Institute of Psychiatry, Psychology and Neuroscience

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Module Title: Introduction to Statistics

Session Title: Hypothesis testing

Topic title: Confidence and significance (II)

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Learning Outcomes

- To understand the idea of hypothesis testing in science
- To understand the role of the sampling distribution

Hypothesis Testing

Let us summarise what we have learned so far:

a) To infer about a **parameter** in the population, we compute a **statistic** in a sample

How many hours per week do you exercise?

Population mean μ=?

Sample mean $\bar{x}$=2.72

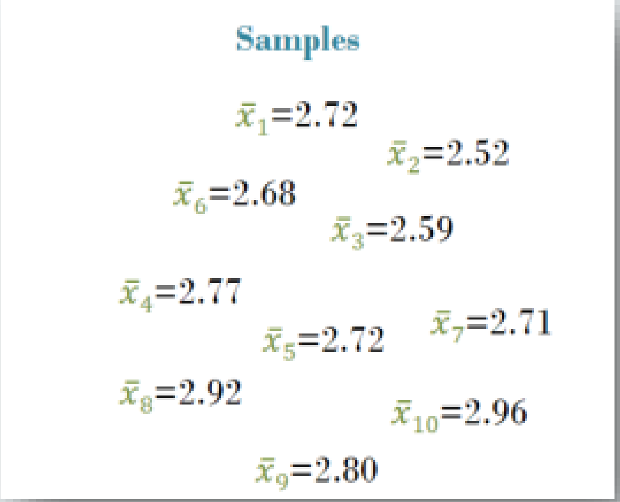
Population

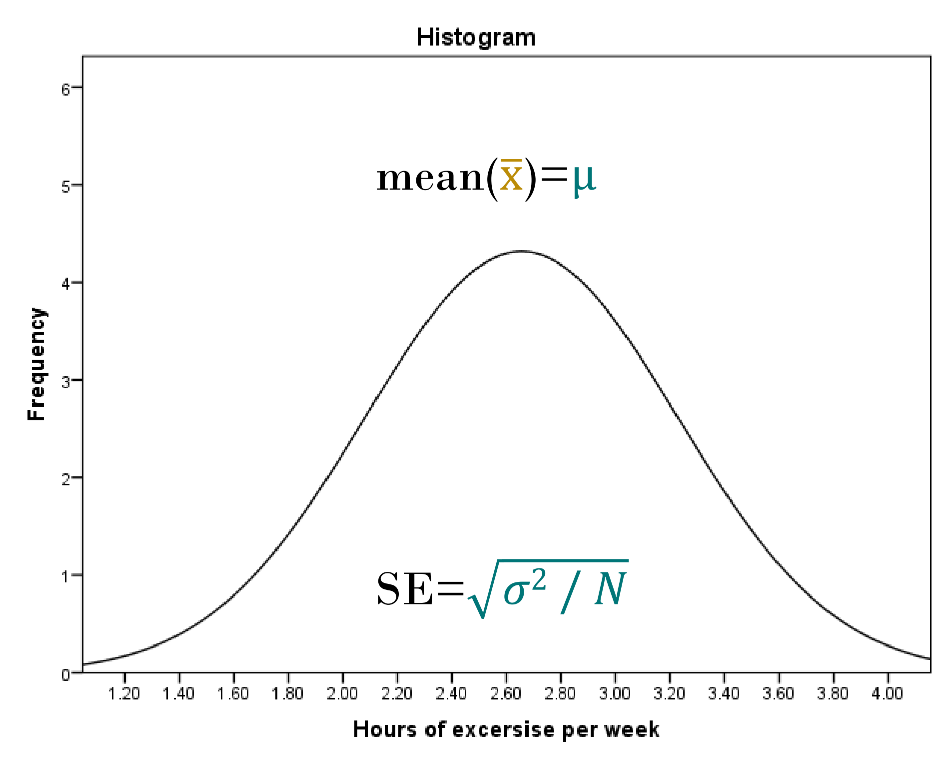
Sample

Hypothesis Testing

b) If we take enough samples from a population, the distribution of the statistic (the **sampling distribution**) will be eventually approximate the normal distribution

How many hours per week do you exercise?





Hypothesis Testing

c) We learned how to state our null and alternative hypotheses

Say, for instance, that for the UK it holds that people exercise on average 2hrs/week. Is this the case for our city?

test value μ_0

How many hours per week do you exercise?

H_0: The mean hours the citizens spend exercising in our city is **equal to the national average: $\mu=\mu_0=2$**.

*H_a: The mean hours the citizens spend exercising in our city is **different than the national average: $\mu \neq \mu_0=2$**.*

Let us know combine all this knowledge to understand hypothesis testing.

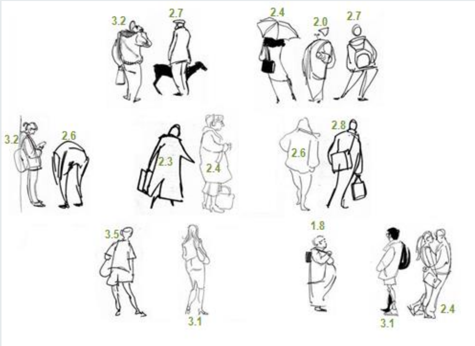
Hypothesis Testing

Step 1: State the hypotheses for the population

City population mean μ=?

H_0: The mean hours the citizens spend exercising is **equal to the national average: $\mu=\mu_0=2$**.
H_a: The mean hours the citizens spend exercising is different than the national average: $\mu \neq \mu_0=2$.

Step 2: Sample from the population and use the correct statistic to estimate the parameter.



Sample mean $\bar{x}=2.66$
Sample stand. dev. $s=0.57$
Sample size $n=31$

Hypothesis Testing

Step 3: Create the sampling distribution assuming that the null hypothesis is true

To draw a normal distribution, we will need its mean (to see where it is located) and standard deviation (to know how spread it is).

- **mean($\bar{x}$)=μ**

H_0: The mean hours the citizens spend exercising is **equal to the national average: $\mu=\mu_0=2$**.

'Under the null hypothesis' the mean: μ=2

- **SE=$\sqrt{\sigma^2 / n}$**

We can estimate this based on our sample:

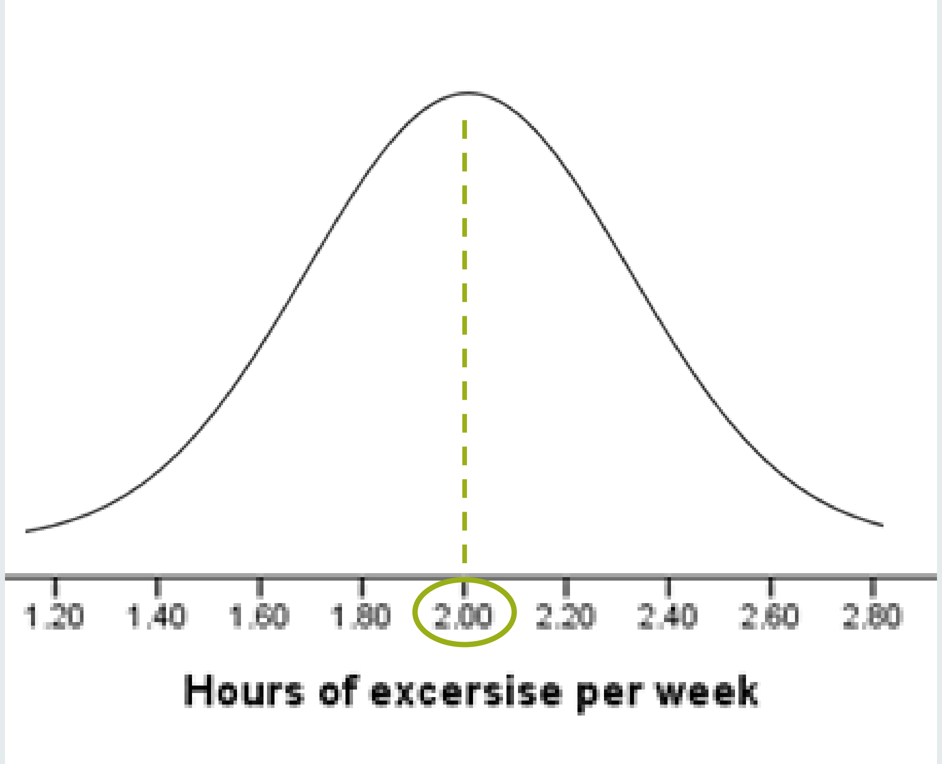
Sample mean	$\bar{x}$=2.66
Sample stand. dev.	s=0.57
Sample size	n= 31

-> $\widehat{SE}=0.57/\sqrt{31} = 0.1$

Hypothesis Testing

Step 3: Create the sampling distribution assuming that the null hypothesis is true

To draw a normal distribution, we will need its mean (to see where it is located) and standard deviation (to know how spread it is).



$\mu=\mu_0=2$

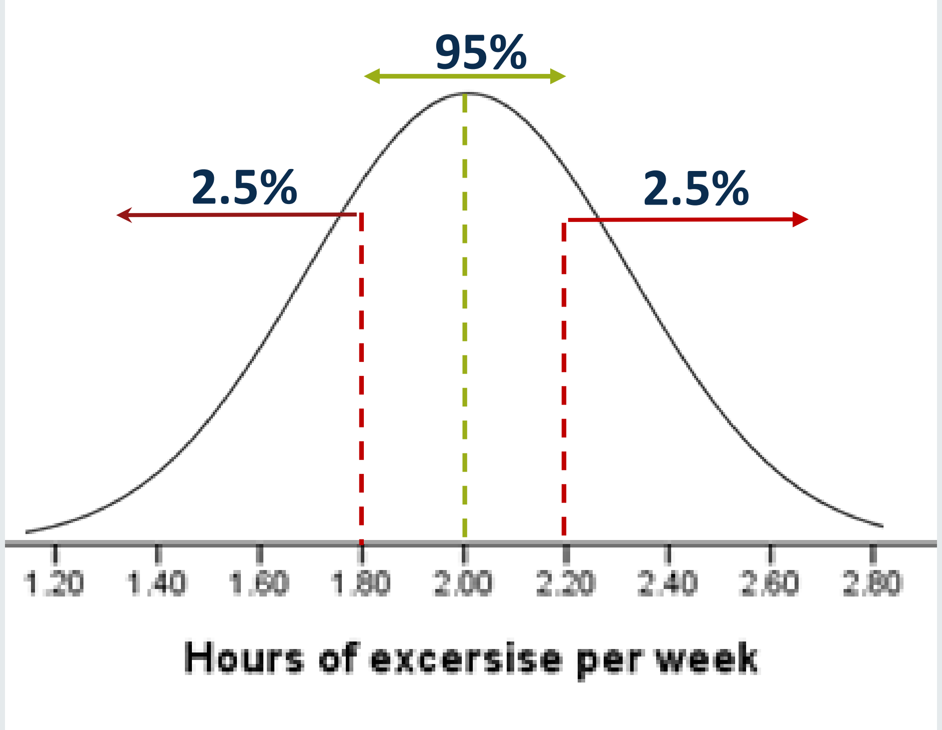
$\widehat{SE}=0.1$

Hypothesis Testing

Step 4: Find the rejection area for the null hypothesis.

$\mu=\mu_0=2$

$\widehat{SE}=0.1$



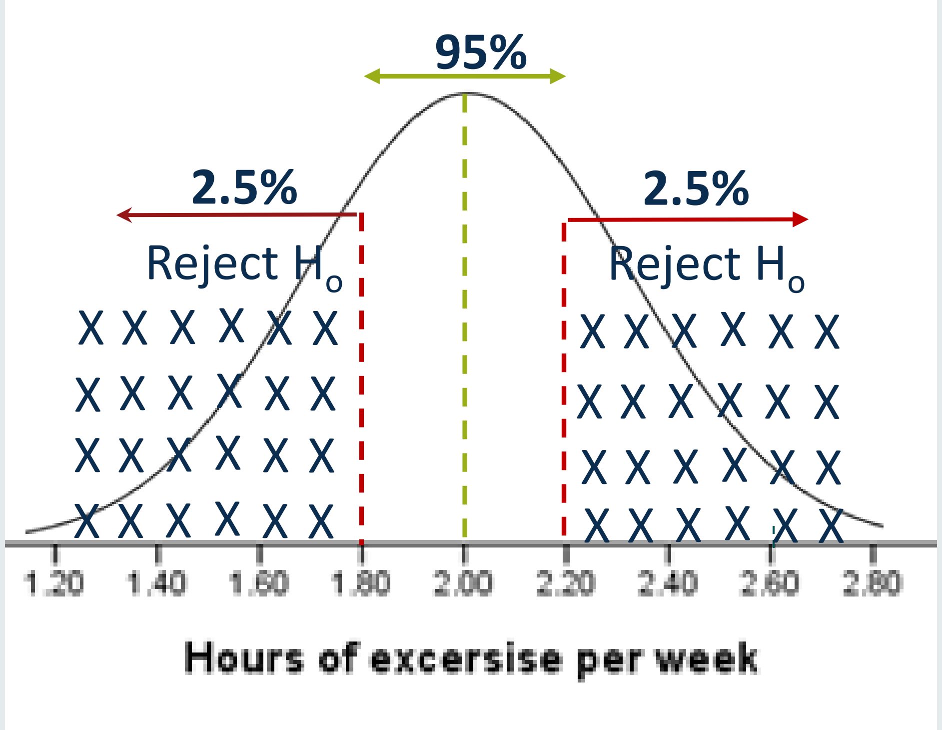
If the null hypothesis is true, then 95% of the values will be between 1.8 and 2.2 (plus-minus 1.96 standard errors from the mean, according to the null).

Thus, there is 2.5% chance for values less than 1.8 and 2.5% chance for values higher than 2.2.

Hypothesis Testing

Step 4: Find the rejection area for the null hypothesis.

$\mu = \mu_0 = 2$
$\widehat{SE} = 0.1$

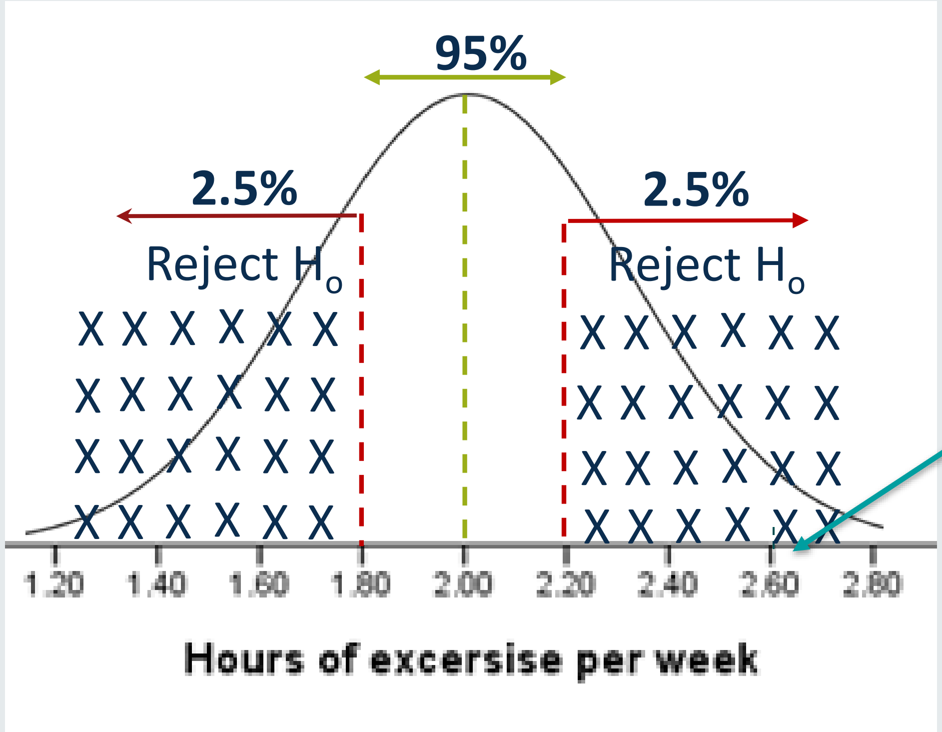


If the value of the statistic (here the sample mean) is not in the 95% interval, then I reject the null hypothesis, as the probability of observing this value if the null hypothesis is true is small.

Hypothesis Testing

Step 4: Find the rejection area for the null hypothesis.

$\mu = \mu_0 = 2$
$\widehat{SE} = 0.1$



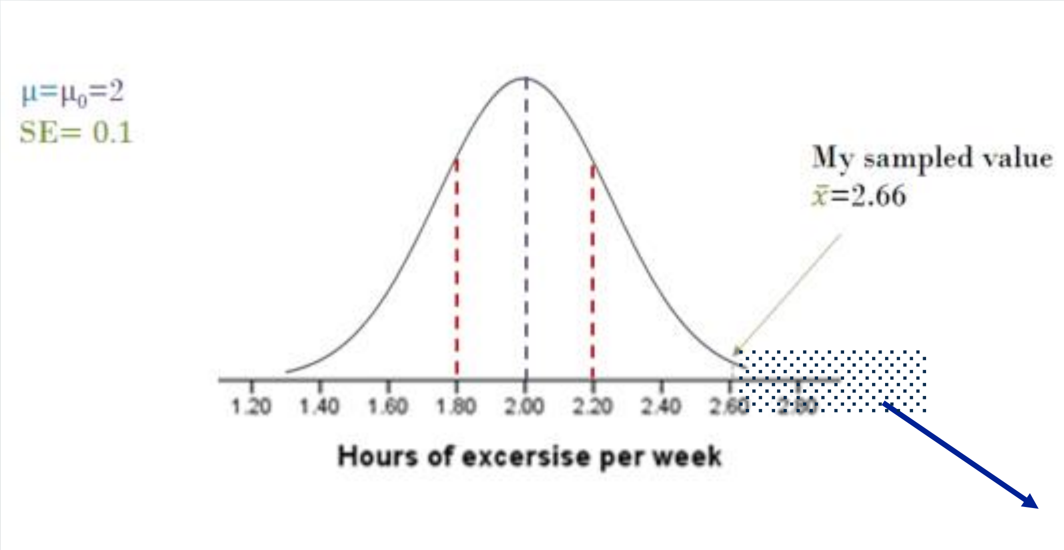
My sampled value $\bar{x}=2.66$

If the null was true, most likely (95% of the times), we wouldn't observe such a value.

If the value of the statistic (here the sample mean) is not in the 95% interval, then I reject the null hypothesis, as the probability of observing this value, if the null hypothesis is true, is small.

Hypothesis Testing

If fact, what we do is to compute **the probability of observing a value equal or more extreme than our sampled value**, under the null hypothesis. This probability is called the **p-value**.



observing a value
of 2.66 or larger

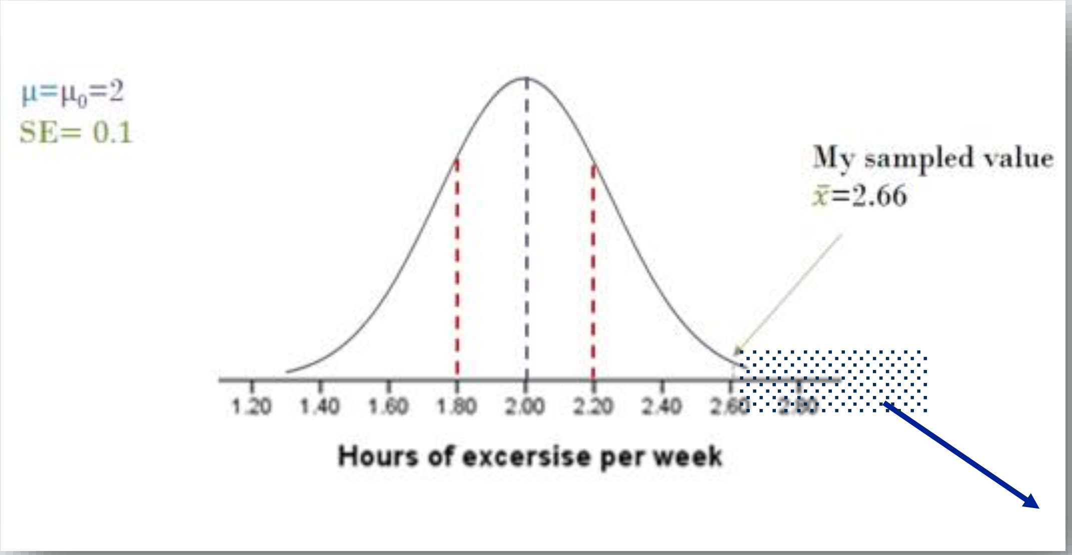
Based on the p-value, we decide if we will reject the null or not.

If the p-value is equal or less than 0.05, we reject the null hypothesis (our value is in the rejection area).

If the p-value is larger than 0.05, we do not reject the null hypothesis (our value is NOT in the rejection area).

Hypothesis Testing

If fact, to make things easier and quicker, what we do is to compute **the probability of observing a value equal or more extreme than our sampled value**, under the null hypothesis. This probability is called the **p-value**.



observing a value of 2.66 or larger

The p-value is the **probability** P-value=P($\bar{x}$≥=2.66| Ho is true)

The p-value estimates the probability of **type 1 error**.

That is why we want the p-value to be small: to be able to reject the null with low risk.

Hypothesis Testing

We will be repeating this procedure for all tests that we will learn in this course!

Step 1: Create the **null** and the **alternative** hypothesis for the population parameter.

Step 2: **Sample** from the population and compute the correct **statistic** to **estimate** the parameter.

Step 3: Create the **sampling distribution** for this statistic, under the null.

Step 4: Find the **rejection area**.

Step 5: Check if your **sampled** value **falls** in the rejection area.

Knowledge Check

The average income in a field for men is £40000 per year. A researcher wants to test if that is the case for women as well. She estimated values in a sample of 100 women being $\bar{x}$=£35.2K and SD=£2K.

a) State the null and the alternative hypothesis.

H_0: $\mu = \mu_0$ =£40000 ⟷ μ-£40000=0

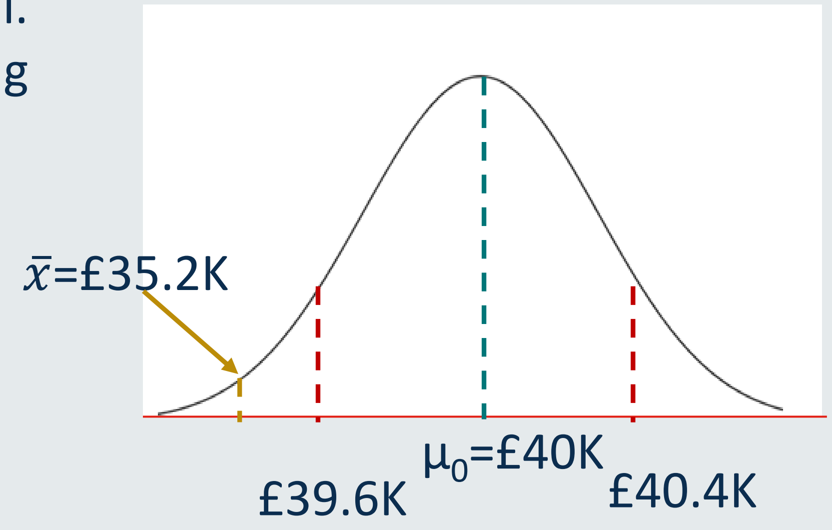
H_α: $\mu \neq \mu_0$ =£40000 ⟷ μ-£40000 ≠ 0

b) Using the sample values, compute the rejection area.

μ_0=£40000 $\quad se = \frac{2000}{10} = £200$

$LL: \ \mu_0 - 1.96 \ * \text{se} = £39.6\text{K}$

$UL: \ \mu_0 + 1.96 \ * \text{se} = £40.4\text{K}$



Under the null hypothesis, the 95% CI is [39.6, 40.4]. Our sampled average income is outside this internal, therefore we reject the null hypothesis.

c) Are the average income for men and women statistically different?

The sampled value was in the rejection area, so we reject the null hypothesis in favour of the alternative.

Reflection

Reflecting on your own research projects

What is your main variable of interest in your project?
How would you summarise this variable?
What would be the hypotheses you want to test?
What would you expect for your sample statistics (mean, median, proportion) in order to reject your null hypothesis?

Thank you

Please contact your module leader or the course lecturer of your programme, or visit the module's forum for any questions you may have.

If you have comments on the materials (spotted typos or missing points) please contact Dr Vitoratou:

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