

ch 4.6 正弦稳态电路的相量 分析法

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回顾

$$\sum i = 0$$

$$\sum \dot{I} = 0$$

$$\sum u = 0$$

$$\sum \dot{U} = 0$$

$$u = Ri$$

$$\dot{U} = R\dot{I}$$

$$u = L \frac{di}{dt}$$

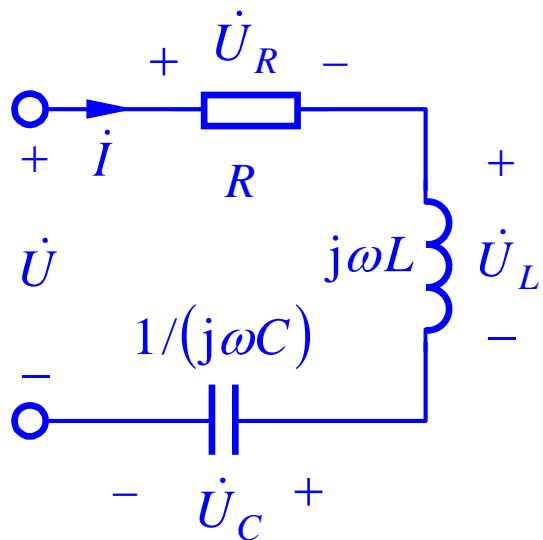
$$\dot{U} = j\omega L\dot{I} = jX_L\dot{I}$$

$$i = C \frac{du}{dt}$$

$$\dot{U} = \frac{1}{j\omega C} \dot{I}$$

$$\dot{U} = K\dot{I}$$

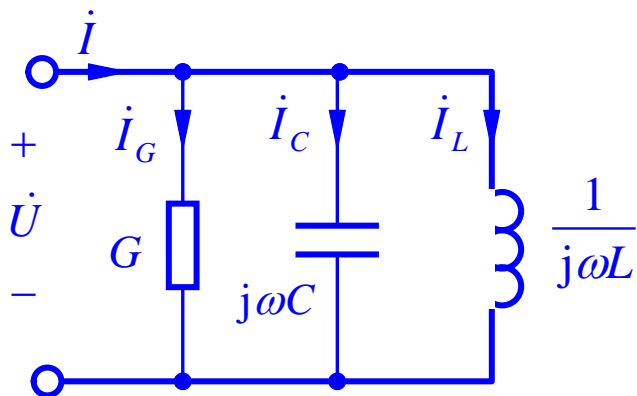
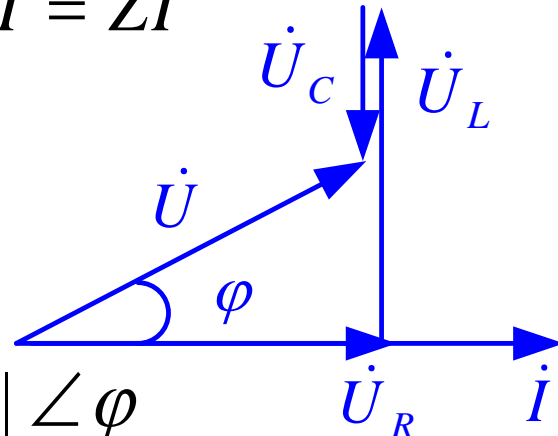
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$$\dot{U} = [R + j(\omega L - \frac{1}{\omega C})]\dot{I} = Z\dot{I}$$

$$Z = \frac{\dot{U}}{\dot{I}} = \frac{U \angle \psi_u}{I \angle \psi_i}$$

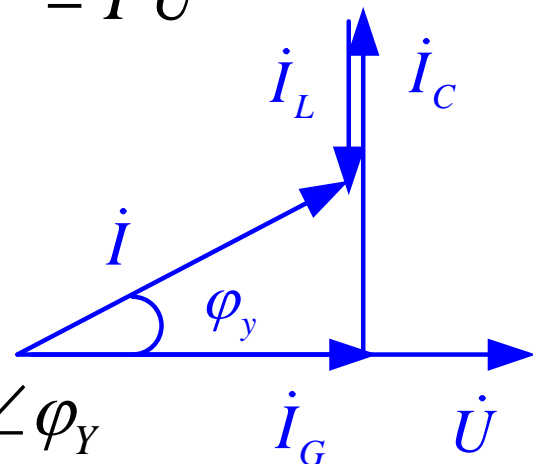
$$= \frac{U}{I} \angle (\psi_u - \psi_i) = |Z| \angle \varphi$$



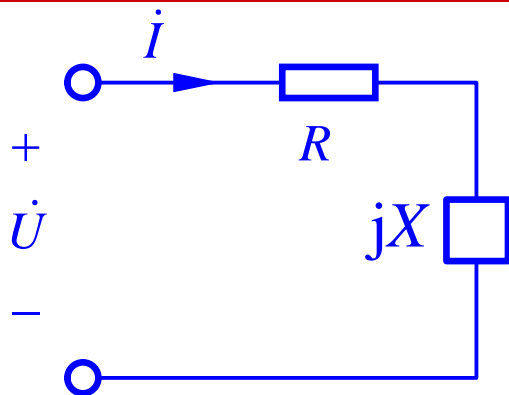
$$\dot{I} = [G + j(\omega C - \frac{1}{\omega L})]\dot{U} = Y\dot{U}$$

$$Y = \frac{\dot{I}}{\dot{U}} = \frac{I \angle \psi_i}{U \angle \psi_u}$$

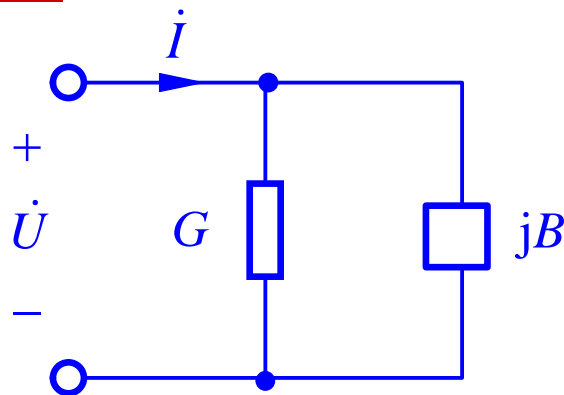
$$= \frac{I}{U} \angle (\psi_i - \psi_u) = |Y| \angle \varphi_Y$$



回顾



(a)



(b)

则

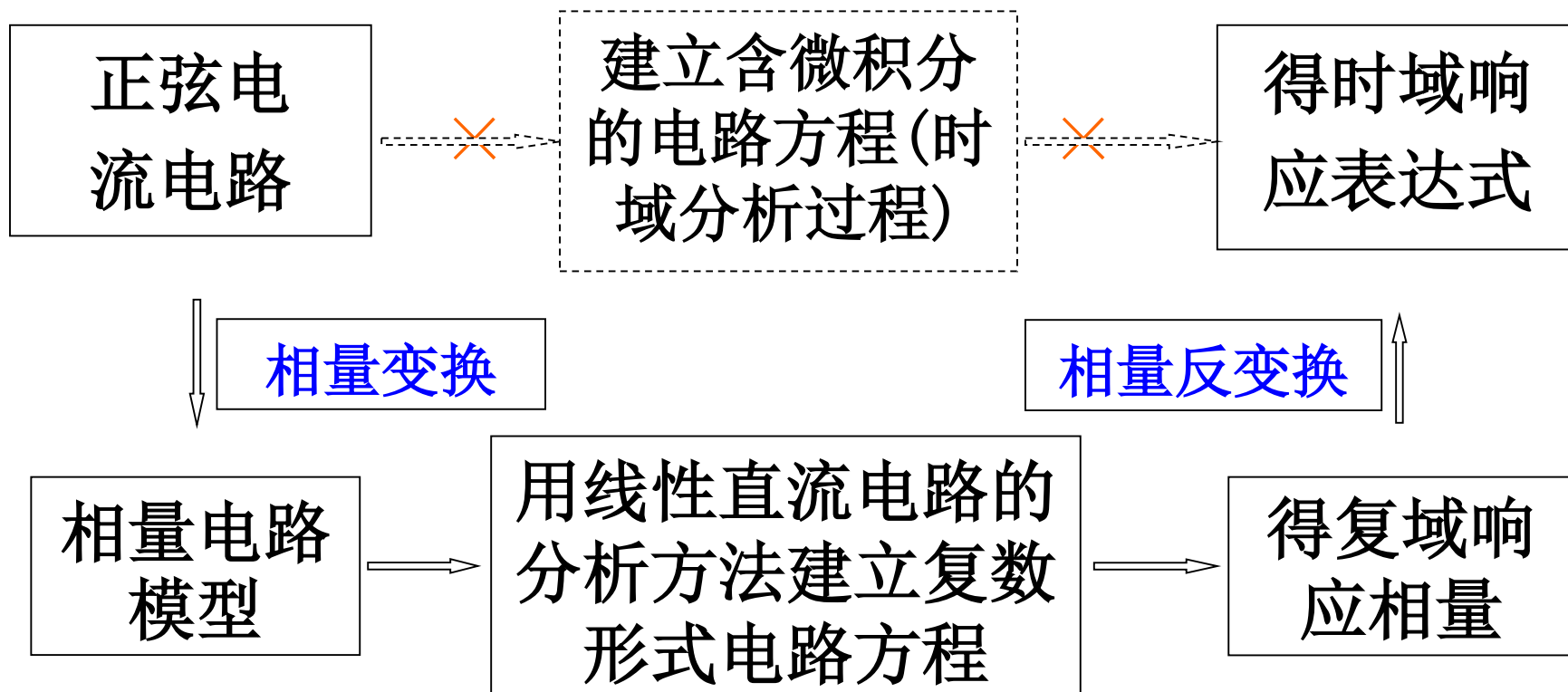
$$Z = R + jX$$
$$Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R}{R^2 + X^2} - j \frac{X}{R^2 + X^2} = G + jB$$
$$G = \frac{R}{R^2 + X^2} \neq \frac{1}{R} \qquad B = -\frac{X}{R^2 + X^2} \neq \frac{1}{X}$$

说明： Y 与 Z 等效是在某一频率下求出的，故等效的 Z 或 Y 与频率有关。

4.6 正弦稳态电路的相量分析法

基本要求：熟练掌握正弦电流电路相量分析法原理及步骤、电路方程和电路定理的相量形式。

正弦电流电路相量分析法原理示意



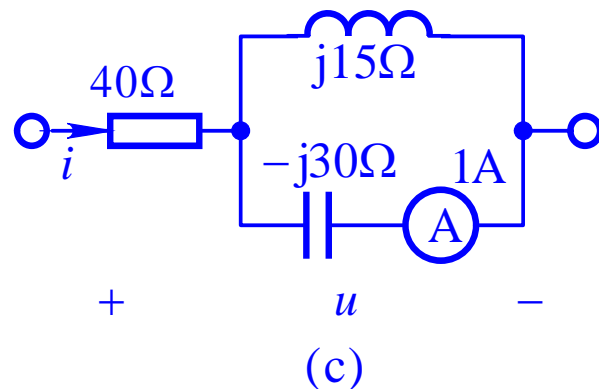
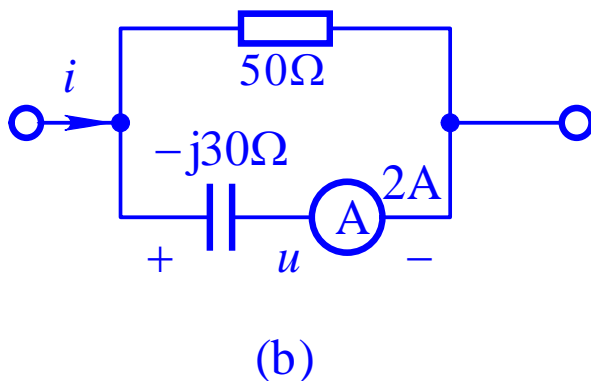
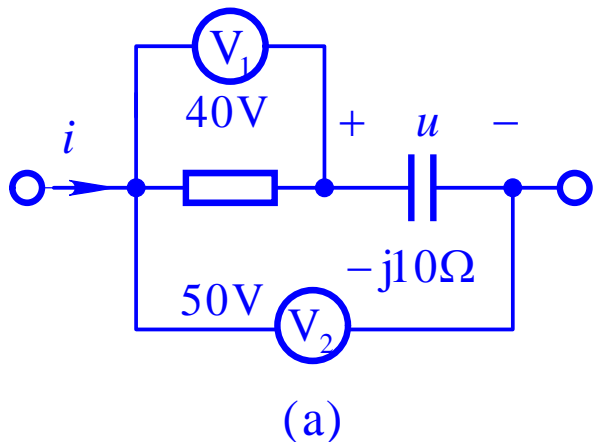
4.6 正弦稳态电路的相量分析法

相量分析法的求解步骤：

- (1) 选取参考相量（正弦量）；
- (2) 构造相量形式电路模型；
 - 1) 将激励和响应用相量形式表示；
 - 2) 计算 X_C 、 X_L ，或阻抗、导纳；
- (3) 按线性直流电路分析方法计算相量电路模型；
- (4) 将所得的电压、电流相量计算结果变换成正弦表达式。

[补充4.4]

图示各电路中已标明电压表和电流表的读数，试求电压 u 和电流 i 的有效值。



【解】

图(a): $\dot{U}_2 = \dot{U}_R + \dot{U}$

$$50\text{V} = \sqrt{(40\text{V})^2 + U^2}$$

$$U = \sqrt{(50\text{V})^2 - (40\text{V})^2} = 30\text{V} \quad I = I_C = \frac{U}{|X_C|} = \frac{30\text{V}}{10\Omega} = 3\text{A}$$

[补充4.4]

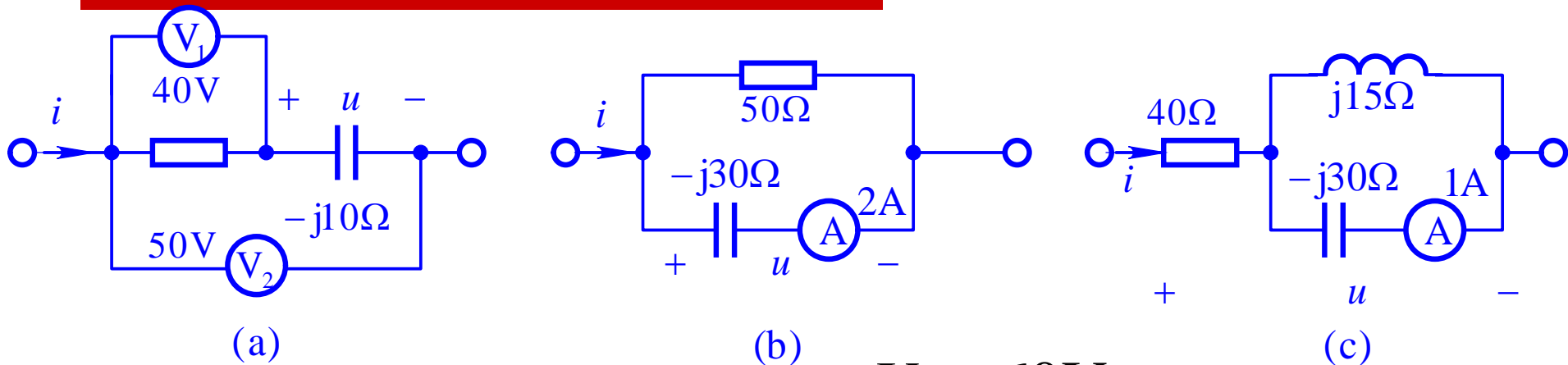


图 (b): $U = 30\Omega \times 2A = 60V$, $I_R = \frac{U}{R} = \frac{60V}{50\Omega} = 1.2A$

$$I = \sqrt{I_C^2 + I_R^2} = \sqrt{2^2 + 1.2^2} A = 2.33A$$

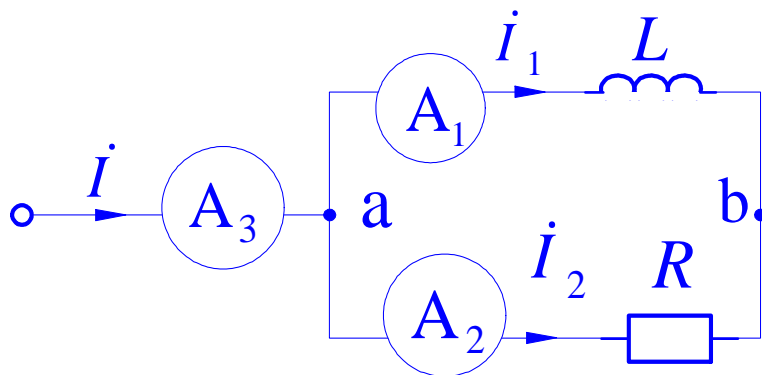
图(c): $U_C = |X_C| I_C = 30\Omega \times 1A = 30V$

$$U_L = U_C = X_L I \Rightarrow I_L = \frac{U_C}{X_L} = \frac{30V}{15\Omega} = 2A$$

$$I = |I_L - I_C| = 1A \quad U = \sqrt{U_C^2 + U_R^2} = \sqrt{30^2 + 40^2} V = 50V$$

[补充4.5]

已知表1的读数是5A， ωL 和 R 数值相等，求表2和表3的读数。



【解】

$$\dot{U}_{ab} = U_{ab} \angle 0^\circ$$

$$\frac{U_{ab}}{R} = \frac{U_{ab}}{\omega L} = 5\text{A}$$

$$\Rightarrow I_1 = I_2 = 5\text{A}$$

$$\dot{I}_1 = 5 \angle -90^\circ \text{ A}$$

L 上电流滞后电压 90°

$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$= -j5 + 5 = 5\sqrt{2} \angle -45^\circ \text{ A}$$

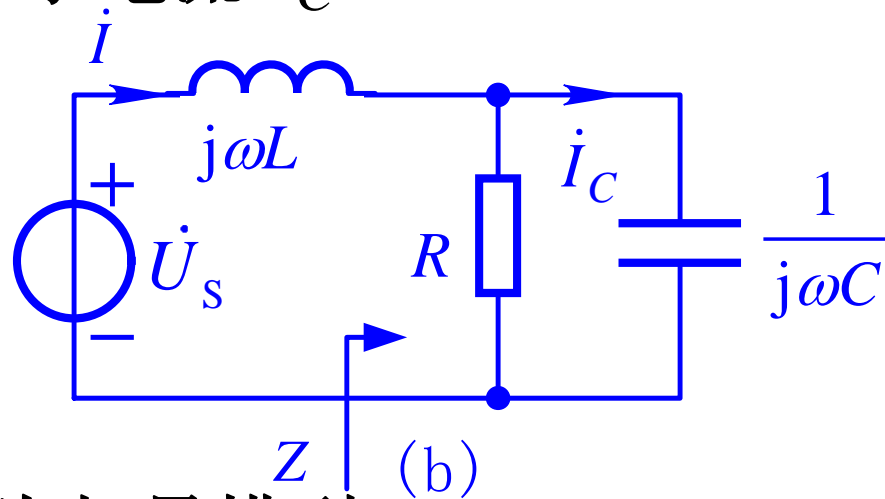
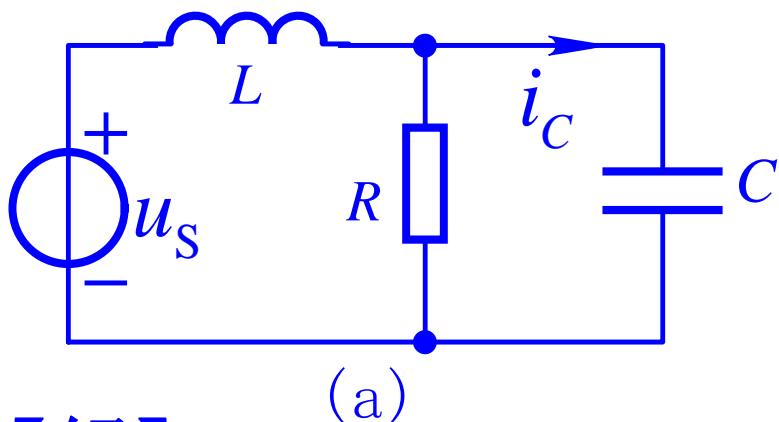
即 表2读数为5A，

表3读数为 $5\sqrt{2}$ A

注意：电流表读数均为有效值，有效值不满足KCL方程，而电流相量是满足KCL方程的。

[例4.9]

设图 (a) 电路 $u_s = 60\sqrt{2} \cos(\omega t + 45^\circ) \text{ V}$, $\omega = 100 \text{ rad/s}$, $C = 10^{-3} \text{ F}$, $R = 10 \Omega$, $L = 0.1 \text{ H}$ 求电流 i_C 。



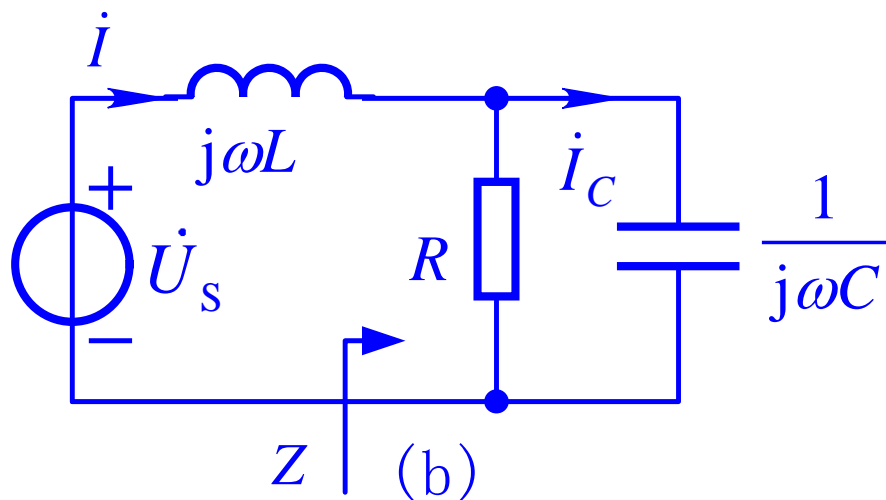
【解】

将时域电路模型变换为相量模型

$$\dot{U}_s = 60 \angle 45^\circ \text{ V}$$

$$Z = R // \frac{1}{j\omega C} = \frac{R \times \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC} = 5(1 - j) \Omega$$

[例4.9]



$$\begin{aligned}\dot{I} &= \frac{\dot{U}_s}{Z + j\omega L} \\ &= \frac{60\angle 45^\circ \text{ V}}{[5(1 - j) + j10]\Omega} = 6\sqrt{2} \text{ A}\end{aligned}$$

$$\dot{I}_C = \frac{R}{R + \frac{1}{j\omega C}} \times \dot{I} = \frac{j\omega RC}{1 + j\omega RC} \times \dot{I} = 6\angle 45^\circ \text{ A}$$

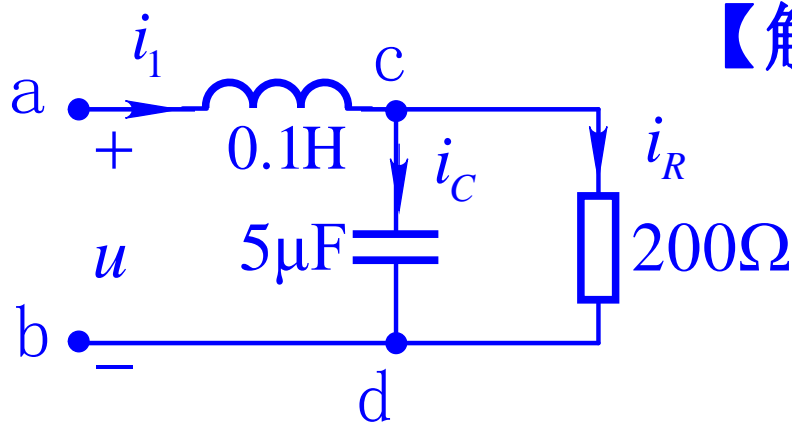
$$i_C = 6\sqrt{2} \cos(100t + 45^\circ) \text{ A}$$

[补充4.6]

在图示电路中已知 $i_R = \sqrt{2} \cos \omega t \text{ A}$, $\omega = 2 \times 10^3 \text{ rad/s}$ 。

(1) 求 ab 端的等效阻抗和等效导纳。

(2) 求各元件的电压、电流及电源电压 u ，并作各电压、电流的相量图。



【解】 $X_L = \omega L$
 $= (2 \times 10^3) \text{ rad/s} \times 0.1 \text{ H} = 200 \Omega$

$$X_C = -\frac{1}{\omega C}$$

$$= -\frac{1}{(2 \times 10^3) \text{ rad/s} \times (5 \times 10^{-6}) \text{ F}} = -100 \Omega$$

$$(1) \quad Z_{cd} = \frac{1}{\frac{1}{R} + j\omega C} = \frac{1}{\frac{1}{200} + j\frac{1}{100}} = 40(1 - j2) \Omega$$

[补充4.6]

$$Z_{ab} = j\omega L + Z_{cd} = 126.49 \angle 71.56^\circ \Omega$$

$$Y_{ab} = \frac{1}{Z_{ab}} = (2.5 - j7.5) \text{mS}$$

$$(2) \dot{U}_{cd} = \dot{I}_R \times R = 200 \angle 0^\circ \text{V}$$

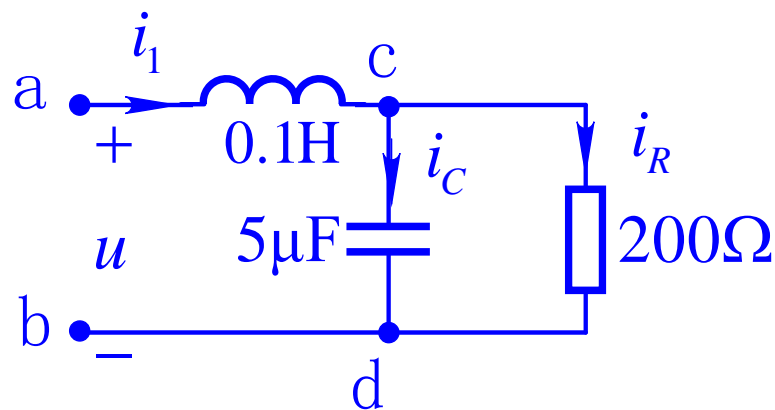
$$\dot{I}_C = j\omega C \dot{U}_{cd} = 2 \angle 90^\circ \text{A}$$

$$\dot{I}_1 = \dot{I}_C + \dot{I}_R = 2.236 \angle 63.43^\circ \text{A}$$

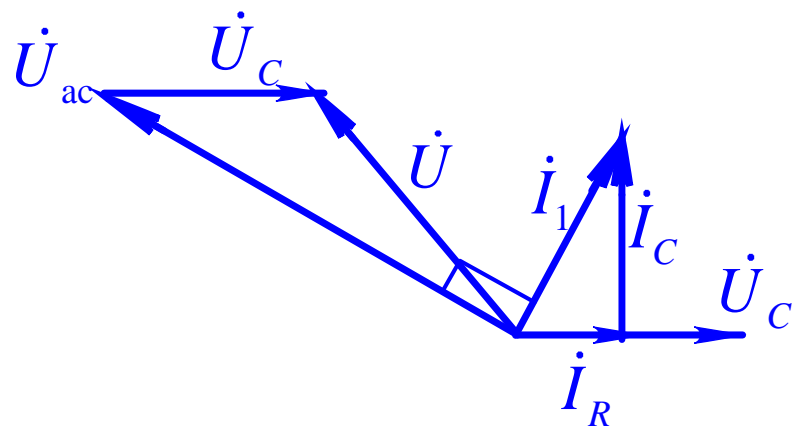
$$\dot{U}_{ac} = j\omega L \times \dot{I}_1 = 447.2 \angle 153.43^\circ \text{V}$$

$$\dot{U} = Z_{ab} \cdot \dot{I}_1 = 282.83 \angle 134.99^\circ \text{V}$$

$$u = 282.83\sqrt{2} \cos(\omega t + 134.99^\circ) \text{V}$$



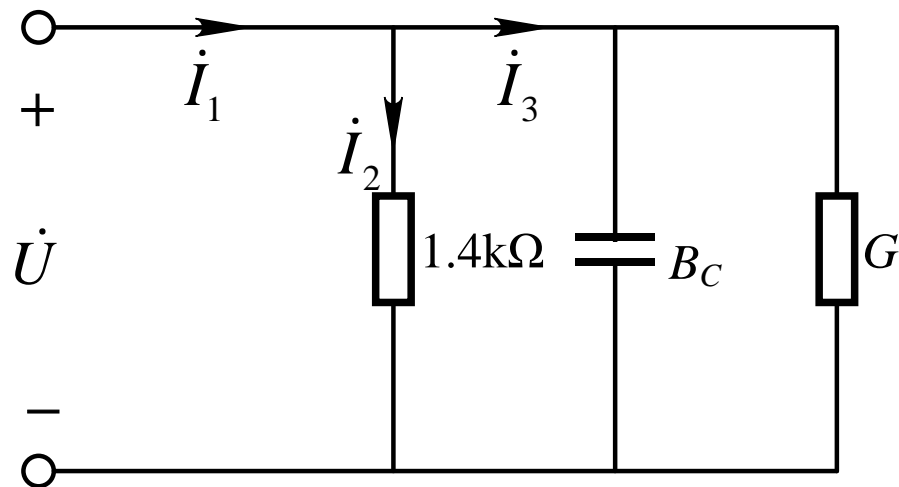
各电压、电流相量图



[补充4.7]

已知 $I_1=0.4\text{A}$, $I_2=0.1\text{A}$,
 $I_3=0.38\text{A}$, 求 B_C 和 G 。

【解】



$$U = 1.4\text{k}\Omega I_2$$

$$I_3 = \sqrt{(B_C U)^2 + (G U)^2} = 0.38\text{A}$$

$$I_1 = \sqrt{(B_C U)^2 + (I_2 + G U)^2} = 0.4\text{A}$$

[补充4.8]

$$R_1 = R_2 = 1\Omega, L_1 = L_2 = 0.01\text{H}, C = 0.01\text{F},$$

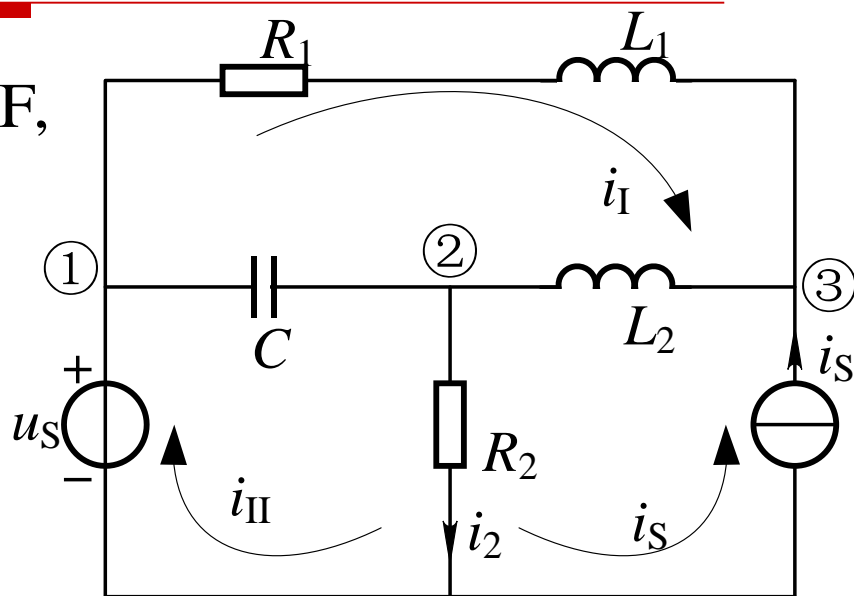
$$u_s = 4\cos(100t - 45^\circ)\text{V},$$

$$i_s = 2.236\sqrt{2}\cos(100t + 153.43^\circ)\text{A}$$

求电流 $i_2(t)$ 。

【解】

回路电流法



$$\dot{U}_s = 2\sqrt{2}\angle 45^\circ\text{V} = (2 - j2)\text{V}$$

$$\dot{I}_s = 2.236\angle 153.43^\circ\text{A} = (-2 + j)\text{A}$$

$$(R_1 + j\omega L_1 + j\omega L_2 + \frac{1}{j\omega C})\dot{I}_I - \frac{1}{j\omega C}\dot{I}_{II} + j\omega L_2\dot{I}_S = 0$$

$$-\frac{1}{j\omega C}\dot{I}_I + (R_2 + \frac{1}{j\omega C})\dot{I}_{II} + R_2\dot{I}_S = \dot{U}_s$$

[补充4.8]

$$\begin{aligned}(R_1 + j\omega L_1 + j\omega L_2 + \frac{1}{j\omega C})\dot{I}_I - \frac{1}{j\omega C}\dot{I}_{II} + j\omega L_2\dot{I}_S &= 0 \\ -\frac{1}{j\omega C}\dot{I}_I + (R_2 + \frac{1}{j\omega C})\dot{I}_{II} + R_2\dot{I}_S &= \dot{U}_S\end{aligned}$$

$$\begin{cases} (1+j)\Omega\dot{I}_I + j\Omega\dot{I}_{II} = (1+2j)V \\ j\Omega\dot{I}_I + (1-j)\Omega\dot{I}_{II} = (4-3j)V \end{cases}$$

$$\begin{cases} \dot{I}_I = -jA = 1\angle -90^\circ A \\ \dot{I}_{II} = 3A \end{cases} \Rightarrow \dot{I}_2 = \dot{I}_{II} + \dot{I}_S = \sqrt{2}\angle 45^\circ A$$

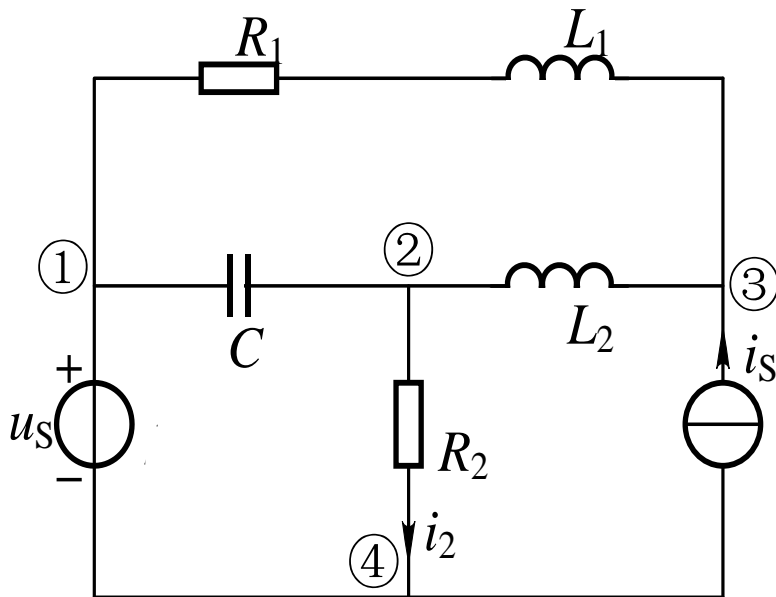
$$i_2(t) = 2\cos(100t + 45^\circ)A$$

[补充4.8]

节点电压法

以节点④为
参考节点

$$\dot{U}_{n1} = \dot{U}_S$$



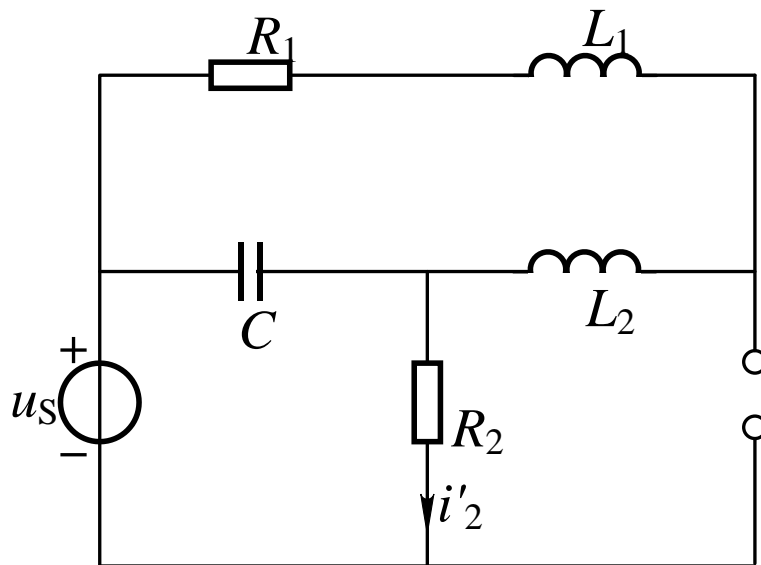
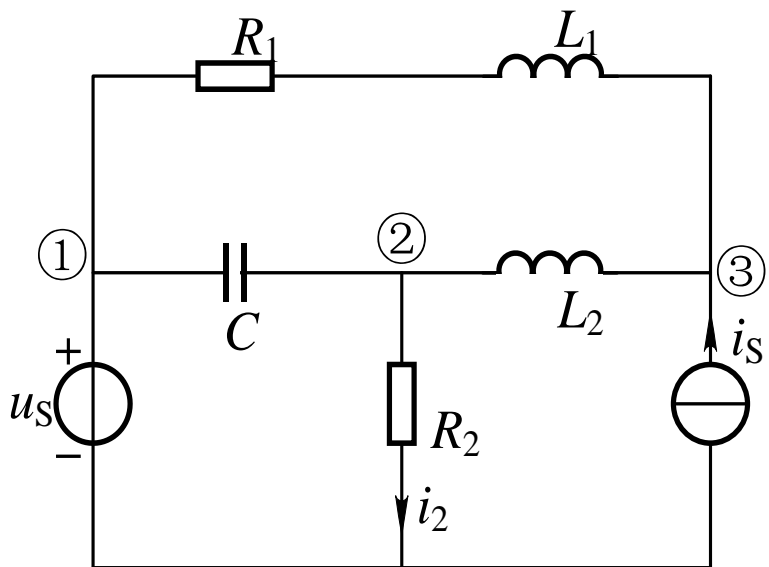
$$-j\omega C\dot{U}_{n1} + (j\omega C + \frac{1}{R_2} + \frac{1}{j\omega L_2})\dot{U}_{n2} - \frac{1}{j\omega L_2}\dot{U}_{n3} = 0$$

$$-\frac{1}{R_1 + j\omega L_1}\dot{U}_{n1} - \frac{1}{j\omega L_2}\dot{U}_{n2} + (\frac{1}{R_1 + j\omega L_1} + \frac{1}{j\omega L_2})\dot{U}_{n3} = \dot{I}_S$$

$$\begin{cases} \dot{U}_{n2} = (1 + j)\text{V} = \sqrt{2}\angle 45^\circ\text{V} \\ \dot{U}_{n3} = (1 - j)\text{V} = \sqrt{2}\angle -45^\circ\text{V} \end{cases} \Rightarrow \dot{I}_2 = \dot{U}_{n2} / R_2 = \sqrt{2}\angle 45^\circ\text{A}$$

[补充4.8]

叠加定理



$$\dot{I}'_2 = \frac{\dot{U}_S}{R_2 + \frac{(R_1 + j\omega L_1 + j\omega L_2) \cdot \frac{1}{j\omega C}}{R_1 + j\omega L_1 + j\omega L_2 + \frac{1}{j\omega C}}} = \frac{(2 - j2)\text{V}}{\frac{3}{1 + j}\Omega} = \frac{4}{3}\text{A}$$

[补充4.8]

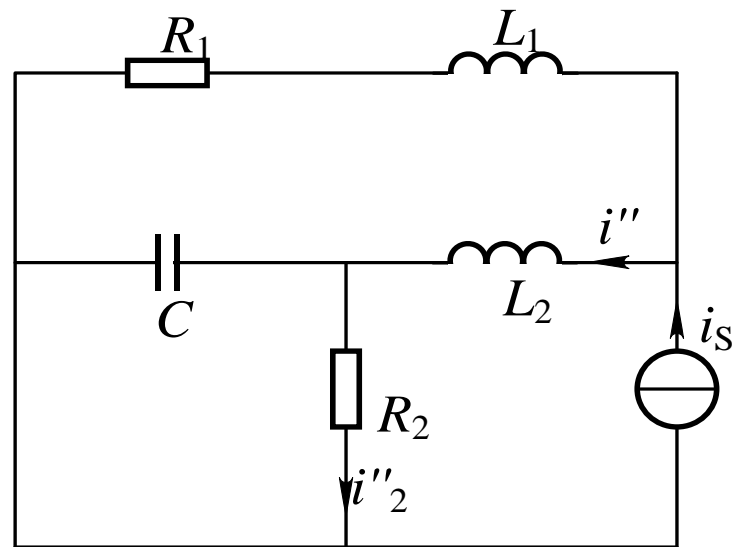
叠加定理

$$Z = j\omega L_2 + \frac{R_2 \cdot \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{1}{1-j} \Omega$$

$$\dot{I}'' = \frac{R_1 + j\omega L_1}{Z + R_1 + j\omega L_1} \dot{I}_s = \frac{2}{3}(-2 + j)A$$

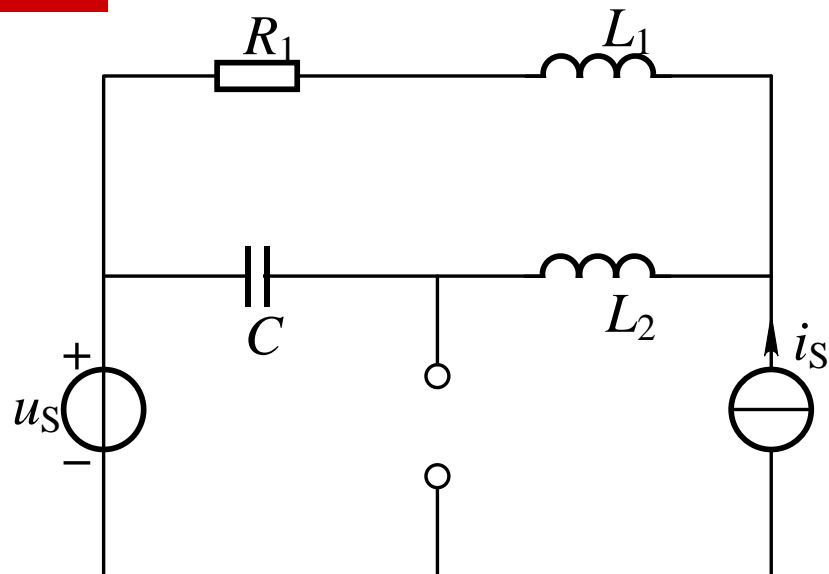
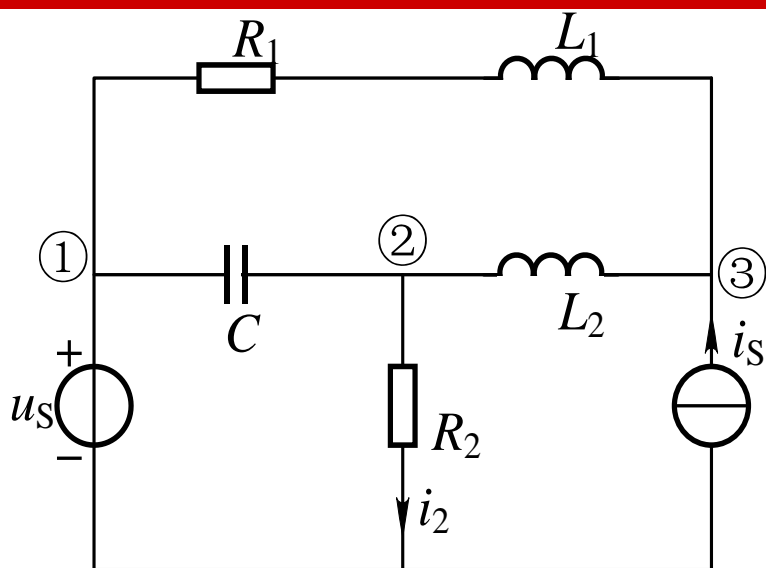
$$\dot{I}_2'' = \frac{\frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} \dot{I}'' = (-\frac{1}{3} + j)A$$

$$\dot{I}_2 = \dot{I}_2' + \dot{I}_2'' = (1 + j)A = \sqrt{2} \angle 45^\circ A$$



[补充4.8]

等效电源定理



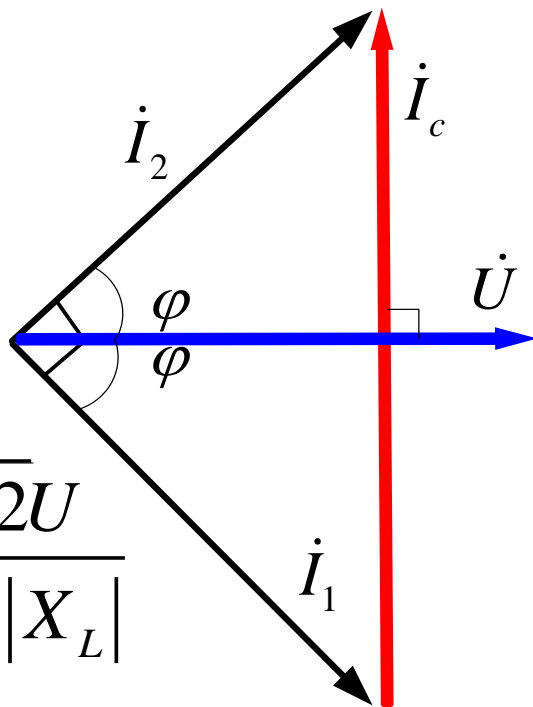
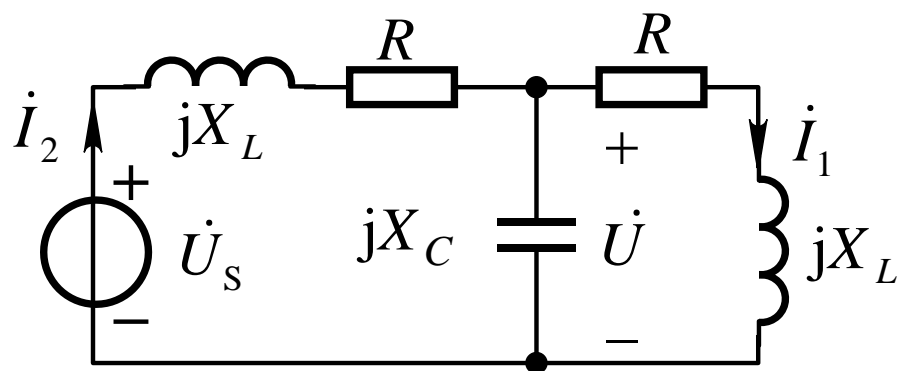
$$\dot{U}_{oc} = \frac{1}{j\omega C} \dot{I}_C + \dot{U}_s = \frac{1}{j\omega C} \cdot \frac{R_1 + j\omega L_1}{R_1 + j\omega L_1 + j\omega L_2 + \frac{1}{j\omega C}} \dot{I}_s + \dot{U}_s = 3V$$

$$Z_i = \frac{(R_1 + j\omega L_1 + j\omega L_2) \cdot \frac{1}{j\omega C}}{R_1 + j\omega L_1 + j\omega L_2 + \frac{1}{j\omega C}} = (0.5 - j1.5)\Omega$$

$$\dot{I}_2 = \frac{\dot{U}_{oc}}{R_2 + Z_i} = \sqrt{2} \angle 45^\circ A$$

[补充4.9]

已知图示电路中的感抗 X_L ，要求 $\dot{I}_2 = j\dot{I}_1$ 。以电压 \dot{U} 为参考相量画出相量图，求电阻 R 和容抗 X_C 。



【解】

$$\varphi = 45^\circ$$

$$R = X_L$$

$$I_C = \sqrt{2}I_1$$

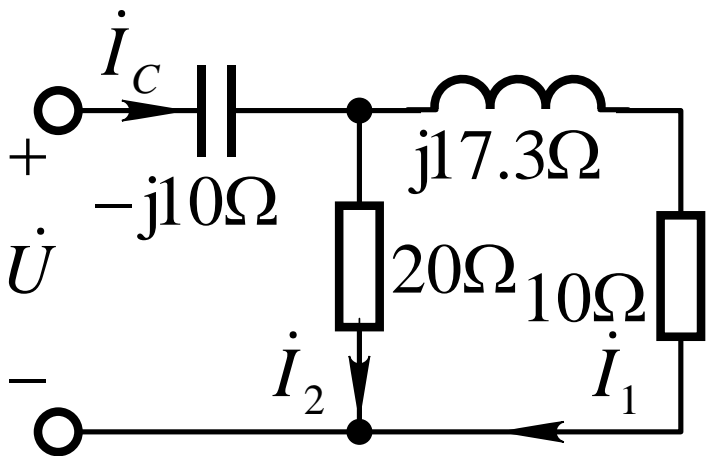
$$\frac{U}{|X_C|} = \frac{\sqrt{2}U}{\sqrt{R^2 + X_L^2}} = \frac{\sqrt{2}U}{\sqrt{2}|X_L|}$$

$$|X_C| = |-X_L|$$

$$X_C = -X_L$$

[补充4.10]

在图示电路中，各元件电压、电流取关联参考方向。
设 $\dot{I}_1 = 1\angle 0^\circ \text{ A}$ ，写出各元件电压、电流相量。



【解】

$$L: \dot{I}_L = \dot{I}_1 = 1\angle 0^\circ \text{ A},$$

$$\dot{U}_L = 17.3\angle 90^\circ \text{ V}$$

$$\dot{U}_2 = (10 + j17.3) \text{ V}$$

$$\dot{I}_2 = \dot{U}_2 / 20\Omega = 1\angle 60^\circ \text{ A}$$

$$R: \dot{I}_R = \dot{I}_1 = 1\angle 0^\circ \text{ A},$$

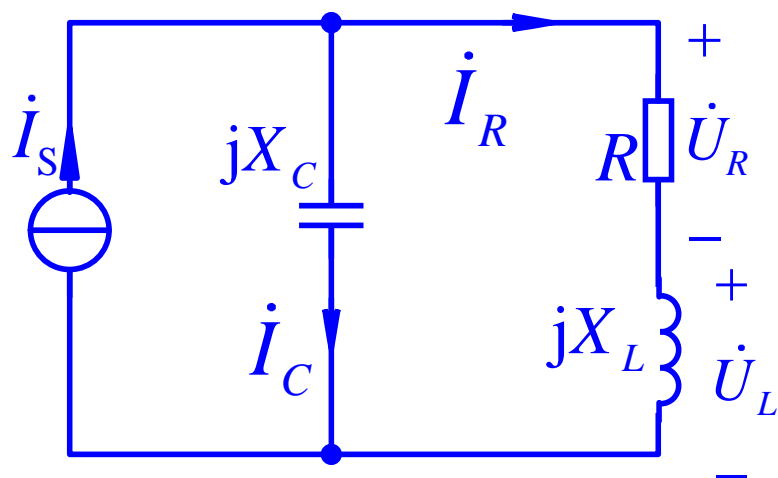
$$\dot{U}_R = 10 \text{ V}$$

$$C: \dot{I}_C = \dot{I}_1 + \dot{I}_2 = 1.732\angle 30^\circ \text{ A}$$

$$\dot{U}_C = -j10\dot{I}_C = 17.32\angle -60^\circ \text{ V}$$

[补充4.12]

已知图示电路中 $U_R = U_L = 10\text{V}$, $R = 10\Omega$, $X_C = -10\Omega$, 求 I_S .



【解】

$$\text{设 } \dot{U}_R = 10\angle 0^\circ \text{ V},$$

$$\text{则 } \dot{U}_L = 10\angle 90^\circ \text{ V}$$

$$\dot{I}_R = \dot{U}_R / R = 1\angle 0^\circ \text{ A}$$

$$\dot{I}_C = \frac{\dot{U}_R + \dot{U}_L}{jX_C} = \frac{10 + j10}{-j10} = (-1 + j) \text{ A}$$

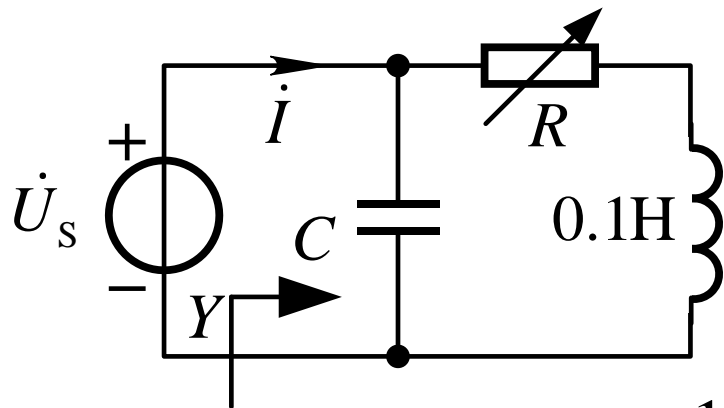
$$\dot{I}_S = \dot{I}_R + \dot{I}_C = 1\angle 0^\circ - 1 + j = j = 1\angle 90^\circ \text{ A}$$

$$I_S = 1 \text{ A}$$

[补充4.13]

图示电路， $\dot{U}_s = 10\text{V}$ ，角频率 $\omega = 10^3\text{rad/s}$ 。要求无论 R 怎样改变，电流有效值 I 始终不变，求 C 的值，并分析电流 I 的辐角变化情况。

【解】



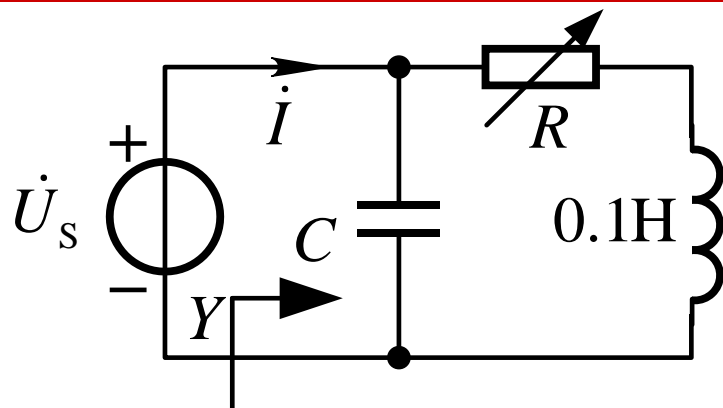
$$Y = j\omega C + \frac{1}{R + j\omega L} = \frac{j\omega RC + 1 - \omega^2 LC}{R + j\omega L} = \omega C \frac{jR - \omega L + \frac{1}{\omega C}}{R + j\omega L}$$

$$R^2 + \left(-\omega L + \frac{1}{\omega C}\right)^2 = R^2 + (\omega L)^2$$

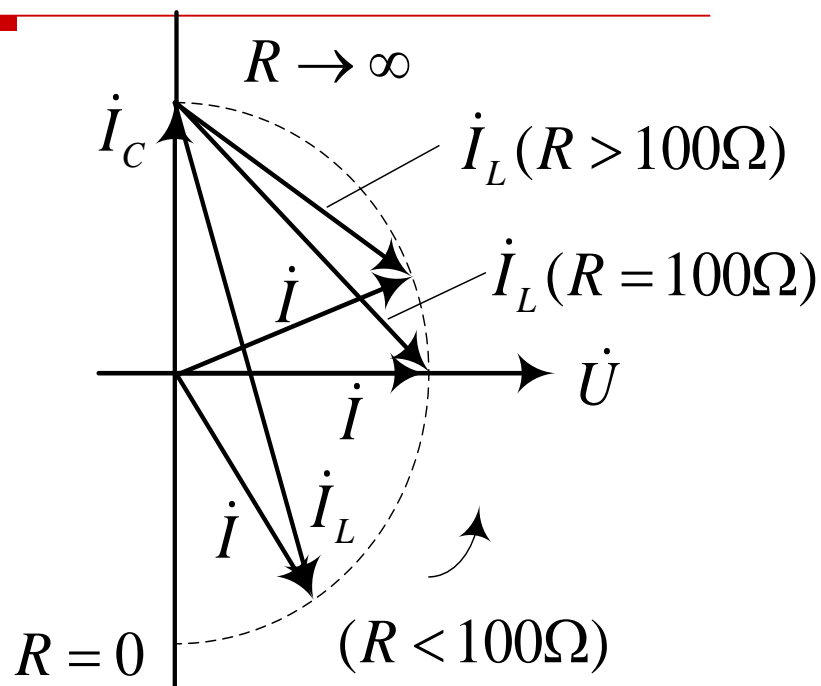
$$\frac{-2L}{C} + \frac{1}{(\omega C)^2} = 0$$

$$C = \frac{1}{2\omega^2 L} = 5\mu\text{F}$$

分子分母
模相等时
满足条件



$$X_C = -200\Omega, \quad X_L = 100\Omega$$



当 $R = 0$, \dot{I} 滞后 \dot{U}_s 为 -90° ;

当 $0 < R < 100\Omega$, \dot{I} 滞后 \dot{U}_S 为从 -90° 向 0 变化;

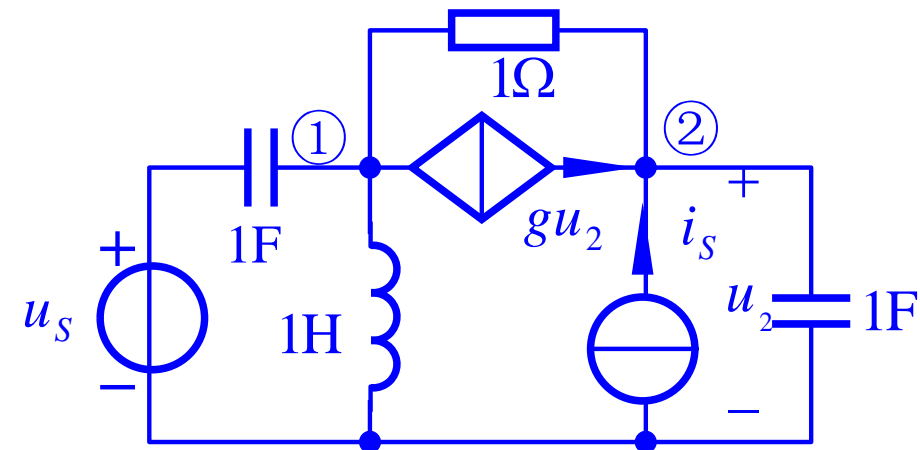
当 $R = 100\Omega$ ， \dot{I} 与 \dot{U}_s 同相位；

当 $R > 100\Omega$, i 超前 U_s 为从 0 向 90° 变化;

当 $R \rightarrow \infty$, \dot{I} 越前 \dot{U}_s 为 90° 。

[补充4.14]

已知图示电路中 $g = 1\text{S}$, $u_S = 10\sqrt{2} \sin \omega t \text{V}$, $i_S = 10\sqrt{2} \cos \omega t \text{A}$, $\omega = 1\text{rad/s}$ 。求受控电流源的电压 u_{12} 。



【解】

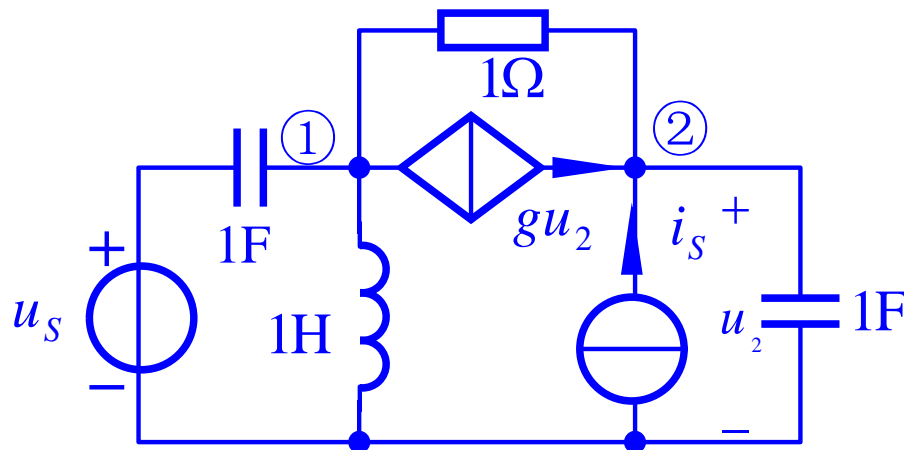
$$\dot{U}_S = 10 \angle -90^\circ \text{ V},$$

$$\dot{I}_S = 10 \angle 0^\circ \text{ A}$$

列写节点电压方程：

$$\begin{cases} n_1 : \left(j\omega C_1 + \frac{1}{j\omega L} + \frac{1}{R} \right) \dot{U}_{n_1} - \frac{1}{R} \dot{U}_{n_2} = j\omega C_1 \dot{U}_S - g \dot{U}_2 \\ n_2 : -\frac{1}{R} \dot{U}_{n_1} + \left(j\omega C_2 + \frac{1}{R} \right) \dot{U}_{n_2} = \dot{I}_S + g \dot{U}_2 \end{cases} \quad \dot{U}_2 = \dot{U}_{n_2}$$

[补充4.14]



$$\begin{cases} n_1 : \left(j\omega C_1 + \frac{1}{j\omega L} + \frac{1}{R} \right) \dot{U}_{n1} - \frac{1}{R} \dot{U}_{n2} = j\omega C_1 \dot{U}_S - g \dot{U}_2 \\ n_2 : -\frac{1}{R} \dot{U}_{n1} + \left(j\omega C_2 + \frac{1}{R} \right) \dot{U}_{n2} = \dot{I}_S + g \dot{U}_2 \end{cases} \quad \dot{U}_2 = \dot{U}_{n2}$$

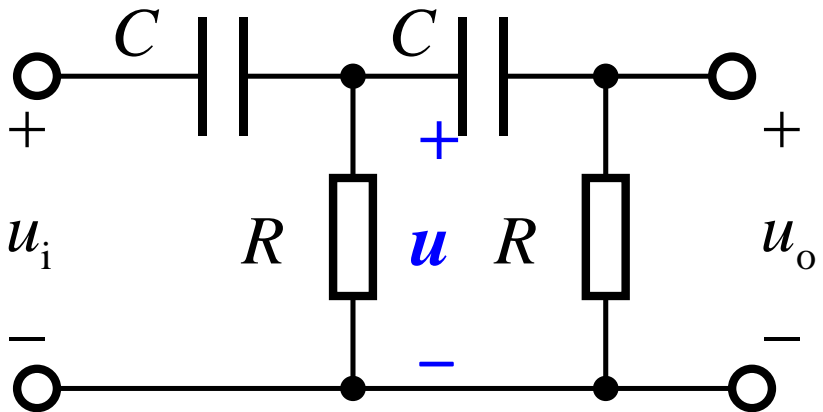
解得 $\dot{U}_{n1} = 10\text{V} \quad \dot{U}_{n2} = -j20\text{V}$

$$\dot{U}_{12} = \dot{U}_{n1} - \dot{U}_{n2} = (10 + j20)\text{V} = 22.36 \angle 63.43^\circ \text{V}$$

$$u_{12} = 22.36\sqrt{2} \cos(\omega t + 63.43^\circ) \text{V}$$

[补充4.15]

在图示 RC 移相电路中设 $R = 1/(\omega C)$ ，试求输出电压 u_o 和输入电压 u_i 的相位差。



【解】

$$\begin{aligned}\frac{\dot{U}_o}{\dot{U}} &= \frac{R}{R + 1/j\omega C} \\ &= \frac{R}{R - jR} = \frac{1}{1 - j}\end{aligned}$$

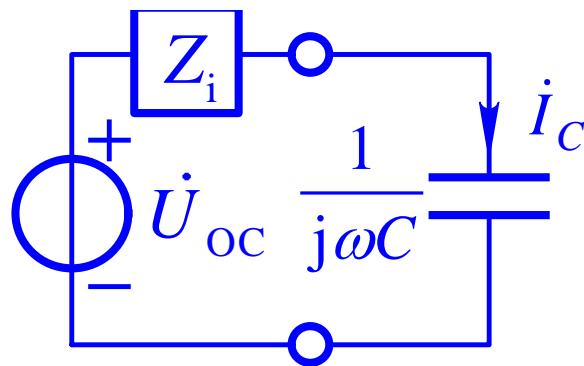
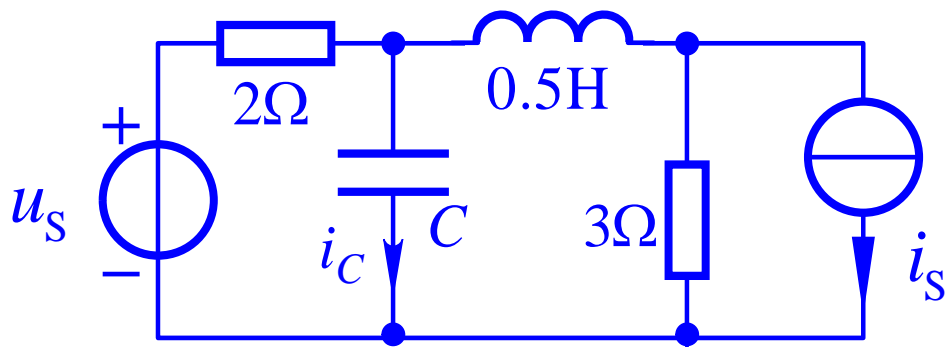
$$\begin{aligned}\frac{\dot{U}}{\dot{U}_i} &= \frac{\frac{R(R + 1/j\omega C)}{R + R + 1/j\omega C}}{1/j\omega C + \frac{R(R + 1/j\omega C)}{R + R + 1/j\omega C}} \\ &= \frac{1}{3}(1 + j)\end{aligned}$$

$$\frac{\dot{U}_o}{\dot{U}_i} = \frac{\dot{U}_o}{\dot{U}} \times \frac{\dot{U}}{\dot{U}_i} = \frac{1}{1 - j} \times \frac{1 + j}{3} = \frac{1}{3}j$$

u_o 超前于 u_i 的相位差为 90°

[例4.12]

图示电路中, $C=0.05\text{F}$ 时, $i_C = 5\sqrt{2} \cos(10t - 60^\circ)\text{A}$, 求当 $C=0.25\text{F}$ 时, $i_C = ?$



$$Z_i = \frac{2 \times (3 + j5)}{2 + 3 + j5} \Omega = (1.6 + j0.4) \Omega$$

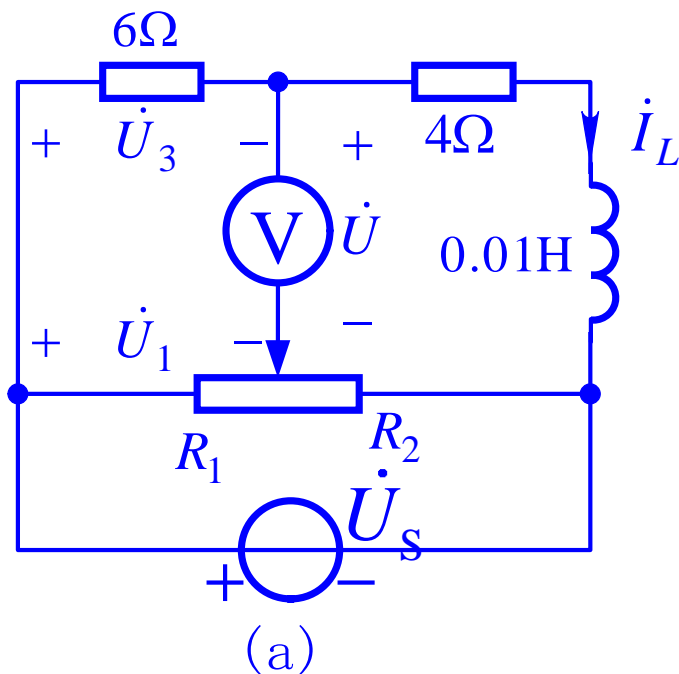
$$\text{当 } C = 0.05\text{F} \text{ 时, } \dot{U}_{oc} = (Z_i + \frac{1}{j\omega C}) \dot{I}_C = (Z_i - j2) \times 5 \angle -60^\circ = 8\sqrt{2} \angle -105^\circ \text{ V}$$

$$\text{当 } C = 0.25\text{F} \text{ 时, } \dot{I}_C = \frac{\dot{U}_{oc}}{Z_i + 1/j\omega C} = 5\sqrt{2} \angle -105^\circ \text{ A}$$

$$i_C = \sqrt{2} \times 5\sqrt{2} \cos(10t - 105^\circ) \text{ A} = 10 \cos(10t - 105^\circ) \text{ A}$$

[例4.13]

图示电路，正弦电压源角频率为 $\omega=1000\text{rad/s}$ ，电压表为理想的。求 R_1/R_2 为何值时，电压表的读数为最小？



【解】

设 $R_1/R_2=r$ ， R_1 分得分压为

$$\dot{U}_1 = \frac{R_1 \dot{U}_s}{R_1 + R_2} = \frac{r}{r+1} \dot{U}_s \quad (1)$$

6 Ω 电阻电压为

$$\dot{U}_3 = \frac{6\dot{U}_s}{(6+4) + j\omega L} = \frac{6\dot{U}_s}{10 + j10} \quad (2)$$

电压表两端电压为

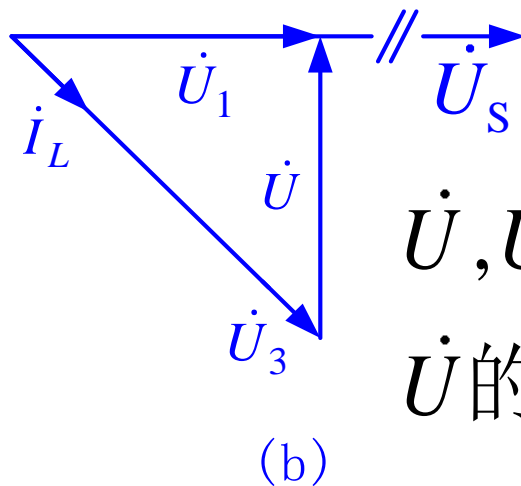
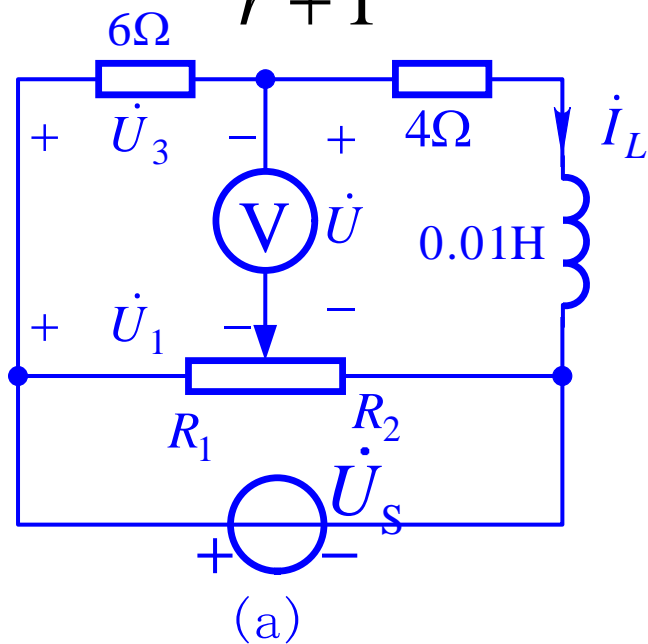
$$\dot{U} = -\dot{U}_3 + \dot{U}_1 = \left(\frac{r}{r+1} - 0.3 + j0.3 \right) \dot{U}_s \quad (3)$$

[例4.13]

$$\dot{U} = -\dot{U}_3 + \dot{U}_1 = \left(\frac{r}{r+1} - 0.3 + j0.3 \right) \dot{U}_s \quad (3)$$

实部为零时电压表的读数便是最小

$$\frac{r}{r+1} - 0.3 = 0 \quad \text{即} \quad r = \frac{R_1}{R_2} = \frac{3}{7}$$



\dot{U}, \dot{U}_1 为垂直关系时,
 \dot{U} 的长度最短。