

1. Software Specifications, Chapter 4.

In addition to the logical quantifiers (\forall and \exists) there exists a numeric quantifier ($\#$), which returns (not a logical value TRUE/ FALSE, but) a number: the number of elements for which the quantified predicate is true. For example, in reference to the array below:

1	2	3	4	5	6	7	8	9	10	11
0.9	3.14	2.1	0.9	3.1	5.7	2.89	0.9	2.5	0.9	1.2

the following equations hold:

$$(\#k: a[k] = 0.9) = 4,$$

$$(\#k: a[k] \geq 3) = 3,$$

$$(\#k: a[k] \geq 2) = 6,$$

$$(\#k: a[k] \leq 10) = 11.$$

Use the ($\#$) quantifier to help write the following specification: place in x the smallest value of a and in k a median index where the smallest value appears in a . In the above array, x gets value 0.9 and k gets value 4 or 8.

1. Several clauses

- Place in x a value that exceeds all the cells of a :
 $M_1 = \{(s, s') \mid \forall h: 1 \leq h \leq N: x' \geq a[h]\}$
- Place in x a value of the array
 $M_2 = \{(s, s') \mid \exists h: 1 \leq h \leq N: x' = a[h]\}$
- Place in k an index of x in a :
 $M_3 = \{(s, s') \mid x' = a[k']\}$
- Ensure that k' is the smallest index:
 $M_4 = \{(s, s') \mid \forall h: 1 \leq h < k': x' \neq a[h]\}$

Overall specification:
 $M = M_1 \cap M_2 \cap M_3 \cap M_4$

$M_1 = \{(s, s') \mid (\forall h: 1 \leq h \leq N: x' \leq a[h]) \wedge (\exists l: 1 \leq l \leq N: x' = a[l])\}$
 $M_2 = \{(s, s') \mid ((\#j: a[j] = x') \geq 3) \wedge (\exists h: a[h] = x' \wedge h < k') \wedge (\exists l: a[l] = x' \wedge l > k') \wedge a[k'] = x')\}$

Overall specification:
 $M = M_1 \cap M_2$

2. Correctness Verification, Chapter 5.

Consider the following program on integer variables x, y, z .

```

mult:
{z=0;
  while (y!=0)
    {if (y%2==0) {x=2*x; int1: y=y/2;}
     else       {z=z+x; int2: y=y-1;}}}

```

We propose to prove the correctness of this program with respect to:

Precondition: $x = x_0 \wedge y = y_0$.

Postcondition: $z = x_0 \times y_0$.

- Propose a formula for the loop invariant, inv .
- Propose a formula for the intermediate assertion at label *int1*.
- Propose a formula for the intermediate assertion at label *int2*.
- Prove the correctness of this program with respect to the proposed specification.

②

$$V: \{x=x_0, y=y_0\} \{z=0, \text{ while } (y \neq 0)\}$$

$$\text{if } (y \% 2 == 0) \bullet \{x = 2 \times x;$$

$$\text{int1: } y = y / 2;\}$$

$$\text{else } \{z = z + x; \text{ int2: } y = y - 1;\}$$

$$\} \{z = x_0 \times y_0\}$$

$$\text{into} = x = x_0 \wedge y = y_0 \wedge z = 0$$

We must prove 2 forms

$$V_0: \{x=x_0 \wedge y=y_0\} \{z=0\} \rightarrow \{x=x_0 \wedge y=y_0 \wedge z=0\}$$

→ We must prove $\{x=x_0 \wedge y=y_0\} \rightarrow \{x=x_0 \wedge y=y_0 \wedge z=0\}$ ✓

$$V_1: \{x=x_0 \wedge y=y_0 \wedge z=0\} \text{ while } (y \neq 0) \{ \text{if } (y \% 2 == 0) \{ x = 2 \times x;$$

$$\text{int1: } y = y / 2;\}$$

$$\text{else } \{ z = z + x; \text{ int2: } y = y - 1;\} \}$$

$$\} \{ z = x_0 \times y_0 \}$$

$$\text{inv} = \{x \times y + z = x_0 \times y_0\}$$

To prove V_1 , we must prove

$$V_{10}: \{x=x_0 \wedge y=y_0 \wedge z=0\} \rightarrow \{x \times y + z = x_0 \times y_0\} \text{ This is a tautology ✓}$$

$$V_{11}: \{x \times y + z = x_0 \times y_0 \wedge y \neq 0\} \text{ if } (y \% 2 == 0) \{ x = 2 \times x;$$

$$\text{int1: } y = y / 2;\}$$

$$\text{else } \{ z = z + x; \text{ int2: } y = y - 1;\}$$

$$\} \{ x \times y + z = x_0 \times y_0 \}$$

according to the alternation Statement Rule:

$$V_{110} \{xxy+z = x_0xy_0 \wedge y' = 0 \wedge y\%2 = 0\} \\ x = 2 * x; \text{int } 1: y = y/2; \{xxy+z = x_0xy_0\}$$

according to the statement Rule:

$$\text{int } 1: \{xxy+z = 2x_0xy_0 - z\} \\ V_{1100} \{xxy+z = x_0xy_0 \wedge y' = 0 \wedge y\%2 = 0\} \\ \{x = 2 * x; \{xxy+z = 2x_0xy_0 - z\}\}$$

then we must prove

$$\{xxy+z = x_0xy_0 \wedge y' = 0 \wedge y\%2 = 0\} \rightarrow \\ \{2x_0xy_0+z = 2x_0xy_0 - z\}$$

~~then we must prove~~

$$V_{1101} \{xxy+z = 2x_0xy_0 - z\} \quad y = y/2; \{xxy+z = x_0xy_0\}$$

then we must prove

$$\{xxy+z = 2x_0xy_0 - z\} \rightarrow \{2xy/2 + z = x_0xy_0\}$$

~~then we must prove~~

$$V_{111} \{xxy+z = x_0xy_0 \wedge y' = 0 \wedge y\%2 = 1\} \\ z = z+x; \text{int } 2: y = y+1; \{xxy+z = x_0xy_0\}$$

according to the sequence Statement Rule:

$$\text{int } 2: \{xxy+z = x_0xy_0 + x\}$$

$$V_{1110} \{xxy+z = x_0xy_0 \wedge y' = 0 \wedge y\%2 = 1\} \quad z = z+x \\ \{xxy+z = x_0xy_0 + x\}$$

then we must prove

$$\{xxy+z = x_0xy_0 \wedge y' = 0 \wedge y\%2 = 1\}$$

$$\rightarrow \{x+y+z+x = x_0xy_0 + x\}$$

$$V_{1111} \{xxy+z = x_0xy_0 + x\} \quad y = y+1; \{xxy+z = x_0xy_0\}$$

then we must prove $\{xxy+z = x_0xy_0 + x\}$

$$\rightarrow \{xx(y+1)+z = x_0xy_0\}$$

$$V_{12} \{xxy+z = x_0xy_0 \wedge y = 0\} \rightarrow \{z = x_0xy_0\}$$

Prove up

3. Functional Criteria of Test Data Generation, Chapters 8, 9.

- a. Consider the sorting program given on page 29 of chapter 8. Generate the following mutants of this program:

- m6: obtained by replacing (N-2) by (N-1) in line 4.
- m7: obtained by replacing (N-1) by (N-2) in line 7.
- m8: obtained by replacing {minval=a[j]} by {minval=a[i]} in line 6.

Run program P and each of the mutants (m6, m7, m8) on the test data T given in page 30 and draw a table similar to that of page 35. Did the test data distinguish all the mutants? If it did not, is that because undistinguished mutants are equivalent to P or because the test data is inadequate? If the test data is inadequate, propose other tests.

- b. Consider the following symbols, and write code to generate these symbols randomly, according to the proposed probability distribution. Run this code in a loop that iterates 100 000 times and show how many of each symbol it produces.

a	b	c	d	e	f	g	h	i	J
0.12	0.08	0.1	0.09	0.11	0.15	0.06	0.03	0.01	0.25

③.a)

T	m6	m7	m8
t ₁	True	True	True
t ₂	True	True	True
t ₃	True	True	True
t ₄	True	False	True
t ₅	True	True	True
t ₆	True	True	True
t ₇	True	False	True
t ₈	True	False	True
mutant distinguished?	No	Yes	No

All mutants not distinguished by the test data are equivalent to P.

3- b.

```

1  import random
2
3  def generate_symbol():
4      ra = random.random()
5      if ra < 0.12:
6          return 'a'
7      elif ra < 0.2:
8          return 'c'
9      elif ra < 0.3:
10         return 'c'
11     elif ra < 0.39:
12         return 'f'
13     elif ra < 0.5:
14         return 'e'
15     elif ra < 0.65:
16         return 'f'
17     elif ra < 0.71:
18         return 'g'
19     elif ra < 0.74:
20         return 'h'
21     elif ra < 0.75:
22         return 'i'
23     else:
24         return 'j'
25
26 if __name__ == '__main__':
27     result = {'a' : 0, 'b' : 0, 'c' : 0, 'd' : 0, 'e' : 0, 'f' : 0, 'g' : 0, 'h' : 0, 'i' : 0, 'j' : 0}
28     for _ in range(100_000):
29         random_symbol = generate_symbol()
30         result[random_symbol] = result[random_symbol] + 1
31     print(result)

```

```

PS C:\VSCodeworkSpace\Python> & C:/Users/dnjisd/AppData/Local/Programs/Python/Python37/python.exe c:/VSCodeworkSpace/Python/test1.py
{'a': 11994, 'b': 0, 'c': 17987, 'd': 0, 'e': 11032, 'f': 23872, 'g': 6027, 'h': 3043, 'i': 965, 'j': 25080}
PS C:\VSCodeworkSpace\Python>

```

4. Structural Criteria of Test Data Generation, Chapter 10.

- a. Consider the following program on integer variables x , y , z (and implicit variables is and os , for input stream and output stream).

```
mult:
{read(x); read(y); z=0;
 while (y!=0)
   {if (y%2==0) {x=2*x; y=y/2;}
    else      {z=z+x; y=y-1;}}}
```

Compute the path function then the path condition of the following paths:

```
p0: read(x); read(y); z=0; ((y!=0)?false);
p1: read(x); read(y); z=0;
    ((y!=0)?true); ((y%2==0)? true) {x=2*x; y=y/2;}
    ((y!=0)?true); ((y%2==0)? false) {z=z+x; y=y-1;}
    ((y!=0)?false);
p2: read(x); read(y); z=0;
    ((y!=0)?true); ((y%2==0)? false) {z=z+x; y=y-1;}
    ((y!=0)?true); ((y%2==0)? true) {x=2*x; y=y/2;}
    ((y!=0)?true); ((y%2==0)? false) {z=z+x; y=y-1;}
    ((y!=0)?false);
p3: read(x); read(y); z=0;
    ((y!=0)?true); ((y%2==0)? true) {x=2*x; y=y/2;}
    ((y!=0)?true); ((y%2==0)? true) {x=2*x; y=y/2;}
    ((y!=0)?true); ((y%2==0)? false) {z=z+x; y=y-1;}
    ((y!=0)?false);
p4: read(x); read(y); z=0;
    ((y!=0)?true); ((y%2==0)? true) {x=2*x; y=y/2;}
    ((y!=0)?true); ((y%2==0)? false) {z=z+x; y=y-1;}
    ((y!=0)?true); ((y%2==0)? true) {x=2*x; y=y/2;}
    ((y!=0)?false);
```

-
- b. Draw a table of definitions and uses of the program for variables x , y , z . (you may want to write the program one statement per line, and number the lines for ease of reference).
- c. Choose a du-path in this program for variable z , then generate test data to exercise the selected path.

④ P0. path function

path condition

$$\left\{ (s, s') \mid \begin{array}{l} \text{length}(is) \geq 2 \wedge h(t(is)) = 0 \wedge \\ x' = h(is) \wedge y' = h(t(is)) \wedge \\ is' = t^2(is) \wedge z' = 0 \wedge os' = os \end{array} \right\} \quad \text{length}(is) \geq 2 \wedge h(t(is)) = 0$$

P1. path function

path condition

$$\left\{ (s, s') \mid \begin{array}{l} \text{length}(is) \geq 2 \wedge h(t(is)) = 2 \wedge \\ x' = h(is) \times 2 \wedge y' = (h(t(is))/2) \wedge \\ is' = t^2(is) \wedge z' = 2 \times h(is) \wedge os' = os \end{array} \right\} \quad \text{length}(is) \geq 2 \wedge h(t(is)) = 2$$

P2. path function

path condition

$$\left\{ (s, s') \mid \begin{array}{l} \text{length}(is) \geq 2 \wedge h(t(is)) = 3 \wedge \\ x' = h(is) \times 3 \wedge y' = (h(t(is))/3) \wedge \\ is' = t^2(is) \wedge z' = 3 \times h(is) \wedge os' = os \end{array} \right\} \quad \text{length}(is) \geq 2 \wedge h(t(is)) = 3$$

P3. path function

path condition

$$\left\{ (s, s') \mid \begin{array}{l} \text{length}(is) \geq 2 \wedge h(t(is)) = 4 \wedge \\ x' = h(is) \times 4 \wedge y' = (h(t(is))/4) \wedge \\ is' = t^2(is) \wedge z' = 4 \times h(is) \wedge os' = os \end{array} \right\} \quad \text{length}(is) \geq 2 \wedge h(t(is)) = 4$$

P. P4. path function: \emptyset

path condition: \emptyset

b. Line		X				Z			
		D	C	P	T	D	C	P	T
1	{read(x);	V							
2	read(y);					V			
3	z = 0;							V	
4	while (y != 0)					V			
5	{if (y % 2 == 0)					V	V		
6	{x = 2 * x;	V	V						
7	y = y / 2;					V	V		
8	} else {								
9	z = z + x;	V						V	V
10	y = y + 1;					V	V		
11	}								
12	}			V		V			V

c. for C:

definitions: lines 3, 9

uses line 9

definition use path: [3, 4, 8, 9]

pre-path: empty

post-path: infinity, we choose one at will.
 $y = y + 1; ((y \neq 0) ? \text{false});$

Test Data: is = (3, 1, ...);

5. Test Oracles and Test Driver Design, Chapters 11, 12.

Consider the space defined by an integer variable x , an integer array $a[1..N]$, and an index variable k (between 0 and $N+1$).

- Write a specification R that provides for placing in k an index where x occurs in a , assuming that x does occur in a .
- Write an (acceptance testing) oracle that checks for correctness with respect to specification R .
- Write a program P that searches for x in a starting at the lower end of the array, assuming that x does appear in a .
- Write a (fault repair) oracle that checks that program P does perform as intended by the programmer (re: question c).

6. Test Outcome Analysis, Chapter 13.

Consider a program whose execution history is recorded in the following table:

Number of faults repaired	Number of executions before the next failure
0	12
1	31
2	30
3	98
4	342
5	875
6	2321

- Using the reliability model $MTTF_N = MTTF_0 \times R^N$, estimate the MTTF of this product once the 7th fault has been repaired.
- Estimate the probability that this product will run for 2400 times without failure.

(6)

a.	N	Inter-Failure Run	Log
	0	12	1.08
	1	31	1.49
	2	30	1.48
	3	98	1.99
	4	342	2.53
	5	875	2.94
	6	2321	3.37

$\log(MTTF_0) = 0.97 \rightarrow MTTF_0 = 9.33$
 $\log R = 0.39 \rightarrow R = 2.45$
 $\therefore MTTF_7 = 9.33 \times 2.45^7 = 4944$

b. $MTTF = 2321$
 $\therefore MTTF = \int_0^{\infty} t \lambda e^{-\lambda t} dt = \frac{1}{\lambda}$
 $\therefore \lambda = \frac{1}{MTTF} = 0.0017$
 $R = e^{-\lambda t} = e^{-\frac{2400}{2321}} = 0.356$
 \therefore the probability is 0.356