Homework 2 学号 姓名

1. Give regular expressions for the following languages.

   i) The set of all strings with an equal number of 0's and l's, such that no prefix has two more 0's than l's, nor two more l's than 0's.

   前缀.

   $(01+10)^*$

   ii) $L = \{a^n b^m : n < 4, m \leq 3\}$.

   $(a+\varepsilon)^3(b+\varepsilon)^3$

2. Prove $L = \{0^n | n$ is a perfect square$\}$ is not regular.

   $\{0^n | n$是完全平方数$\}$ 泵引理.

   $\{0^{k^2} | k>0\}$ →

3. If L is a language, and a is a symbol, then L/a, the quotient of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if L is regular, so is L/a. Hint: Start with a DFA for L and consider the set of accepting states.

   $L.\ A=(Q,\Sigma,\delta,q_0,F)$

   $\hat{\delta}(q_0, wa) \in F$ 有 $\delta(\hat{\delta}(q_0,w),a) \in F$.

   则 $q = \hat{\delta}(q_0,w)$

   $\forall q = \hat{\delta}(q_0,w)$. 有 $\delta(q,a) \in F$.

   $B=(Q,\Sigma,\delta,q_0,F')$.

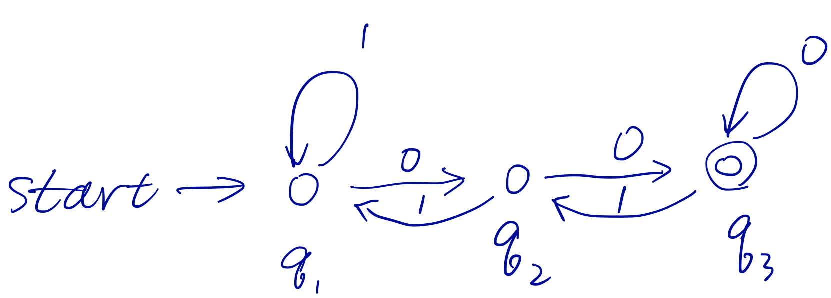
4. Here is a transition table for a DFA:

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

   a) Give all the regular expressions $R_{ij}^{(0)}$, $R_{ij}^{(1)}$ and $R_{ij}^{(2)}$. Try to simplify the expressions as much as possible. Note: Think of state q_i as if it were the state with integer number i.

   b) Give a regular expression for the language of the automaton.

   00+



3. If L is a language, and a is a symbol, then L/a, the quotient of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if L is regular, so is L/a. Hint: Start with a DFA for L and consider the set of accepting states.

证明：令 $L=L(M)$. 其中 $M=(Q,\Sigma,\delta,q_0,F)$.

构造 $M'=(Q,\Sigma,\delta,q_0,F')$

其中 $F'=\{q \mid q\in Q$ 且 $\delta(q,a)\in F$ 且 $a\in\Sigma\}$.

下证 $L(M')$ 为 L/a 语言.

$\forall w\in L(M')$ 有 $\hat{\delta}(q_0,w)\in F'$ 即 $\delta(\hat{\delta}(q_0,w),a)\in F$

$\therefore w\in L/a$

再证 $L(M')$ 是正则的.

$\forall w\in L/a$. 有 $wa\in L$. 有 $\hat{\delta}(q_0,wa)\in F$.

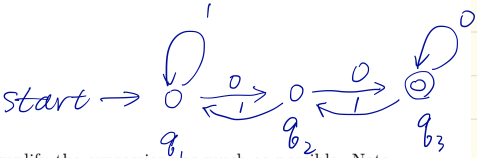
即 $\delta(\hat{\delta}(q_0,w),a)\in F$. 从而有 $\hat{\delta}(q_0,w)\in F'$ $w\in L(M')$

故 L 为正则的 $\therefore L(M')$ 是正则的. 从而 L/a 正则.

4. Here is a transition table for a DFA:

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

DFA → RE



a) Give all the regular expressions $R_{ij}^{(0)}$, $R_{ij}^{(1)}$ and $R_{ij}^{(2)}$. Try to simplify the expressions as much as possible. Note: Think of state q_i as if it were the state with integer number i.

b) Give a regular expression for the language of the automaton.

复习公式

$$\begin{cases} R_{ij}^{k} = R_{ij}^{k-1} \cup R_{ik}^{k-1}(R_{kk}^{k-1})^* R_{kj}^{k-1} \\ R_{ij}^{0} = \begin{cases} \{a \mid \delta(q_i, a) = q_j\}, & i \neq j \\ \{a \mid \delta(q_i, a) = q_j\} \cup \{\varepsilon\}. & i = j \end{cases} \end{cases}$$

进而

	$k=0$	$k=1$	$k=2$	
R_{11}^k	$\varepsilon+1$	1^*	$1^*(\varepsilon+0(11^*0)^*11^*)$	$(1+01)^*$
R_{12}^k	0	1^*0	$1^*0(11^*0)^* \overset{?}{=} (1^*0)^* = (1+01)^*0$	
R_{13}^k	ϕ	ϕ	$1^*0(11^*0)^*0$	$= (1+01)^*00$
R_{21}^k	1	11^*	$(11^*0)^*11^*$	
R_{22}^k	ε	$\varepsilon+11^*0$	$(11^*0)^*$	
R_{23}^k	0	0	$(11^*0)^*0$	
R_{31}^k	ϕ	ϕ	$1(11^*+0)^*11^*$	
R_{32}^k	1	1	$1(11^*0)^*$	
R_{33}^k	$\varepsilon+0$	$\varepsilon+0$	$\varepsilon+0+1(11^*0)^*0$	$\varepsilon+0+(1+01)^*00+10$

$$R_{13}^{3} = R_{13}^{2} + R_{13}^{2}(R_{33}^{2})^* R_{33}^{2}$$

$$= 1^*0(11^*0)^*0 + 1^*0(11^*0)^*0(\varepsilon+0+1(11^*0)^*0)^*(\varepsilon+\cdots$$

$$= 1^*0(11^*0)^*0(0+1(11^*0)^*0)^*$$

$$= (1+01)^*00(0+10+(1+01)^*00)^*$$