CIS 315, Intermediate Algorithms
Spring 2020

Sample Recurrence for Assignment 5

1 recall

The situation for this programming assignment is that we have

- coins of denominations $d_1, d_2, \ldots, d_n$
- a target value T
- where all denominations are distinct (though this is not important to the problem or solution)
- and 5 coins of each denomination

The problem is to write code that will determine the *maximum* number of coins that will add up to the target value T (exactly). Here we describe a recurrence relation to get you started on the code part.

2 subproblem

Define $MC(t, k)$ to be the maximum number of coins chosen from at most 5 each of $d_1, d_2, \ldots, d_k$ that add up to exactly t. The valid ranges of t and k are $0 \leq t \leq T$ and $0 \leq k \leq n$ (although for convenience below we allow $t < 0$ to be considered).

3 recurrence

The idea for inputs t and k (for coins $d_1, d_2, \ldots d_k$) is to try all allowed amounts of coin d_k and test the effects of that while using coins $d_1, d_2, \ldots d_{k-1}$ on the remaining value (using $MC(*, k-1)$ recursively). If i of the d_k coins are used, the remaining target value is $t - i \cdot d_k$, so i should be added to $MC(t - i \cdot d_k,\ k-1)$.

$$MC(t,k) = \begin{cases} 0 & \text{if } t = 0 \\ -\infty & \text{if } t < 0 \\ -\infty & \text{if } k = 0 \text{ and } t > 0 \\ \max\{\ i + MC(t - i \cdot d_k, k-1) \mid 0 \leq i \leq 5\ \} & \text{otherwise} \end{cases}$$

4 target value

Want to return $MC(T, n)$. In this formulation it is not possible to get a set of coins to add up to T if $MC(T, n) < 0$.