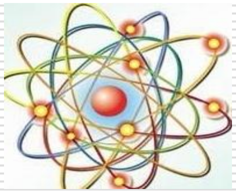

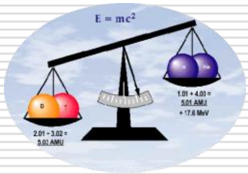

第7章 频率特性和谐振现象

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7.3 *RLC*串联电路的频率特性

基本要求：了解*RLC*串联电路的网络函数及其频率特性。

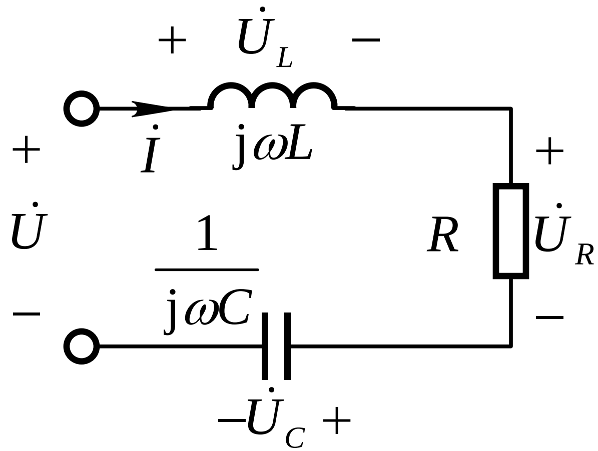
主要内容

一、以电阻电压为响应的网络函数

二、以电容电压为响应的网络函数

三、以电感电压为响应的网络函数

7.3 *RLC*串联电路的频率特性



$$H_R(\mathrm{j}\omega)=\frac{\dot{U}_R}{\dot{U}}=\frac{R}{R+\mathrm{j}[\omega L-1/(\omega C)]}$$

$$H_C(\mathrm{j}\omega)=\frac{\dot{U}_C}{\dot{U}}=\frac{1/(\mathrm{j}\omega C)}{R+\mathrm{j}\omega L+1/(\mathrm{j}\omega C)}$$

$$H_L(\mathrm{j}\omega)=\frac{\dot{U}_L}{\dot{U}}=\frac{\mathrm{j}\omega L}{R+\mathrm{j}\omega L+1/(\mathrm{j}\omega C)}$$

一、以电阻电压为响应的网络函数

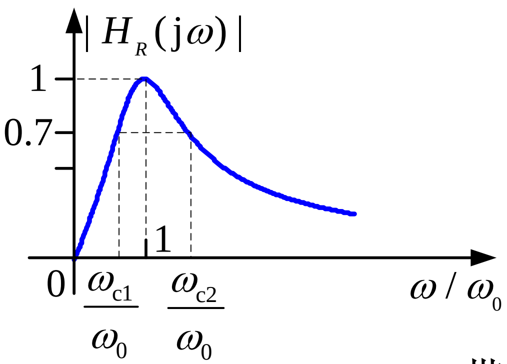
$$H_R(\mathrm{j}\omega)=\frac{\dot{U}_R}{\dot{U}}=\frac{R}{R+\mathrm{j}[\omega L-1/(\omega C)]}$$

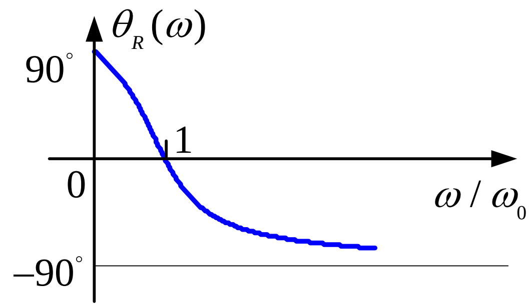
$$\left|H_R(\mathrm{j}\omega)\right|=\frac{1}{\sqrt{1+\frac{1}{R^2}\left(\omega L-\frac{1}{\omega C}\right)^2}}=\frac{1}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}$$

$$\theta_R(\omega)=-\arctan Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$

ω/ω_0	$\left\|H_R(\mathrm{j}\omega)\right\|$	$\theta_R(\omega)$
0	0	90°
1	1	0°
⋮	⋮	⋮
⋮	⋮	⋮
∞	0	-90°

一、以电阻电压为响应的网络函数





***RLC*带通电路的频率特性**

$$|H_R(\mathrm{j}\omega)| = \frac{1}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}$$

$$\theta_R(\omega) = -\arctan Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$

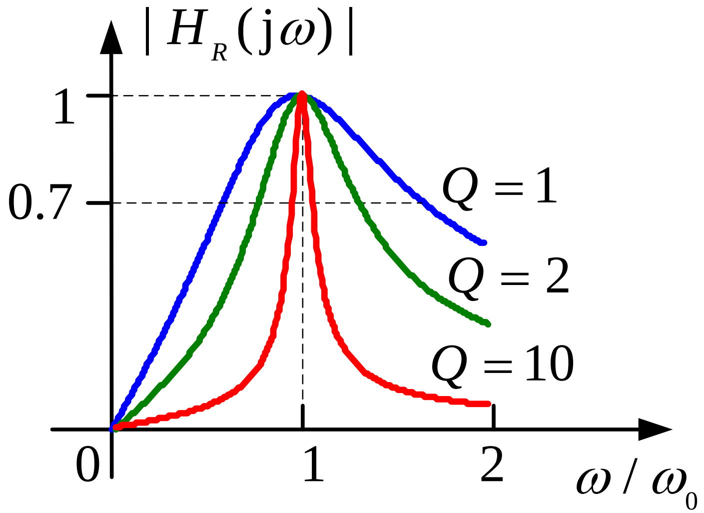
1. $|H_R(\mathrm{j}\omega)|$ 具有带通特性

$$\frac{1}{\sqrt{1+Q^2\left(\frac{\omega_c}{\omega_0}-\frac{\omega_0}{\omega_c}\right)^2}} = \frac{1}{\sqrt{2}} \xrightarrow{\text{截止频率}} \begin{cases} \omega_{c1} = \omega_0\left(-\frac{1}{2Q}+\sqrt{\frac{1}{4Q^2}+1}\right) \\ \omega_{c2} = \omega_0\left(\frac{1}{2Q}+\sqrt{\frac{1}{4Q^2}+1}\right) \end{cases}$$

通带宽度 $\Delta\omega = \omega_{c2} - \omega_{c1} = \frac{\omega_0}{Q}$

一、以电阻电压为响应的网络函数

2. 频率特性与品质因数的关系



$$\Delta\omega = \omega_{c2} - \omega_{c1} = \frac{\omega_0}{Q}$$

(1)*Q*值越大，截止频率处的曲线越陡，频率选择性越好，带宽越窄。

(2)*Q*值越小，带宽越宽，选择性能越差。

***RLC*串联电路，频率选择性与带宽存在矛盾。**

二、以电容电压为响应的网络函数

$$H_C(\mathrm{j}\omega)=\frac{\dot{U}_C}{\dot{U}}=\frac{1/(\mathrm{j}\omega C)}{R+\mathrm{j}\omega L+1/(\mathrm{j}\omega C)}$$

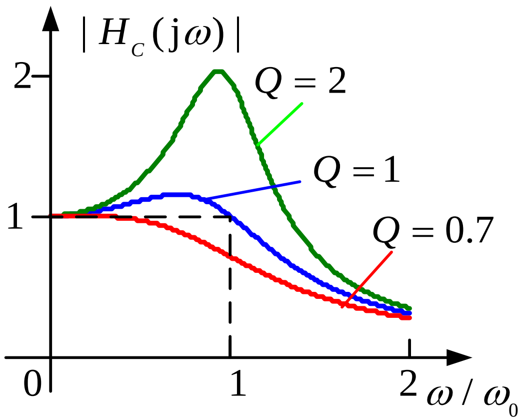
$$=\frac{1}{(1-\omega^2 LC)+\mathrm{j}\omega CR}=\frac{1}{[1-\left(\dfrac{\omega}{\omega_0}\right)^2]+\mathrm{j}\dfrac{1}{Q}\left(\dfrac{\omega}{\omega_0}\right)}$$

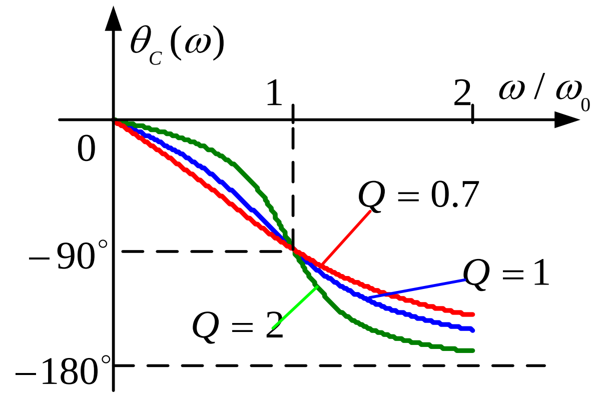
$$|H_C(\mathrm{j}\omega)|=\frac{1}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_0}\right)^2\right]^2+\dfrac{1}{Q^2}\left(\dfrac{\omega}{\omega_0}\right)^2}}$$

$$\theta_C(\omega)=-\arctan\frac{1}{Q(\dfrac{\omega_0}{\omega}-\dfrac{\omega}{\omega_0})}$$

ω/ω_0	$\lvert H_C(\mathrm{j}\omega)\rvert$	$\theta_C(\omega)$
0	1	0°
1	Q	-90°
⋮	⋮	⋮
⋮	⋮	⋮
∞	0	-180°

二、以电容电压为响应的网络函数





*RLC*低通电路的滤波特性

$$|H_C(j\omega)| = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]^2 + \frac{1}{Q^2}\left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\theta_C(\omega) = -\arctan\frac{1}{Q(\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0})}$$

$|H_C(j\omega)|$具有低通特性

三、以电感电压为响应的网络函数

$$H_L(\mathrm{j}\omega)=\frac{\dot{U}_L}{\dot{U}}=\frac{\mathrm{j}\omega L}{R+\mathrm{j}\omega L+1/(\mathrm{j}\omega C)}$$

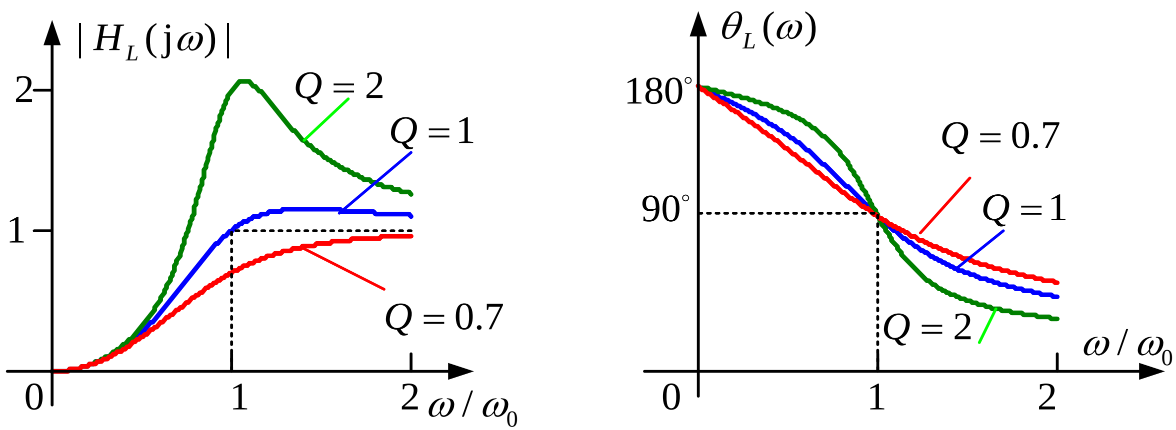
$$=\frac{1}{[1-1/(\omega^2 LC)]-\mathrm{j}R/(\omega L)}=\frac{1}{\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]-\mathrm{j}\frac{1}{Q}\left(\frac{\omega_0}{\omega}\right)}$$

$$|H_L(\mathrm{j}\omega)|=\frac{1}{\sqrt{\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]^2+\frac{1}{Q^2}\left(\frac{\omega_0}{\omega}\right)^2}}$$

$$\theta_L(\omega)=-\arctan\frac{-1}{Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}$$

ω/ω_0	$\lvert H_L(\mathrm{j}\omega)\rvert$	$\theta_L(\omega)$
0	0	180°
1	Q	90°
⋮	⋮	⋮
⋮	⋮	⋮
∞	1	0°

三、以电感电压为响应的网络函数



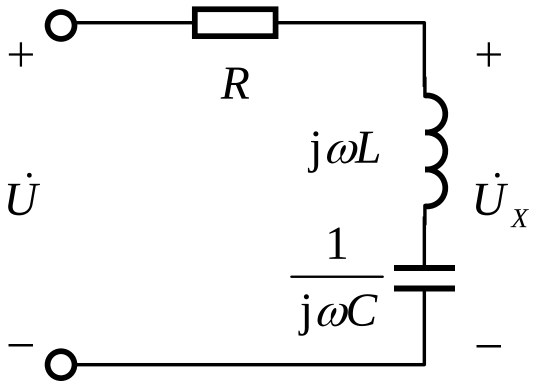
*RLC*高通电路频率特性

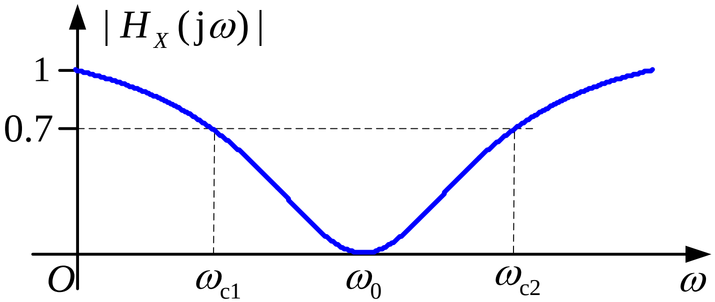
$$|H_L(\mathrm{j}\omega)| = \frac{1}{\sqrt{\left[1-\left(\frac{\omega_0}{\omega}\right)^2\right]^2 + \frac{1}{Q^2}\left(\frac{\omega_0}{\omega}\right)^2}}$$

$$\theta_L(\omega) = -\arctan\frac{-1}{Q\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}$$

$\left|H_L(\mathrm{j}\omega)\right|$具有高通特性

思考：*RLC*串联电路能否实现带阻特性？





$$H_X(\mathrm{j}\omega)=\frac{\dot{U}_X}{\dot{U}}=\frac{\mathrm{j}[\omega L-1/(\omega C)]}{R+\mathrm{j}[\omega L-1/(\omega C)]}$$

$$\omega_{c1}=\omega_0\left(-\frac{1}{2Q}+\sqrt{\frac{1}{4Q^2}+1}\right)\qquad \omega_{c2}=\omega_0\left(\frac{1}{2Q}+\sqrt{\frac{1}{4Q^2}+1}\right)$$