Homework 1

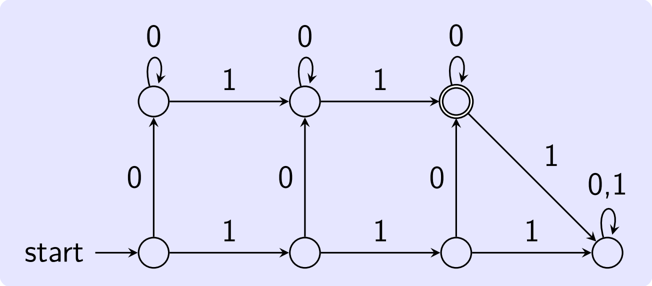
Give DFA's accepting the languages over the alphabet $\{0, 1\}$.

1. the set of all strings with at least one 0 and exactly two 1's.(所有以 01 开始或结尾的串.)

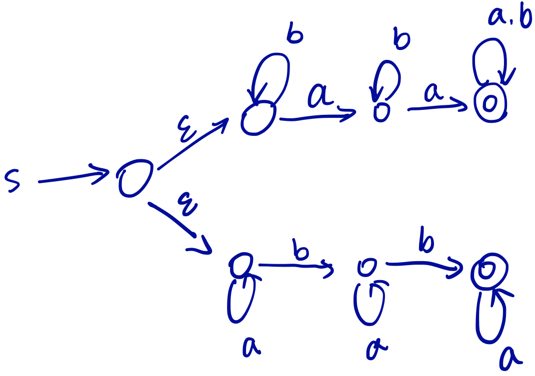
Homework 1

Give DFA's accepting the languages over the alphabet $\{0, 1\}$.

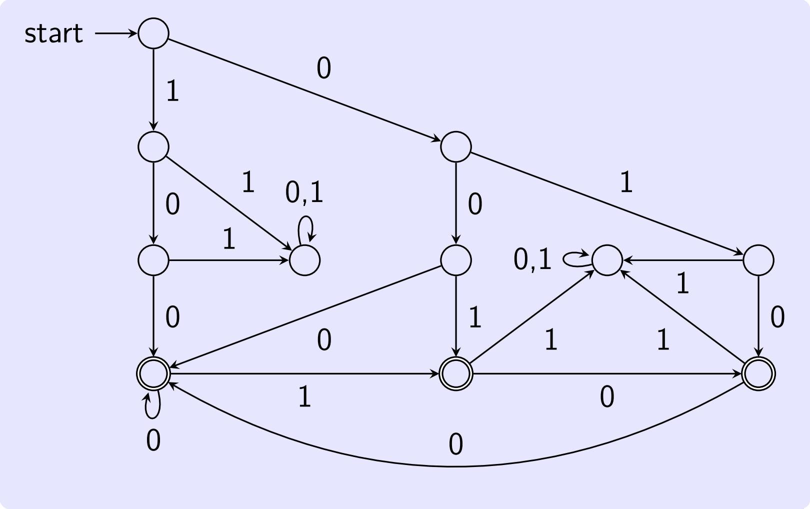
1. the set of all strings with at least one 0 and exactly two 1's.(所有以 01 开始或结尾的串.)



2. The set of all strings such that each block of three consecutive symbols contains at least two 0's.(任何 3 个连续的字符都至少有两个 0.)

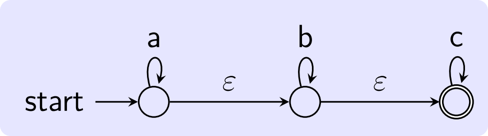


2. The set of all strings such that each block of three consecutive symbols contains at least two 0's.(任何 3 个连续的字符都至少有两个 0.)



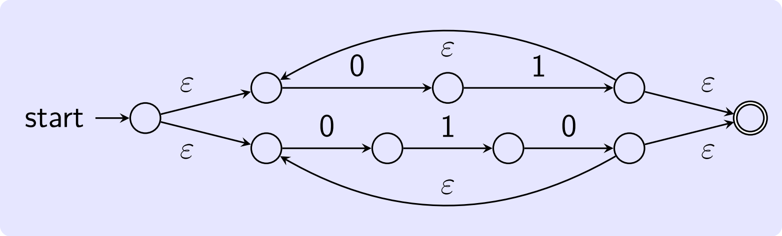
3. The set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.

3. The set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.



4. The set of strings that consist of either 01 repeated one or more times or 010 repeated one or more times.

4. The set of strings that consist of either 01 repeated one or more times or 010 repeated one or more times.



Design regular expression:

5. The set of all strings of 0's and 1's not containing 101 as a substring.

Design regular expression:

5. The set of all strings of 0's and 1's not containing 101 as a substring.

$$0^*(1+000^*)^*0^*$$

or

$$(0+\varepsilon)(1+000^*)^*(0+\varepsilon)$$

Homework 2

1. Prove that language $L =\{0^n \mid n$ is a power of 2$\}$ is not regular.

证明思路：泵引理，反证法。取 $w = 0^{2^N}$, 则
$2^N < |xy^2z| < 2^N + N < 2^N + 2^N = 2^{(N+1)}$

2. If L is a language, and a is a symbol, then L/a, the quotient of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if L is regular, so is L/a. Hint: Start with a DFA for L and consider the set of accepting states.

2. If L is a language, and a is a symbol, then L/a, the quotient of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if L is regular, so is L/a. Hint: Start with a DFA for L and consider the set of accepting states.

证明.

设 $M = (Q, \Sigma, \delta, q_0, F)$, $L = L(M)$, 构造
$M' = (Q, \Sigma, \delta, q_0, F')$, 其中
$F' = \{q \mid \delta(q, a) \in F\}, q \in Q, a \in \Sigma$
往证 $L(M') = L/a$
$\because \forall w \in L(M')$ 即 $\hat{\delta}(q_0, w) \in F'$ 即
$\delta(\hat{\delta}(q_0, w), a) \in F \therefore w \in L/a$
又 $\because \forall w \in L/a$ 有 $wa \in L$ 即 $\hat{\delta}(q_0, wa) \in F$ 即
$\delta(\hat{\delta}(q_0, w), a) \in F$ 即 $\hat{\delta}(q_0, w) \in F'$ $\therefore w \in L(M')$ $\square$

Design context-free grammars for the following languages:

3. The set $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

Design context-free grammars for the following languages:

3. The set $\{a^ib^jc^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

$$
\begin{aligned}
S &\to \overbrace{A_1C \mid A_2C}^{a^ib^jc^*} \mid \overbrace{AB_1 \mid AB_2}^{a^*b^jc^k} \\
A_1 &\to aA_1b \mid aA_1 \mid a \quad \text{a的数量多于b的数量} \\
A_2 &\to aA_2b \mid A_2b \mid b \quad \text{a的数量少于b的数量} \quad \Big\} \; a^ib^j \;\; i \neq j \\
C &\to Cc \mid \varepsilon \\
B_1 &\to bB_1c \mid bB_1 \mid b \\
B_2 &\to bB_2c \mid B_2c \mid c \\
A &\to Aa \mid \varepsilon
\end{aligned}
$$

(注意：C 若为 $Cc \mid c$ 则不能产生 a, c 同时为 0 个，或 b, c)

4. The set of all strings over $\{0, 1\}$ with twice as many 0's as 1's.

$S \rightarrow S0S0S1S \mid S0S1S0S \mid S1S0S0S \mid \varepsilon$

$S \rightarrow 0S0S1 \mid 0S1S0 \mid 1S0S0 \mid SS \mid \varepsilon$

5. The set of all strings over $\{a, b\}$ that are **not** of the form ww, for some string w. Explain how your grammar works. You needn't prove it's correctness formally.

5. The set of all strings over $\{a, b\}$ that are **not** of the form ww, for some string w. Explain how your grammar works. You needn't prove it's correctness formally.

串长为奇数，显然不是 ww (文法中的 $A \mid B$)。串长为偶数 $2n$，至少存在一对儿距离为 n 的两个不同字符。如 $aaaabbbb = \underline{aa\check{a}ab}\ \underline{b\check{b}b}$ 或 $aabaaa = \underline{aa\check{b}aa}\ \underline{\check{a}}$.

$$
\begin{aligned}
S &\to \overbrace{A \mid B}^{\text{奇数}} \mid AB \mid BA \\
A &\to XAX \mid a \\
B &\to XBX \mid b \\
X &\to a \mid b
\end{aligned}
$$

Exercise 2.2.5 b)

The set of all strings whose tenth symbol from the right end is a 1.

利用五元组的方式给出(状态转移图需要记录的状态过多)

$D=(Q,\Sigma,\delta,q_0,F)$

$Q=\{x_1x_2\cdots x_n \mid 0\le n\le 10,\ x_i\in\{0,1\}$ 且 $i\in\{1,2,\cdots,n\}\}$

$\Sigma=\{0,1\}.\quad q_0=\varepsilon$

$$\delta(x_1x_2\cdots x_n,\ y)=\begin{cases} x_1x_2\cdots x_n y, & 0\le n<10 \\ x_2x_3\cdots y, & n=10 \end{cases}$$

$F=\{1x_2x_3\cdots x_{10} \mid x_i\in\{0,1\}$ 且 $i\in\{2,3,\cdots,10\}\}$

Exercise 2.2.5 b)

The set of all strings whose tenth symbol from the right end is a 1.

$A = (Q, \Sigma, \delta, q_0, F)$, 其中 $\Sigma = \{0, 1\}$

$Q = \{\overline{x_1 x_2 \ldots x_n} \mid 1 \le n \le 10, x_i \in \{0, 1\}, i \in \{1, \ldots, n\}\}$

$$\delta(\overline{x_1 x_2 \ldots x_n}, y) = \begin{cases} \overline{x_1 x_2 \ldots x_n y} & \text{if } n < 10 \\ \overline{x_2 \ldots x_n y} & \text{if } n = 10 \end{cases}$$

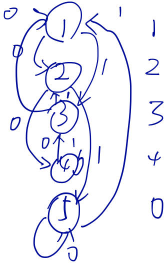
$q_0 = \overline{\varepsilon}$ ($q_0 \notin Q$ 故有点小问题)

$F = \{\overline{1 x_2 \ldots x_{10}} \mid x_i \in \{0, 1\}, i \in \{2, \ldots, 10\}\}$

Exercise 2.2.6 a)

从而0101是不接收的串

The set of all strings beginning with a 1 that, when interpreted as binary integer, is a multiple of 5. for example, strings 101(5), 1010(10), and 1111(15) are in the language; 0, 100(4) and 111(7) are not.



Exercise 2.2.6 a)

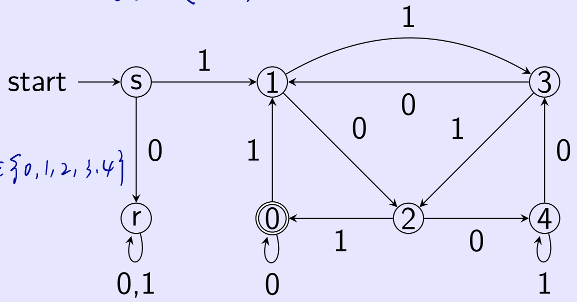
$\delta(q_2, 0) = q_4$ $i \bmod 5 = 2$

$(2i) \bmod 5 = 4$

$\delta(q_2, 1) = q_0$ $(2i+1) \bmod 5 = 0$

	0	1
→s	r	q1
*q0	q0	q1
q1	q2	q3
q2	q4	q0
q3	q1	q2
q4	q3	q4
r	r	r

mod 5 余 i, $i \in \{0,1,2,3,4\}$



Exercise 2.2.6 b)

The set of all strings that, when interpreted *in reverse* as a binary integer, is divisible by 5. Examples of string in the language are 0, 10011(25), 1001100(25), and 0101(10).

Exercise 2.2.6 b)

Solutions: $A = (Q, \Sigma = \{0,1\}, \delta, q_0, F)$

$$Q = \left\{ (x,y) \;\middle|\; \begin{array}{ll} x \in \{0,1,2,3\}, & \mathrm{len}(w) \mod 4 \\ y \in \{0,1,2,3,4\}, & \mathrm{bin}(\overleftarrow{w}) \mod 5 \end{array} \right\} \cup \{q_0\}$$

从右向左解释为2进制数

mod5的余数

$$f(x) \stackrel{def}{=} \left\{ \begin{array}{r|cccc} x & 0 & 1 & 2 & 3 \\ f(x) & 1 & 2 & 4 & 3 \end{array} \right\}$$

1001100

$$\delta \begin{cases} \delta((x,y),0) &= ((x+1) \mod 4, y) \\ \delta((x,y),1) &= ((x+1) \mod 4, (y + f(x)) \mod 5) \\ \delta(q_0, 0) &= (1,0) \\ \delta(q_0, 1) &= (1,1) \end{cases}$$

接收0,只增加长度

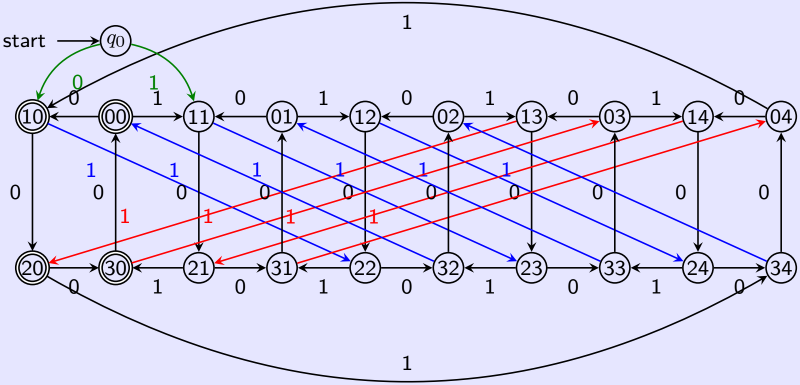
接收1,增加长度

改变mod5的余数

$$F = \{(x,0) \mid x \in \{0,1,2,3\}\}$$

	mod 5
1	1
2	2
4	4
8	3
16	1
32	2
64	4
128	3

Exercise 2.2.6 b)



Exercise 2.2.7

Let A be a DFA and q a particular state of A, such that $\delta(q,a)=q$ for all input symbols a. Show by induction on the length of the input that for all input strings w, $\hat{\delta}(q,w)=q$.

施归纳于w的长度

① 当 $w=\varepsilon$ 时. 显然成立

② 当 $|w|<n$ 时. 假设均成立. 往证 $|w|=n$ 时成立

令 $w=xa$, 则有 $\hat{\delta}(q,x)=q$

由 $\forall a\in\Sigma$. 有 $\delta(q,a)=q$

从而 $\delta(\hat{\delta}(q,x),a)=q$

即 $\hat{\delta}(q,xa)=q$ 有 $\hat{\delta}(q,w)=q$

Exercise 2.2.7

Let A be a DFA and q a particular state of A, such that $\delta(q, a) = q$ for all input symbols a. Show by induction on the length of the input that for all input strings w, $\hat{\delta}(q, w) = q$.

首先，对于 $|w| = 0$ 的 w，显然成立。

假设对所有 $|w| < n$ 的串 w 成立，则当 $|w| = n$ 时，令 $w = xa$，有

$$
\begin{aligned}
\hat{\delta}(q, w) &= \hat{\delta}(q, xa) \\
&= \delta(\hat{\delta}(q, x), a) \\
&= \delta(q, a) \\
&= q
\end{aligned}
$$

Exercise 2.2.8

Let A be a DFA and a a particular input symbol of A, such that for all states q of A we have $\delta(q, a) = q$.

a) Show by induction on n that for all $n \geq 0$, $\hat{\delta}(q, a^n) = q$, where a^n is the string consisting of n a's.

Exercise 2.2.8

Let A be a DFA and a a particular input symbol of A, such that for all states q of A we have $\delta(q, a) = q$.

a) Show by induction on n that for all $n \geq 0$, $\hat{\delta}(q, a^n) = q$, where a^n is the string consisting of n a's.

归纳基础 $\hat{\delta}(q, a^0) = \hat{\delta}(q, \varepsilon) = q$, 归纳递推
$\hat{\delta}(q, a^{n+1}) = \hat{\delta}(q, a^n a) = \delta(\hat{\delta}(q, a^n), a) = \delta(q, a) = q$

Exercise 2.2.8

Let A be a DFA and a a particular input symbol of A, such that for all states q of A we have $\delta(q, a) = q$.

a) Show by induction on n that for all $n \geq 0$, $\hat{\delta}(q, a^n) = q$, where a^n is the string consisting of n a's.

归纳基础 $\hat{\delta}(q, a^0) = \hat{\delta}(q, \varepsilon) = q$, 归纳递推
$\hat{\delta}(q, a^{n+1}) = \hat{\delta}(q, a^n a) = \delta(\hat{\delta}(q, a^n), a) = \delta(q, a) = q$

b) Show that either $\{a\}^* \subseteq L(A)$ or $\{a\}^* \cap L(A) = \emptyset$.

Exercise 2.2.8

Let A be a DFA and a a particular input symbol of A, such that for all states q of A we have $\delta(q, a) = q$.

a) Show by induction on n that for all $n \geq 0$, $\hat{\delta}(q, a^n) = q$, where a^n is the string consisting of n a's.

归纳基础 $\hat{\delta}(q, a^0) = \hat{\delta}(q, \varepsilon) = q$, 归纳递推
$\hat{\delta}(q, a^{n+1}) = \hat{\delta}(q, a^n a) = \delta(\hat{\delta}(q, a^n), a) = \delta(q, a) = q$

b) Show that either $\{a\}^* \subseteq L(A)$ or $\{a\}^* \cap L(A) = \emptyset$.

证 $q_0 \in F \Leftrightarrow \{a\}^* \subseteq L(A)$ 即可.

Exercise 2.2.9

Let $A = (Q, \Sigma, \delta, q_0, \{q_f\})$ be a DFA, and suppose that for all a in Σ we have $\delta(q_0, a) = \delta(q_f, a)$

a) Show that for all $w \neq \varepsilon$ we have $\hat{\delta}(q_0, w) = \hat{\delta}(q_f, w)$.

归纳法

Exercise 2.2.9

Let $A = (Q, \Sigma, \delta, q_0, \{q_f\})$ be a DFA, and suppose that for all a in Σ we have $\delta(q_0, a) = \delta(q_f, a)$

a) Show that for all $w \neq \varepsilon$ we have $\hat{\delta}(q_0, w) = \hat{\delta}(q_f, w)$.

$|w| = 1$ 显然成立，假设 $|w| < n$ 成立，当 $|w| = n$ 时，令 $w = za$，有

$$\begin{aligned}\hat{\delta}(q_0, w) &= \hat{\delta}(q_0, za) = \delta(\hat{\delta}(q_0, z), a) \\ &= \delta(\hat{\delta}(q_f, z), a) = \hat{\delta}(q_f, za) \\ &= \hat{\delta}(q_f, w)\end{aligned}$$

Exercise 2.2.9

Let $A = (Q, \Sigma, \delta, q_0, \{q_f\})$ be a DFA, and suppose that for all a in Σ we have $\delta(q_0, a) = \delta(q_f, a)$

a) Show that for all $w \neq \varepsilon$ we have $\hat{\delta}(q_0, w) = \hat{\delta}(q_f, w)$.

b) Show that if x is a nonempty string in $L(A)$, then for all $k > 0$, x^k (i.e. x written k times) is also in $L(A)$.

归纳法

Exercise 2.2.9

Let $A = (Q, \Sigma, \delta, q_0, \{q_f\})$ be a DFA, and suppose that for all a in Σ we have $\delta(q_0, a) = \delta(q_f, a)$

a) Show that for all $w \neq \varepsilon$ we have $\hat{\delta}(q_0, w) = \hat{\delta}(q_f, w)$.

b) Show that if x is a nonempty string in $L(A)$, then for all $k > 0$, x^k (i.e. x written k times) is also in $L(A)$.

如果 $x \in L(A)$，则有 $\hat{\delta}(q_0, x) = q_f$，即 $k = 1$ 成立；假设 $k = n - 1$ 时，$x^k \in L(A)$ 成立，那么当 $k = n$ 时

$$\hat{\delta}(q_0, x^n) = \hat{\delta}(\hat{\delta}(q_0, x^{n-1}), x) = \hat{\delta}(q_f, x) = \hat{\delta}(q_0, x) = q_f$$

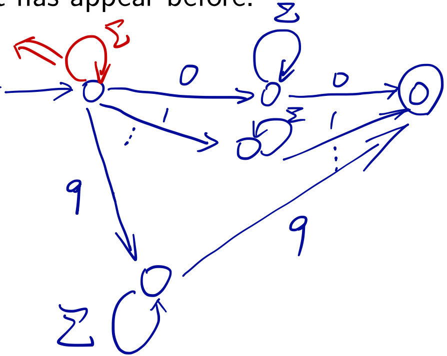
Exercise 2.3.4

Give NFA, try to take advantage of nondeterminism as much as possible.

尝试利用非确定性

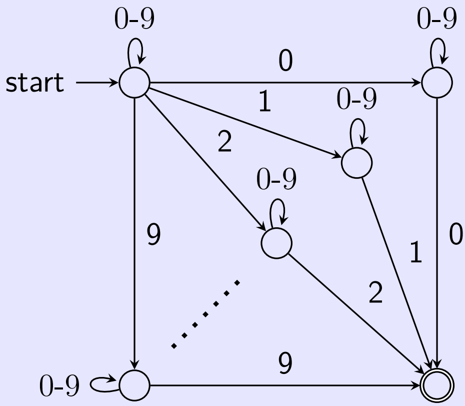
a) The set of strings over alphabet $\{0, 1, \cdots, 9\}$ such that the final digit has appear before.

如果没有这个会出现第1个字符必须与最后1个字符相同



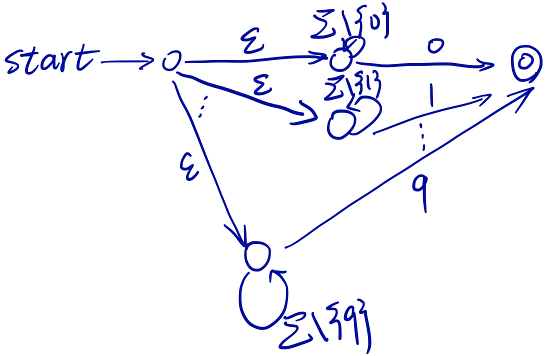
Exercise 2.3.4

a) The set of strings over alphabet $\{0, 1, \cdots, 9\}$ such that the final digit has appear before.



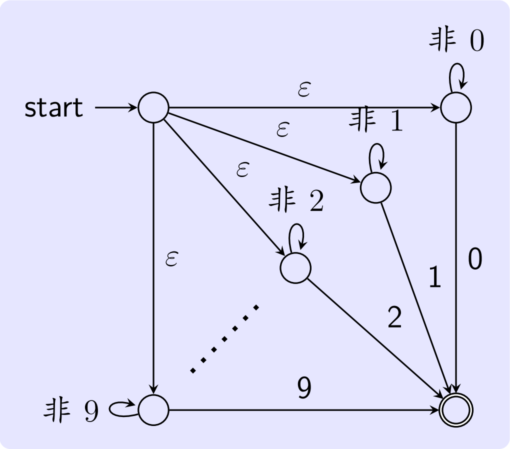
Exercise 2.3.4

b) The set of strings over alphabet $\{0, 1, \cdots, 9\}$ such that the final digit has *not* appeared before.



Exercise 2.3.4

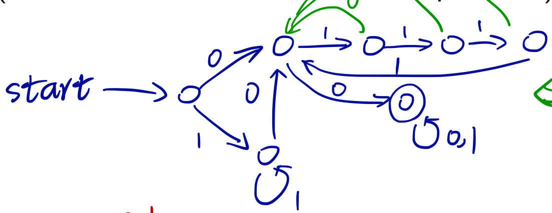
b) The set of strings over alphabet $\{0, 1, \cdots, 9\}$ such that the final digit has *not* appeared before.



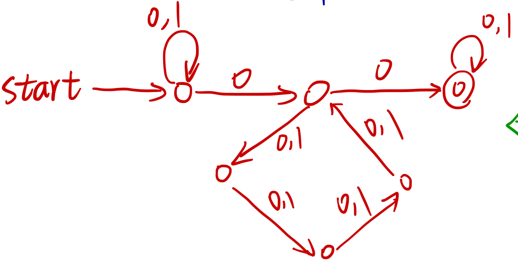
2个0间隔的位置是4的倍数(注意0是4的倍数)

c) The set of strings of 0's and 1's such that there are two 0's separated by a number of positions that is a mutiple of 4. (Note that 0 is an allowable multiple of 4.)

0100



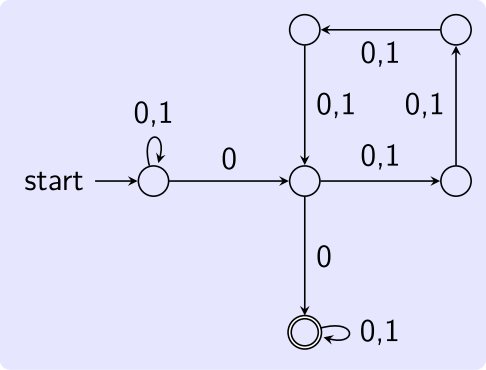
⇐ DFA. 没有利用非确定性



⇐ NFA

Exercise 2.3.4

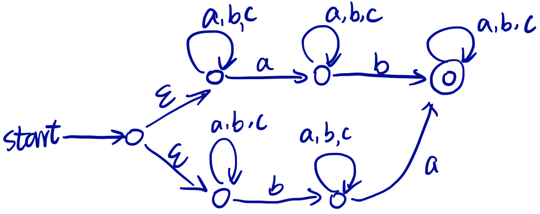
c) The set of strings of 0's and 1's such that there are two 0's separated by a number of positions that is a mutiple of 4. (Note that 0 is an allowable multiple of 4.)



Exercise 3.1.1

书写正则表达式(regular expression)

a) The set of strings over alphabet $\{a, b, c\}$ containing at least one a and at least one b.



$$(a+b+c)^*\big(a(a+b+c)^*b + b(a+b+c)^*a\big)(a+b+c)^*$$

Exercise 3.1.1

a) The set of strings over alphabet $\{a, b, c\}$ containing at least one a and at least one b.

$$(a + b + c)^*(a(a + b + c)^*b + b(a + b + c)^*a)(a + b + c)^*$$

Exercise 3.1.1

a) The set of strings over alphabet $\{a, b, c\}$ containing at least one a and at least one b.

$$(a + b + c)^*(a(a + b + c)^*b + b(a + b + c)^*a)(a + b + c)^*$$

b) The set of strings of 0's and 1's whose tenth symbol from the right end is 1.

$(0+1)^* \, 1 \, (0+1)^9$

Exercise 3.1.1

a) The set of strings over alphabet $\{a, b, c\}$ containing at least one a and at least one b.

$$(a+b+c)^*(a(a+b+c)^*b+b(a+b+c)^*a)(a+b+c)^*$$

b) The set of strings of 0's and 1's whose tenth symbol from the right end is 1.

$$(0+1)^*1(0+1)^9$$

Exercise 3.1.1

a) The set of strings over alphabet $\{a, b, c\}$ containing at least one a and at least one b.

$$(a+b+c)^*(a(a+b+c)^*b+b(a+b+c)^*a)(a+b+c)^*$$

b) The set of strings of 0's and 1's whose tenth symbol from the right end is 1.

$$(0+1)^*1(0+1)^9$$

c) The set of strings of 0's and 1's with at most one pair of consecutive 1's.

$(0+10)^*11(0+01)^*$

确定仅有11的串 → $(0+10)^*(\varepsilon+1+11)(0+01)^*$

Exercise 3.1.1

a) The set of strings over alphabet $\{a, b, c\}$ containing at least one a and at least one b.

$$(a + b + c)^*(a(a + b + c)^*b + b(a + b + c)^*a)(a + b + c)^*$$

b) The set of strings of 0's and 1's whose tenth symbol from the right end is 1.

$$(0 + 1)^*1(0 + 1)^9$$

c) The set of strings of 0's and 1's with at most one pair of consecutive 1's.

$$(0 + 10)^*(\varepsilon + 1 + 11)(0 + 01)^*$$

Exercise 3.1.2

Write regular expressions for the following languages:

a) The set of all strings of 0's and 1's such that every pair of adjacent 0's appears before any pair of adjacent 1's.

1001 ✓ 0101 ✓ 1100 ✗ ε ✓. 1010

$(10+0)^*(1+01)^*$

以0结尾的上式无法识别

从而 $(10+0)^*(1+01)^*(0+\varepsilon)$

类似 $(1+\varepsilon)(0+01)^*(1+01)^*(0+\varepsilon)$

Exercise 3.1.2

Write regular expressions for the following languages:

a) The set of all strings of 0's and 1's such that every pair of adjacent 0's appears before any pair of adjacent 1's.

$$(0 + 10)^*(01 + 1)^*(\varepsilon + 0)$$

Exercise 3.1.2

Write regular expressions for the following languages:

a) The set of all strings of 0's and 1's such that every pair of adjacent 0's appears before any pair of adjacent 1's.

$$(0 + 10)^*(01 + 1)^*(\varepsilon + 0)$$

b) The set of strings of 0's and 1's whose number of 0's is divisible by five.

$(1^*01^*01^*01^*01^*01^*)^*$ X

更簡練

$\Rightarrow (01^*01^*01^*01^*0+1)^*$

$1^*(1^*01^*01^*01^*01^*01^*)^*1^*$

Exercise 3.1.2

Write regular expressions for the following languages:

a) The set of all strings of 0's and 1's such that every pair of adjacent 0's appears before any pair of adjacent 1's.

$$(0 + 10)^*(01 + 1)^*(\varepsilon + 0)$$

b) The set of strings of 0's and 1's whose number of 0's is divisible by five.

$$(01^*01^*01^*01^*0 + 1)^*$$

Exercise 3.1.3

a) The set of all strings of 0's and 1's not containing 101 as a substring.

2个1之间有0或不少于2个的0.

$$0^*(1+000^*)^*0^*$$

Exercise 3.1.3

a) The set of all strings of 0's and 1's not containing 101 as a substring.

$0^*(1 + 000^*)^*0^*$ or $(0 + \varepsilon)(1 + 000^*)^*(0 + \varepsilon)$ or

$(0 + \varepsilon)(1 + 00 + 000)^*(0 + \varepsilon)$

Exercise 3.1.3

a) The set of all strings of 0's and 1's not containing 101 as a substring.

$0^*(1 + 000^*)^*0^*$ or $(0 + \varepsilon)(1 + 000^*)^*(0 + \varepsilon)$ or

$(0 + \varepsilon)(1 + 00 + 000)^*(0 + \varepsilon)$

b) The set of all strings with an equal number of 0's and 1's, such that no prefix has two more 0's than 1's, nor two more 1's than 0's.

11 x   101 x

$(01 + 10)^*$

Exercise 3.1.3

a) The set of all strings of 0's and 1's not containing 101 as a substring.

$0^*(1 + 000^*)^*0^*$ or $(0 + \varepsilon)(1 + 000^*)^*(0 + \varepsilon)$ or

$(0 + \varepsilon)(1 + 00 + 000)^*(0 + \varepsilon)$

b) The set of all strings with an equal number of 0's and 1's, such that no prefix has two more 0's than 1's, nor two more 1's than 0's.

$(01 + 10)^*$

Exercise 3.1.3

a) The set of all strings of 0's and 1's not containing 101 as a substring.

$0^*(1+000^*)^*0^*$ or $(0+\varepsilon)(1+000^*)^*(0+\varepsilon)$ or

$(0+\varepsilon)(1+00+000)^*(0+\varepsilon)$

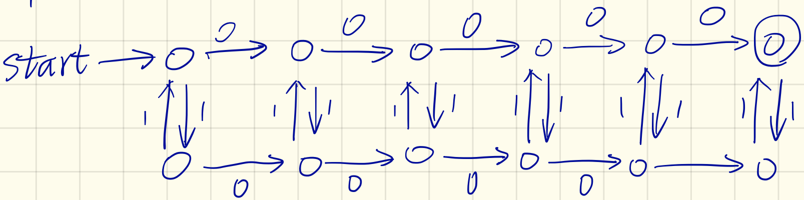
b) The set of all strings with an equal number of 0's and 1's, such that no prefix has two more 0's than 1's, nor two more 1's than 0's.

$(01+10)^*$

c) The set of all strings of 0's and 1's whose number of 0's is divisible by five and whose number of 1's is even.

c) The set of all strings of 0's and 1's whose number of 0's is divisible by five and whose number of 1's is even.

DFA.



5个0和奇数个1组成的串.

00000

$(11)^*$

$(11)^*0(11)^*0(11)^*0(11)^*0(11)^*0(11)^* = A$

$(1^*01^*01^*01^*01^*01^*)$

https://math.stackexchange.com/questions/1665392/regular-expression-for-strings-with-even-number-of-1s-and-number-of-0s-divisib

[redacted]

1 Answer

active oldest votes

2

There probably *isn't* a good way to construct a regular expression for this.

Of course you can always fall back to the systematic algorithm for converting DFAs to regexes, which leads to a horribly huge expression.

As a semi-systematic method, you can start by creating regexes A for "strings with exactly five 0s and an even number of 1s" and B for "strings with exactly five 0s and an odd number of 1s", and then put them together as

$$(A \mid BA^*B)^* \mid (11)^*$$

Somewhat shorter (but more complex to argue for) would be to let P and Q be regexes for strings with exactly five 0s and even/odd numbers of 1s *that both begin and end with* 0, and put them together as

$$\left(P \mid (1 \mid Q)P^*(1 \mid Q)\right)^*$$

share cite improve this answer

edited Feb 21 '16 at 11:45

answered Feb 21 '16 at 11:38

Henning Makholm

219k ● 14 ■ 281 ▲ 507

add a comment

even after they became useless?
Why aren't Americans simply ca
How do I convince my parents t best in class?
Could the average human surviv thrown back in the wilderness?
Why would a tribe of humans co consistently taller?
Can just anyone control a Broom
more hot questions
question feed

Exercise 3.1.4

Give English descriptions of the languages of the following regular expressions:

a) $(1+\varepsilon)(00^*1)^*0^*$

Exercise 3.1.4

Give English descriptions of the languages of the following regular expressions:

a) $(1+\varepsilon)(00^*1)^*0^*$ $(1+\varepsilon)(0+01)^*$

没有连续的 1

Exercise 3.1.4

Give English descriptions of the languages of the following regular expressions:

a) $(1+\varepsilon)(00^*1)^*0^*$

没有连续的 1

b) $(0^*1^*)^*000(0+1)^*$

$\Updownarrow$

$(0+1)^*000(0+1)^*$

Exercise 3.1.4

Give English descriptions of the languages of the following regular expressions:

a) $(1+\varepsilon)(00^*1)^*0^*$

没有连续的 1

b) $(0^*1^*)^*000(0+1)^*$

有 3 个连续 0 的串

Exercise 3.1.4

Give English descriptions of the languages of the following regular expressions:

a) $(1+\varepsilon)(00^*1)^*0^*$

没有连续的 1

b) $(0^*1^*)^*000(0+1)^*$

有 3 个连续 0 的串

c) $(0+10)^*1^*$

Exercise 3.1.4

Give English descriptions of the languages of the following regular expressions:

a) $(1+\varepsilon)(00^*1)^*0^*$

没有连续的 1

b) $(0^*1^*)^*000(0+1)^*$

有 3 个连续 0 的串

c) $(0+10)^*1^*$

任何连续 1 以后没有 0

Exercise 4.1.2

Prove that the following are not regular languages.

d) The set of strings of 0's and 1's whose length is a perfect square.

$w = 0^{N^2}$ pump lemma

Exercise 4.1.2

Prove that the following are not regular languages.

d) The set of strings of 0's and 1's whose length is a perfect square.

取 $w = 0^{N^2}$

Exercise 4.1.2

Prove that the following are not regular languages.

d) The set of strings of 0's and 1's whose length is a perfect square.

取 $w = 0^{N^2}$

e) The set of strings of 0's and 1's that are of the form ww, that is some string repeated.

$0^N 1^N 0^N 1^N$

Exercise 4.1.2

Prove that the following are not regular languages.

d) The set of strings of 0's and 1's whose length is a perfect square.

取 $w = 0^{N^2}$

e) The set of strings of 0's and 1's that are of the form ww, that is some string repeated.

取 $w = 0^N 1 0^N 1$

Exercise 4.2.2

If L is a language, and a is a symbol, then L/a, the quotient of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if L is regular, so is L/a. Hint: Start with a DFA for L and consider the set of accepting states.

Exercise 4.2.2

If L is a language, and a is a symbol, then L/a, the quotient of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if L is regular, so is L/a. Hint: Start with a DFA for L and consider the set of accepting states.

令 $L = L(M)$, 其中 $M = (Q, \Sigma, \delta, q_0, F)$

构造 $M' = (Q, \Sigma, \delta, q_0, F')$，其中 $F' = \{q \mid \delta(q, a) \in F\}$, $q \in Q$, $a \in \Sigma$. 先证明 $L(M') = L/a$, 再说明 $L(M')$ 正则

$\because \forall w \in L(M')$ 即 $\delta(q_0, w) \in F'$ 即 $\delta(\delta(q_0, w), a) \in F$, $\therefore w \in L/a$ 又 $\because \forall w \in L/a$ 有 $wa \in L$ 即 $\delta(q_0, wa) \in F$ 即 $\delta(\delta(q_0, w), a) \in F$ 即 $\delta(q_0, w) \in F'$ $\therefore w \in L(M')$

Exercise 4.2.6 a)

Show that the regular languages are closed under the following operations:

$\min(L) = \{w \mid w$ is in L, but no proper prefix of w is in $L\}$.

Exercise 4.2.6 a)

min(L) = $\{w \mid w$ is in L, but no proper prefix of w is in $L\}$.

由 $M = (Q, \Sigma, \delta, q_0, F)$ 构造 $M' = (Q, \Sigma, \delta', q_0, F)$ 其中

$$\delta'(q, a) = \begin{cases} \delta(q, a) & \text{if } q \notin F \\ \varnothing & \text{if } q \in F \end{cases} \tag{1}$$

证明 $L(M') = \min(L)$

1° $\forall w \in L(M')$ 存在转移序列 $q_0 q_1 \cdots q_n \in F$ 使 M' 接受 w 其中 $q_i \notin F, 0 \le i \le n-1$ $\therefore w \in \min(L)$

2° $\forall w \in \min(L)$ 有 $w \in L$，如果 M 接受 w 的状态序列为 $q_0 q_1 \cdots q_n \in F$ 则显然 $q_i \notin F, 0 \le i \le n-1$(因为否则，$w$ 有 L 可接受的前缀) $\therefore w \in L(M')$

Exercise 4.2.6 a)

$\min(L) = \{ w \mid w$ is in L, but no proper prefix of w is in $L\}$.

用封闭性证明

$$\min(L) = L - L\Sigma^{+}$$

Exercise 4.2.6 b)

$\max(L) = \{ \ w \mid w$ is in L and for no x other than ε is wx in $L\}$

Exercise 4.2.6 b)

max(L) $= \{\ w \mid w$ is in L and for no x other than ε is wx in $L\}$

由 $M = (Q, \Sigma, \delta, q_0, F)$ 构造 $M' = (Q, \Sigma, \delta, q_0, F')$ 其中

$$F' = \{f \mid f \in F, \forall x \in \Sigma^+, \hat{\delta}(f, x) \notin F\}$$

则 $L(M') = \max(L)$

Exercise 4.2.6 b)

max(L) = $\{\ w \mid w$ is in L and for no x other than ε is wx in $L\}$

利用封闭性。如果 $\Sigma = \{a, b, \cdots\}$，设 $\Gamma = \{a, \hat{a}, b, \hat{b}, \cdots\}$，定义同态 $h\ (\Gamma \to \Sigma^*)$ 和 $g\ (\Gamma \to \Sigma^*)$：

$$\begin{array}{ll} h(a) = a & g(a) = a \\ h(\hat{a}) = a & g(\hat{a}) = \varepsilon \\ h(b) = b & g(b) = b \\ h(\hat{b}) = b & g(\hat{b}) = \varepsilon \end{array}$$

那么

$$\max(L) = L - g(h^{-1}(L) \cap (a+b)^*(\hat{a}+\hat{b})^+)$$

Exercise 4.2.6 c)

init(L)={ w | for some x, wx is in L }

用同样的同态 h 和 g，则

$$\text{init}(L) = g(h^{-1}(L) \cap (a+b)^*(\hat{a}+\hat{b})^*)$$

Exercise 4.2.6 c)

init(L)=$\{\ w \mid$ for some x, wx is in $L\ \}$

由 $M = (Q, \Sigma, \delta, q_0, F)$ 构造 $M' = (Q, \Sigma, \delta, q_0, Q - Q')$ 其中

$Q' = \{\ q \mid q \in Q$, 没有从 q 到终态的路径 $\}$.

$$q \in Q - Q' \ \Leftrightarrow\ \exists x,\ \hat{\delta}(q, x) \in F$$

$$\forall w \in \Sigma^*, \hat{\delta}(q_0, w) \in Q - Q' \Leftrightarrow \exists x,\ \hat{\delta}(\hat{\delta}(q_0, w), x) \in F$$

即 $L(M') = \mathrm{init}(L)$.

Exercise 5.1.3

Show that every regular laugnage is a context-free laugnage.
Hint: Construct a CFG by induction on the number of operators in the regular expression.

Exercise 5.1.3

Show that every regular laugnage is a context-free laugnage.
Hint: Construct a CFG by induction on the number of operators in the regular expression. $(1+\varepsilon)(0+111^*)^*(1+\varepsilon)$

证明：对正则表达式 R 中运算符的个数 n 进行归纳。(1) 当 $n=0$ 时，R 只能是 ε, $\emptyset$ 或 a ($a \in \Sigma$)，可以构造仅有一条产生式的文法 $S \to \varepsilon$，$S \to \emptyset$ 或 $S \to a$ 得到。(2) 假设当 $n \leq m$ 时成立，当 $n = m+1$ 时，R 的形式只能由表达式 R_1 和 R_2 由连接、并或闭包形成： $S \to ABA$ $D \to 11E$

$A \to 1 | \varepsilon$ $E \to E1 | \varepsilon$

(i) 若 $R = R_1 + R_2$，则 R_1 和 R_2 中运算符都不超过 m，所以都存在文法 G_1 和 G_2，分别开始于 S_1 和 S_2，只需构造新产生式和开始符号 $S \to S_1 \mid S_2$，连同 G_1 和 G_2 的产生式，构成 R 的文法； $B \to BC | \varepsilon$

(ii) 若 $R = R_1R_2$，则同理构造 $S \to S_1S_2$ 即可；

(iii) 若 $R = R_1^*$，则构造 $S \to SS_1 \mid \varepsilon$ 即可。 $C \to 0 | D$

Exercise 7.1.7

Suppose G is a CFG with p productions, and no production body longer than n. Show that if $A \overset{*}{\underset{G}{\Rightarrow}} \varepsilon$, then there is a derivation of ε from A of no more than $(n^p - 1)/(n - 1)$ steps. How close can you actually come to this bound?

Exercise 7.1.7

Suppose G is a CFG with p productions, and no production body longer than n. Show that if $A \overset{*}{\underset{G}{\Rightarrow}} \varepsilon$, then there is a derivation of ε from A of no more than $(n^p - 1)/(n - 1)$ steps. How close can you actually come to this bound?

取 $A \overset{*}{\underset{G}{\Rightarrow}} \varepsilon$ 节点数最少的派生树，则任何从根节点到叶子的路径长度不超过 $p-1$。因为否则会有重复变元，可以将重复变元之间的节点去掉，得到节点数更少的派生树。即树的高度最多为 $p-1$，且第 k 层的内节点，最多为 n^k 个，因为产生式右部最长为 n。所以整个树的内节点数最多为 $1+n+n^2+\cdots+n^{p-1}=(n^p-1)/(n-1)$，而内节点数与推导的次数相等。