# Homework 2　　　　班号　　　　学号　　　　姓名

1. Prove that language $L = \{0^n \mid n \text{ is a power of } 2\}$ is not regular.

   证明思路：泵引理，反证法。取 $s = 0^{2^N}$, 则 $2^N < |xy^2z| < 2^N + N < 2^N + 2^N = 2^{(N+1)}$

2. If $L$ is a language, and $a$ is a symbol, then $L/a$, the quotient of $L$ and $a$, is the set of strings $w$ such that $wa$ is in $L$. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if $L$ is regular, so is $L/a$. Hint: Start with a DFA for $L$ and consider the set of accepting states.

   证明：令 $L = L(M)$, 其中 $M = (Q, \Sigma, \delta, q_0, F)$

   构造 $M' = (Q, \Sigma, \delta, q_0, F')$ , 其中 $F' = \{q | \delta(q, a) \in F\}, q \in Q, a \in \Sigma$

   证明 $L(M') = L/a$, $\because \forall w \in L(M')$ 即 $\hat{\delta}(q_0, w) \in F'$ 即 $\delta(\hat{\delta}(q_0, w), a) \in F \therefore w \in L/a$

   又 $\because \forall w \in L/a$ 有 $wa \in L$ 即 $\hat{\delta}(q_0, wa) \in F$ 即 $\delta(\hat{\delta}(q_0, w), a) \in F$ 即 $\hat{\delta}(q_0, w) \in F' \therefore w \in L(M')$

Design context-free grammars for the following languages:

3. The set $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of $a$'s followed by $b$'s followed by $c$'s, such that there are either a different number of $a$'s and $b$'s or a different number of $b$'s and $c$'s, or both.

   $S \to A_1C | A_2C | AB_1 | AB_2$
   $A_1 \to aA_1b | aA_1 | a$
   $A_2 \to aA_2b | A_2b | b$
   $C \to Cc | \varepsilon$
   $B_1 \to bB_1c | bB_1 | b$
   $B_2 \to bB_2c | B_2c | c$
   $A \to Aa | \varepsilon$

   (注意：$Cc|\varepsilon$ 若为 $Cc|c$ 则不能产生 $a, c$ 同时为 0 个，或 $b, c$)

4. The set of all strings over $\{0, 1\}$ with twice as many 0's as 1's.

   $S \to S0S0S1S | S0S1S0S | S1S0S0S | \varepsilon$

5. The set of all strings over $\{a, b\}$ that are **not** of the form $ww$, for some string $w$. Explain how your grammar works. You needn't prove it's correctness formally.

   如果串长为奇数，显然不是 $ww$ 形式 (对应下面文法中的 $A$ 或 $B$)。而对于长度为偶数 $(2n)$ 的串，至少存在一对儿距离为 $n$(串长度的一半) 的两字符不相同。为了能够产生两个不相同字符的距离刚好是整个长度的一半，使用两个变元 $A$ 和 $B$ 分别产生基数长的串，然后合并即可。对 $A$ 或 $B$，在为产生串时，如果增加了两字符间的字符数，那么也要增加两字符外的字符数。如 $aaaabbbb = \underline{aa\check{a}ab}\ \underline{b\check{b}b}$ 或 $aabaaa = \underline{aa\check{b}aa}\ \underline{\check{a}}$.

   $S \to A \mid B \mid AB \mid BA$
   $A \to XAX \mid a$
   $B \to XBX \mid b$
   $X \to a \mid b$