{a^i b^j | i \neq j \text{ and } i \neq 2j\}$

   b) The set of all strings with twice as many 0's as 1's.

2. Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

   a) The set of all strings of 0's and l's such that no prefix has more l's than 0's.

   b) $\{0^n 1^m | n < m < 2n\}$

3. Design a context-free grammar for the language consisting of all strings over $\{a, b\}$ that are **not** of the form ww, for some string w. Explain how your grammar works. You needn't prove it's correctness formally.

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1. Design context-free grammars for the following languages:

   a) L = $\{a^i b^j | i \neq j \text{ and } i \neq 2j\}$

   $i \neq j$ and $i \neq 2j$ $S \to aS \mid Sb \mid aA \mid Ab \mid aB \mid 1$

ε	a	aa	ab	abb	aab	aaab
x	√	√	x	√	x	

   b) The set of all strings with twice as many 0's as 1's.

   $i < j$

   $j < i < 2j$

   $i > 2j$

   $A \to aAb \mid aA \mid a$

   $B \to aBb \mid Bb \mid b$

   $C \to aaCb \mid aC \mid a$

2. Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

   a) The set of all strings of 0's and 1's such that no prefix has more 1's than 0's.

   $i = 2j$

   $S \to 00S \mid$

   ~~$A \to aAb \mid Ab \mid b$~~

   $B \to aaAb \mid aA \mid a$

   $C \to aCb \mid aaCb \mid$ ~~$a \mid b$~~ aaabb

   b) $\{0^n 1^m | n < m < 2n\}$

   CFG → PDA

3. Design a context-free grammar for the language consisting of all strings over $\{a, b\}$ that are **not** of the form ww, for some string w. Explain how your grammar works. You needn't prove it's correctness formally.

   $S \to AB \mid BA \mid F$

   $A \to CAC \mid 0$

   $B \to CBC \mid 1$

   $C \to 0 \mid 1$

   $F \to CFC \mid 0 \mid 1$

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1. Design context-free grammars for the following languages:

   a) L = $\{a^i b^j | i \neq j \text{ and } i \neq 2j\}$

   b) The set of all strings with twice as many 0's as 1's.

2. Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

   a) The set of all strings of 0's and 1's such that no prefix has more 1's than 0's.

任何前缀1的数量不多于0的数量.

CFG ⇒ PDA.

$S \to 0S1S \mid 0S \mid \varepsilon$

ε . 0 , 1 01 10 010

✓ ✓ × . ✓ . ×

0, Z_0/0Z_0

0, 0/00

1, 0/ε

ε, 0/ε

ε, Z_0/ε

空栈

   b) $\{0^n 1^m | n < m < 2n\}$

start ⟶

$S \to 0S1 \mid 0S11 \mid 00111$

3. Design a context-free grammar for the language consisting of all strings over $\{a, b\}$ that are **not** of the form ww, for some string w. Explain how your grammar works. You needn't prove it's correctness formally.

1. Design context-free grammars for the following languages:

a) $L = \{a^i b^j | i \neq j \text{ and } i \neq 2j\}$

$i<j$ 或 $j<i<2j$ 或 $i>2j$

三种情况

① $A \to aAb \mid Ab \mid b$

② $B \to aBb \mid aaBb \mid aabbb$

③ $C \to aaCb \mid aC \mid a$

从而有 $S \to A \mid B \mid C$

b) The set of all strings with twice as many 0's as 1's.

类似于 $L = \{w \in \{0,1\}^* \mid 0\text{和}1\text{数量相等}\}$

$S \to S0S1S \mid S1S0S \mid \varepsilon$ (每对0.1相对)

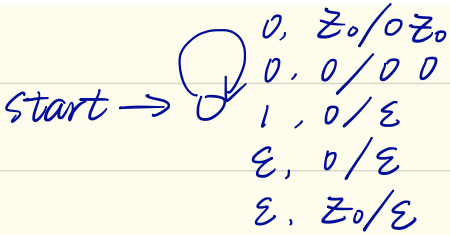
则 每2个0对应1个1即有

$S \to S0S0S1S \mid S0S1S0S \mid S1S0S0S \mid \varepsilon$

2. Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

a) The set of all strings of 0's and 1's such that no prefix has more 1's than 0's.

① 直接画



空栈方式

② $S \to 0S1S \mid 0S \mid \varepsilon$ 先写CFG

构造PDA $P=(\{q_0\}, \{0,1\}, \{0,1,S\}, \delta, q_0, S, \phi)$

其中. $\delta(q_0, \varepsilon, S) = \{(q_0, 0S1S), (q_0, 0S), (q_0, \varepsilon)\}$.

$\delta(q_0, 0, 0) = \{(q_0, \varepsilon)\}$

$\delta(q_0, 1, 1) = \{(q_0, \varepsilon)\}$

b) $\{0^n 1^m | n < m < 2n\}$

$S \to 0S1 \mid 0S11 \mid 00111$

构造PDA. $P=(\{q_0\}, \{0,1\}, \{0,1,S\}, \delta, q_0, S, \phi)$.

$\delta(q_0, \varepsilon, S) = \{(q_0, 0S1), (q_0, 0S11), (q_0, 00111)\}$.

$\delta(q_0, 0, 0) = \{(q_0, \varepsilon)\}$.

$\delta(q_0, 1, 1) = \{(q_0, \varepsilon)\}$.