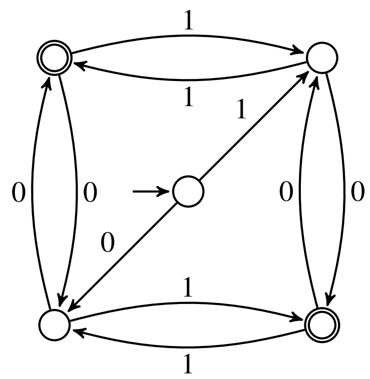
CS 181 EXAM #1 **NAME** ______________________________

SPRING 2015 **UCLA ID** ______________________________

You have 90 minutes to complete this exam. You may assume without proof any statement proved in class.

(3 pts) **1** Give a simple verbal description of the language recognized by the following DFA.

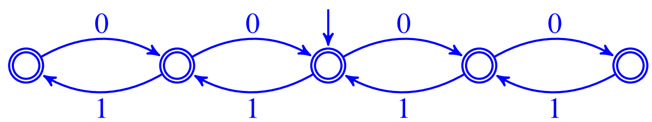


Solution. Nonempty strings of even length.

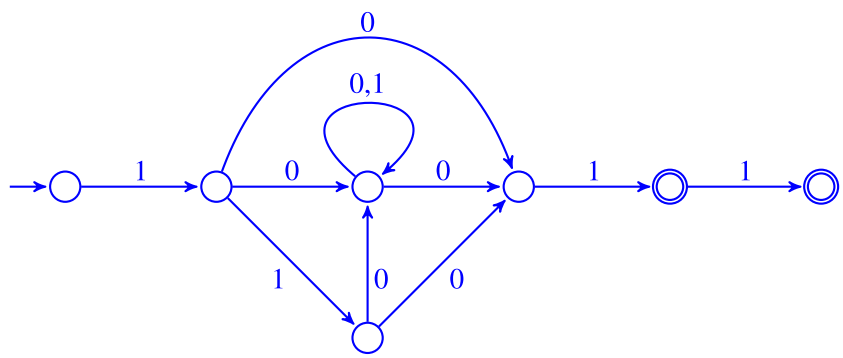
2 Draw NFAs for the following languages over $\{0, 1\}$, taking full advantage of nondeterminism:

(2 pts) **a.** strings such that in every prefix, the numbers of zeroes and ones differ by at most 2;

(2 pts) **b.** strings that begin with 10 or 110, and end with 01 or 011.



a.

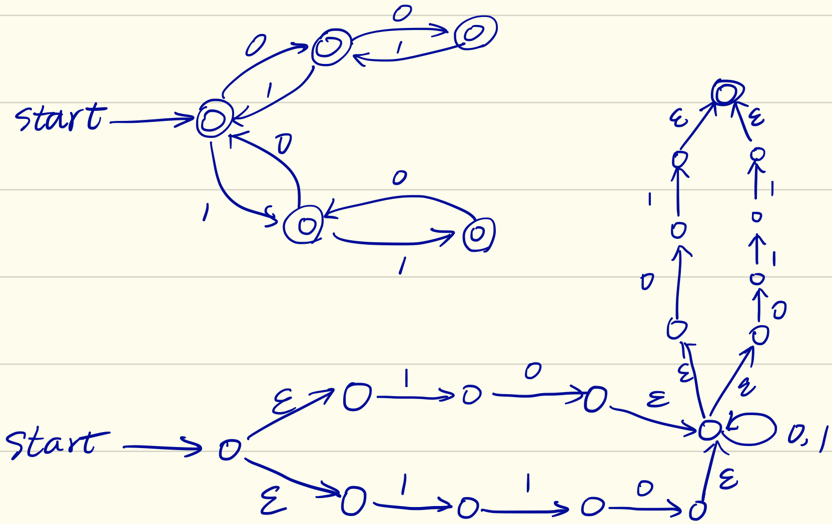


b.

2 Draw NFAs for the following languages over $\{0, 1\}$, taking full advantage of nondeterminism:

a. strings such that in every prefix, the numbers of zeroes and ones differ by at most 2;

b. strings that begin with 10 or 110, and end with 01 or 011.



3 Prove that the following languages over a given alphabet Σ are regular:

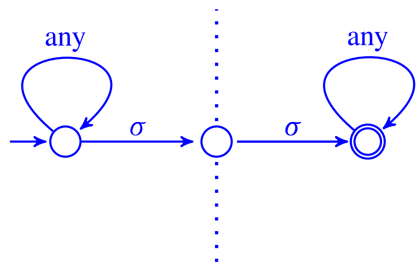
(2 pts) **a.** strings in which no pair of consecutive characters are identical;

(2 pts) **b.** binary strings in which every even-numbered character is a 0;

(2 pts) **c.** the language $\{3, 6, 9, 12, 15, 18, 21, \ldots\}$ over the decimal alphabet, corresponding to natural numbers that are divisible by 3.

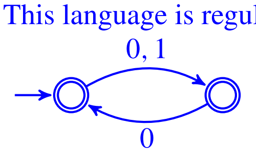
Solution.

a. Let L be the language in the problem statement. Then $\overline{L}$ is the set of all strings that contain a pair of consecutive characters that are identical, corresponding to the following NFA:



This diagram features a branch for each symbol $\sigma \in \Sigma$. Since $\overline{L}$ is regular and regular languages are closed under complement, L must be regular as well.

b. This language is regular because it is recognized by the following NFA:



c. Recall that an integer is divisible by 3 if and only if the sum of its digits is divisible by 3. This suggests the automaton $(\{0, 1, 2\}, \{0, 1, 2, \ldots, 9\}, \delta, 0, \{0\})$ where $\delta(q, \sigma) = (q + \sigma) \bmod 3$. This automaton *almost* works, except that it accepts syntactically invalid strings such as ϵ or 003. To fix this, modify the automaton to only accept strings that start with a nonzero digit: $(\{\epsilon, 0, 1, 2, \text{FAIL}\}, \{0, 1, 2, \ldots, 9\}, \delta, \epsilon, \{0\})$ where

$$\delta(q, \sigma) = \begin{cases} (q + \sigma) \bmod 3 & \text{if } q = 0, 1, 2, \\ \sigma \bmod 3 & \text{if } q = \epsilon \text{ and } \sigma \neq 0, \\ \text{FAIL} & \text{otherwise.} \end{cases}$$

(3 pts) **4** The symmetric difference of two languages L' and L'', denoted $L' \Delta L''$, is the set of strings that belong to L' or L'' but not both. Prove that regular languages are closed under symmetric difference.

Solution. Let L' and L'' be regular. By definition, $L' \Delta L'' = (L' \cap \overline{L''}) \cup (\overline{L'} \cap L'')$. Since regular languages are closed under complement, intersection, and union, it follows that $L' \Delta L''$ is regular.

(3 pts) **5** Prove or argue to the contrary: adding a finite number of strings to a regular language necessarily results in a regular language.

Solution. As shown in class, every finite language is regular. Since regular languages are closed under union, it follows that the union of a regular language with a finite language is regular.

(3 pts) **6** The *circular shift* of a language L is defined as $L^{\circlearrowleft} = \{uv : vu \in L\}$, where u and v stand for arbitrary strings. For example, $\{1234\}^{\circlearrowleft} = \{1234, 2341, 3412, 4123\}$. Prove that regular languages are closed under circular shift.

Let $D = (Q, \Sigma, \delta, q_0, F)$ be a DFA for L. Here is a nondeterministic procedure for deciding whether a given string w is in $L^{\circlearrowleft}$:

Stage 1. Choose a state $q \in Q$ nondeterministically.

Stage 2. Launch the DFA on the input string, starting in state q rather than q_0. No need to wait for the DFA to process all of w; whenever the DFA is in an accept state, you may choose to advance to the next stage.

Stage 3. Run the DFA on the unprocessed portion of w, starting in state q_0 and accepting if you end up in state q.

For any fixed $q \in Q$, stages 2 and 3 can be implemented as an NFA D_q which consists of two copies of the original DFA: the first copy has all states marked as rejecting and has ϵ-transitions added from any state in F to the state q_0 of the second copy, and the second copy has q marked as the only accept state. Now for each $q \in Q$, create a copy of that composed automaton D_q. To obtain an automaton for $L^{\circlearrowleft}$, it remains to introduce a new start state that has, for each $q \in Q$, an ϵ-transition to state q of the copy D_q.

(3 pts) **7** Describe an algorithm that takes as input a DFA and determines whether the automaton recognizes the empty language, $\varnothing$.

We can view a DFA as a directed graph, with the DFA's arrows and states corresponding to edges and vertices. The algorithm is simply to check if the graph contains a path from the start state to an accept state. This can be done either by trying out all candidate paths in a brute force manner, or by using an efficient graph algorithm you may have encountered in CS 180, such as depth- or breadth-first search.