CS 181 EXAM #2 **NAME** ______________________________

SPRING 2015 **UCLA ID** ______________________________

You have 90 minutes to complete this exam. You may assume without proof any statement proved in class.

1 Find a regular expression for each of the following languages over $\{0, 1\}$:

(1 pt) **a.** nonempty strings in which the first and last symbols are different;

(1 pt) **b.** strings in which the number of 0s is even;

(2 pts) **c.** strings not containing the substring 01;

(3 pts) **d.** strings in which the number of 0s and the number of 1s are either both even or both odd.

Solution.

a. $0\Sigma^*1 \cup 1\Sigma^*0$

b. $1^*(01^*01^*)^*$

c. 1^*0^*

d. $(\Sigma\Sigma)^*$

1 Find a regular expression for each of the following languages over $\{0, 1\}$:

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d. strings in which the number of 0s and the number of 1s are either both even or both odd.

a. $1(0+1)^*0 + 0(0+1)^*1$

b. $(01^*0+1)^*$

c. 1^*0^*

d. $((0+1)(0+1))^*$

0/OD

2 Prove or disprove:

(2 pts) **a.** for any regular languages $L_1 \subseteq L_2 \subseteq \ldots \subseteq L_n \subseteq \ldots$, the union $\bigcup_{n=1}^{\infty} L_n$ is regular;

(3 pts) **b.** if L_1 and L_2 are two languages such that the equivalence classes of $\equiv_{L_1}$ are exactly the same as those of $\equiv_{L_2}$, then $L_1 = L_2$.

Solution.

a. False. Let $L_n = 01 \cup 0011 \cup \cdots \cup 0^n 1^n$. Then each L_n is finite and hence regular, whereas their union is the language $\{0^n 1^n : n \geqslant 1\}$, which was shown in class to be nonregular.

b. False. The languages $L_1 = \varnothing$ and $L_2 = \Sigma^*$ have the same set of equivalence classes, namely, a single class Σ^*.

(3 pts) **3** Construct a language that can be recognized by a DFA with 2015 states but not with 2014 states. Prove both claims.

Solution. Let L be the language of binary strings whose length is a multiple of 2015. Then L is recognized by the following DFA with 2015 states: $(\{0, 1, 2, \ldots, 2014\}, \{0, 1\}, \delta, 0, \{0\})$, where $\delta(q, \sigma) = (q + \sigma) \bmod 2015$. We will now show that no smaller DFA exists. For any distinct $i, j \in \{0, 1, 2, \ldots, 2014\}$, we have $0^i \mathbf{0^{2015-i}} \in L$ and $0^j \mathbf{0^{2015-i}} \notin L$. As a result, the 2015 strings $\varepsilon, 0, 00, 000, \ldots, 0^{2014}$ are each in a different equivalence class of $\equiv_L$. By the Myhill-Nerode theorem, any DFA for L must have at least 2015 states.

4 For each of the following languages, determine whether it is regular, and prove your answer:

(2 pts) **a.** binary strings with five times as many 0s as 1s; $1^N 0^{5N}$

(2 pts) **b.** binary strings of the form uvu, where u and v are nonempty strings;

0^*

(3 pts) **c.** strings over the decimal alphabet $\{0, 1, 2, \ldots, 9\}$ with characters in sorted order;

(3 pts) **d.** binary strings such that in every suffix, the number of 0s and the number of 1s differ by at most 2.

Solution.

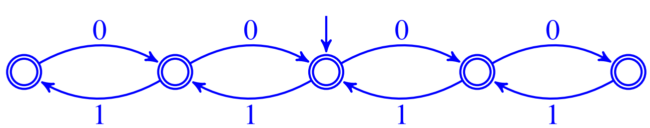
In each part, L stands for the language in question.

a. Nonregular. Let p be arbitrary, and consider the string $0^{5p}1^p$. Let x, y, z be any strings such that y is nonempty, $|xy| \leqslant p$, and $xyz = 0^{5p}1^p$. Then y is a nonempty string of 0s, and therefore the number of 0s in xz is less than five times the number of 1s. Since $xz \notin L$, the language is nonregular by (the contrapositive of) the pumping lemma.

b. Nonregular. We claim that the infinite collection of strings $\varepsilon, 0, 00, \ldots, 0^n, \ldots$ are each in a different equivalence class of $\equiv_L$. Indeed, for $n < N$, the language contains $0^n\mathbf{10}^N\mathbf{1}$ but not $0^N\mathbf{10}^N\mathbf{1}$. By the Myhill-Nerode theorem, L is nonregular.

c. Regular. This language is given by the regular expression $0^*1^*2^*3^* \cdots 9^*$.

d. Regular. Observe that L^R, the reverse of L, is the language of strings with the property that in every *prefix*, the number of 0s and the number of 1s differ by at most 2. This language is regular because it is recognized by the following NFA:



Since regular languages are closed under the reverse operation, L must be regular as well.