1. Design a PDA to accept the set of all strings of 0's and 1's such that no prefix has more 1's than 0's.

2. Design a PDA to accept the language $L = \{0^n1^m \mid 0 < n < m < 2n\}$.

3. Use CFL pumping lemma to show that $L = \{a^ib^jc^k \mid i < j < k\}$ is not a context-free language.

4. Design a Turing machine for the language $L = \{w \mid w \in \{0,1\}^*, w = w^R\}$.

1、宫栈

start $\longrightarrow$ ◯

$0, z_0/0z_0$
$0, 0/00$
$1, 0/\varepsilon$
$\varepsilon, 0/\varepsilon$
$\varepsilon, z_0/\varepsilon$

2. 设计 CFG. G为 $S \rightarrow 0S1 \mid 0S11 \mid 00111$

由此可以构造PDA $P = (\{q\}, \{0,1\}, \{0,1,S\}, \delta, q, S, \phi)$.

其中 $\delta(q, \varepsilon, S) = \{(q, 0S1), (q, 0S11), (q, 00111)\}$.

$\delta(q, 0, 0) = \{(q, \varepsilon)\}$

$\delta(q, 1, 1) = \{(q, \varepsilon)\}$.

3. 假设L是CFL. 则由泵引理可有 N.

取 $\delta = a^{2N} b^{2N+1} c^{2N+2}$, $|\delta| > N$ 且 $\delta = uvwxy$

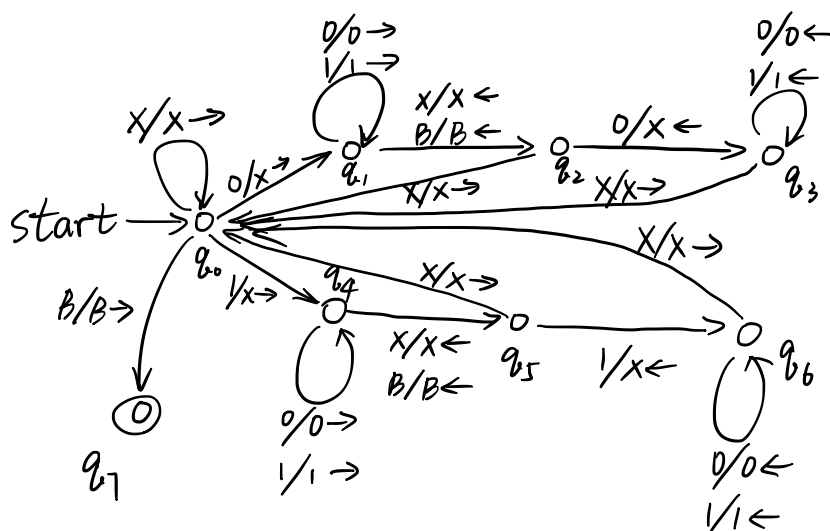其中 $|vwx| \leq N$ 且 $vx \neq \varepsilon$. 则 vwx 仅有以下情况

① vwx 包含 δ 的中点. 则 vwx 仅由b组成

从而 $uv^0wx^0y \notin L$

② vwx 在中点左侧. 则 vwx 由a和b或仅由b组成

从而 $uv^2wx^2y \notin L$

③ vwx 在中点右侧. 则 vwx 由b和c或仅由c组成

从而 $uv^0wx^0y \notin L$

综上假设不成立. 即 L 不是 CFL.

4.

1. Design a PDA to accept the set of all strings of 0's and l's such that no prefix has more l's than 0's.

$$a^{2N} b^{2N+1} c^{2N+2}$$

$$\underbrace{a\,a\,a\cdots a\,a}_{\geq N}\underbrace{b\,b\cdots b}_{N+1}\underbrace{b\,b\,b\cdots bb}_{N-1}\underbrace{c\,c\cdots c}_{2N+2}$$

$$\boxed{6N+3 \atop 3N+\frac{3}{2}} \quad .$$

$$\boxed{2N+2} \quad . \quad b.$$

2. Design a PDA to accept the language $L = \{0^n 1^m \mid 0 < n < m < 2n\}$.

vwx 包含中点，则 } 只能由 b 组成 从而 $uv^0wx^0y \notin L$

vwx 在中点左侧. 包含 a 和 b 或仅包含 b.

有 $uv^2wx^2y \notin L$　a

vwx 在中点右侧 (包含 b 和 c)

有 $uv^0wx^0y \notin L$.

3. Use CFL pumping lemma to show that $L = \{a^i b^j c^k \mid i < j < k\}$ is not a context-free language.

假设满足泵引理、有 N.

$\forall w \in L. |w| \geq N$ 满足泵引理.

取 $w = a^{2N} b^{2N+1} c^{N+2}$　w 可分为 uvwxy　$|vwx| \leq N$

$\underbrace{\quad}_{vwx}$　且 $vx \neq \varepsilon$

只包含 a, 或 b 或 c.

4. Design a Turing machine for the language $L = \{w \mid w \in \{0,1\}^*, w = w^R\}$.

$$\underbrace{a\,a\,a\cdots a}_{N}\,\overbrace{b\,b\,b\cdots b}^{\leftarrow N-1 \rightarrow}\bigm|\underbrace{c\,c\,c\cdots c}_{N+2}$$

中间 b 组标注 $N+1$

如 vwx 在 中点左侧 则有 $uv^2wx^2y \notin L$.

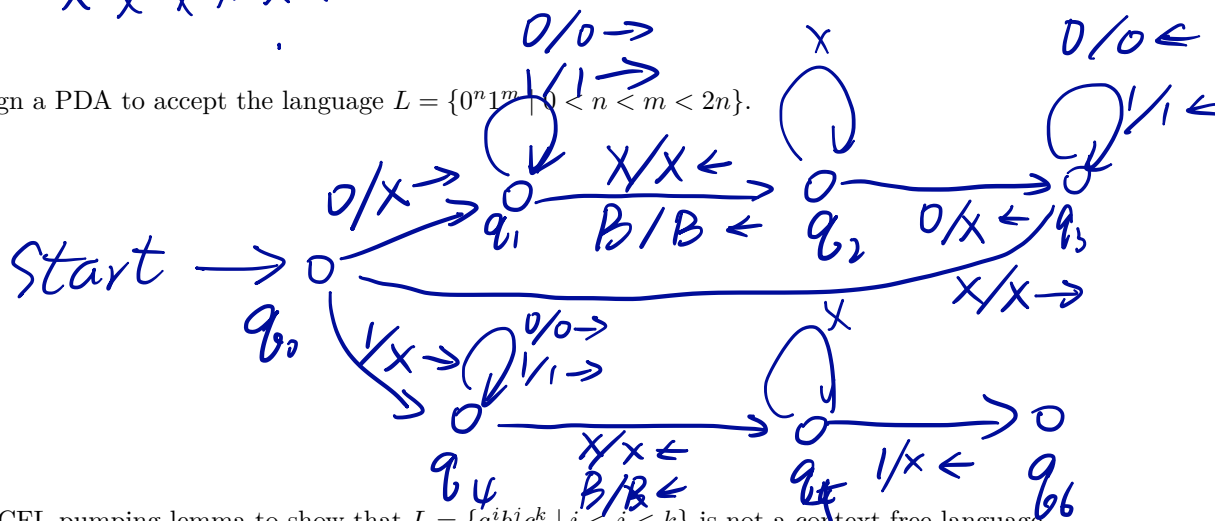如 vwx 在中点右侧. 则有 $uv^0wx^0y \notin L$

如 vwx 包含中点. 则有 $uv^0wx^0y \notin L$.

1. Design a PDA to accept the set of all strings of 0's and 1's such that no prefix has more 1's than 0's.

Ev. 11　101　1010

$\checkmark$　$\checkmark$　$\times$

$B\,001100\,B$

$\times\ \times\ \times\ \times\ \times\ \times$

2. Design a PDA to accept the language $L = \{0^n 1^m \mid 0 < n < m < 2n\}$.

$0/0\rightarrow$
$1/1\rightarrow$
X
$0/0\leftarrow$
$1/1\leftarrow$

$0/X\rightarrow$　$X/X\leftarrow$　$0/X\leftarrow$　$X/X\rightarrow$

Start $\rightarrow$ q_0　q_1　$B/B\leftarrow$　q_2　q_3

$1/X\rightarrow$　$0/0\rightarrow$　$1/1\rightarrow$

q_4　$X/X\leftarrow$　$B/B\leftarrow$　q_5　$1/X\leftarrow$　q_6

3. Use CFL pumping lemma to show that $L = \{a^i b^j c^k \mid i < j < k\}$ is not a context-free language.

$B\,010\,B$　　　0100

$\times\times\times$

$B1B$　　$[0]$

$\uparrow$

$B/1\,011\,B$

$\times\ \times\ \times\times$

4. Design a Turing machine for the language $L = \{w \mid w \in \{0,1\}^*, w = w^R\}$.

$0/0\rightarrow$
$1/1\rightarrow$
$X/X\leftarrow$
$B/B\leftarrow$
$0/X\leftarrow$
$0/0\leftarrow$
$1/1\leftarrow$

$X/X\rightarrow$

$0/X\rightarrow$

Start $\rightarrow$

$X/X\rightarrow$

B/B　$1/X\rightarrow$

$X/X\rightarrow$　$X/X\rightarrow$

$X/X\leftarrow$　$1/X\leftarrow$

$B/B\leftarrow$

$0/0\rightarrow$
$1/1\rightarrow$

$0/0\leftarrow$
$1/1\leftarrow$

Top diagram:

0/0→
1/1→
X/X→
0/X→
X/X←
B/B←
X/X→
0/X←
0/0←
1/1←
start→
X/X→
X/X→
X/X→
B/B
1/X→
X/X→
X/X→
X/X←
B/B←
1/X←
0/0→
1/1→
0/0←
1/1←

Bottom diagram:

0/0→
1/1→
X/X→
0/X→
X/X←
B/B←
0/0←
1/1←
start→
q_1
q_2
X/X→
0/X←
q_3
X/X→
q_0
1/X→
q_4
X/X→
X/X←
B/B←
q_5
1/X←
q_6
B/B→
0/0→
1/1→
0/0←
1/1←
q_7