Programming Homework 4

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1 Question

For the periodic boundary question

$$\begin{cases} u_t = u_{xx}, \quad x \in [0, 2\pi], t \geqslant 0 \\ u(x,0) = 1 + \sin(x) + \sin(10x), \quad x \in [0, 2\pi] \end{cases}$$

(1)Using FTCS scheme to compute the value of u(x,t) at $t = 0, 10^{-2}, 1$ and plot the picture. $\delta = \frac{\Delta t}{\Delta x^2} = \frac{1}{2}, \quad$ N=100. Then let $\delta = 0.8$ and $\delta = 1$, compute the value of u(x,t) at $t = 0, 10^{-2}, 1$ and plot the picture.Compare the difference and get the conclusion.

(2)Using BTCS scheme to compute the value of u(x,t) at $t = 0, 10^{-2}, 1$ and plot the picture. $\Delta t = \Delta x$, $N = 100$. N is the number of space division.

2 Algorithm

2.1 FTCS

Here, using FTCS scheme, for every node, we can get the approximate formula

$$v_j^{n+1} = v_j^n + \frac{\Delta x}{\Delta x^2}(v_{j+1}^n - 2v_j^n + vj - 1^n) \quad j = 0, 1, \ldots, N$$

and we have periodic boundary condition

$$v_j^n = v_{j+N}^n \quad for 0 \leq n \leq \frac{T}{t_n}$$

We can iterate, and get the numerical solution.

2.2 BTCS

We use BTCS scheme, and we can get the approximate formula

$$v_j^{n+1} = v_j^n + \frac{\Delta x}{\Delta x^2}(v_{j+1}^{n+1} - 2v_j^{n+1} + vj - 1^{n+1}) \quad j = 0, 1, \dots, N$$

Then we get the implicit scheme, and we can write in matrix form

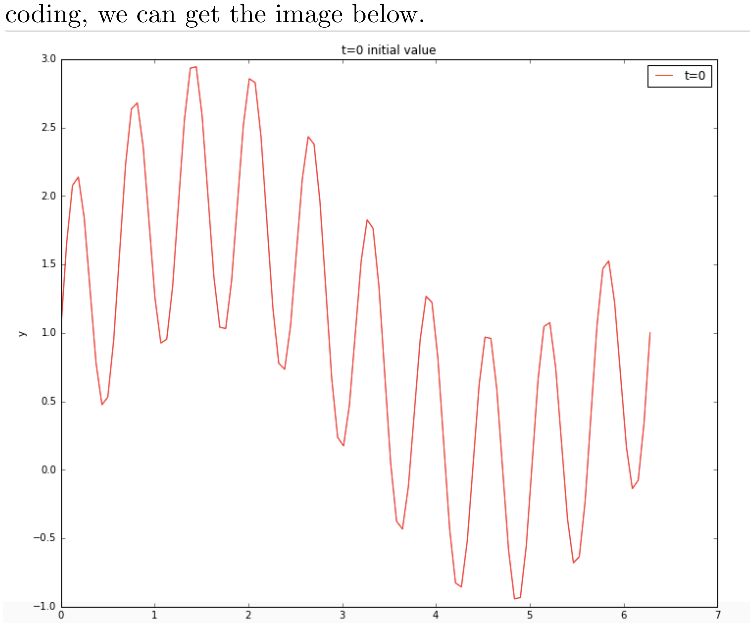
$$Av^{n+1} = v^n$$

$$A = \begin{pmatrix} 1+2\delta & -\delta & 0 & \dots & -\delta \\ -\delta & 1+2\delta & -\delta & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ -\delta & 0 & \dots & -\delta & 1+2\delta \end{pmatrix}$$

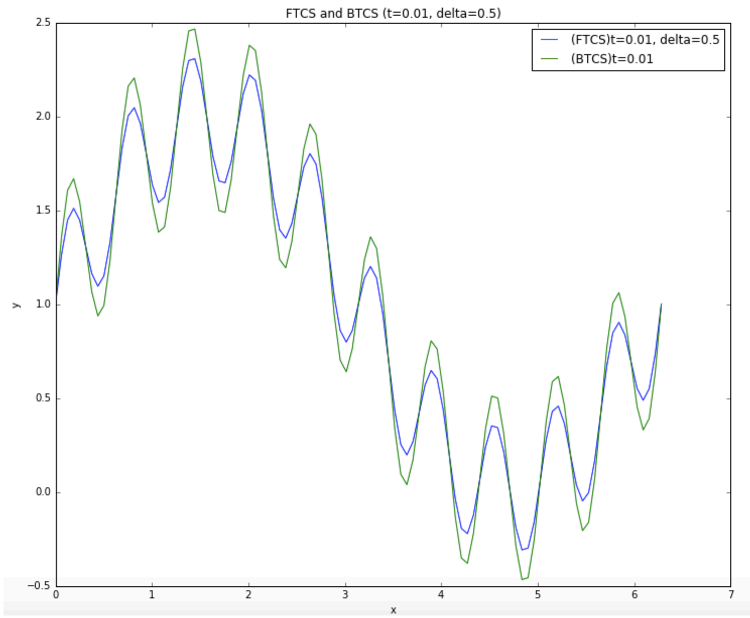
Then we can solve this linear equations by compute the inverse of matrix A, and then we can get the numerical solution by $v^{n+1} = A^{-1}v^n$

3 Results and Analysis

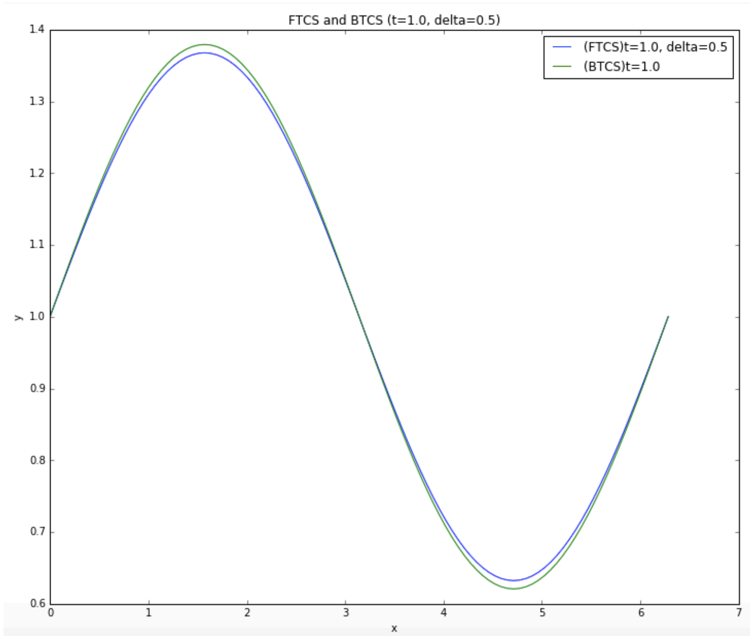
By coding, we can get the image below.



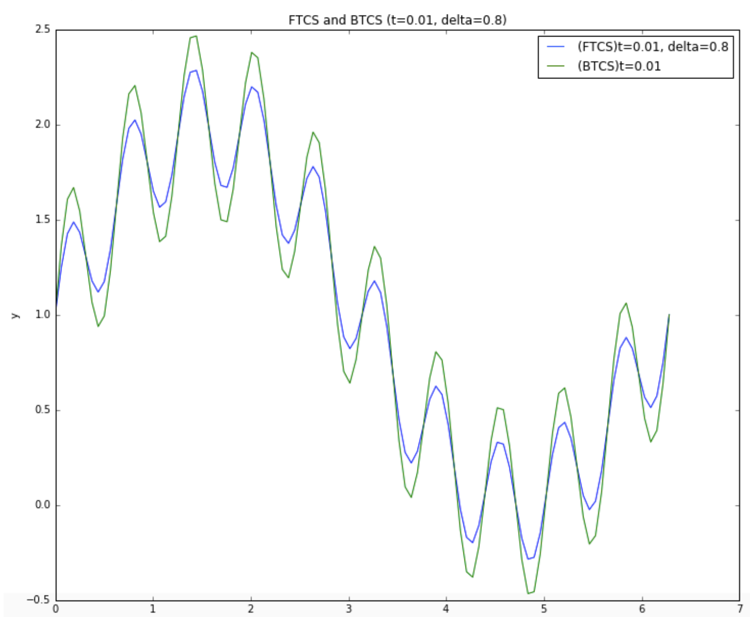
First, we can get the value of the function at t=0. Whatever scheme, its initial value is the same, so we plot only one picture. We can see the result.



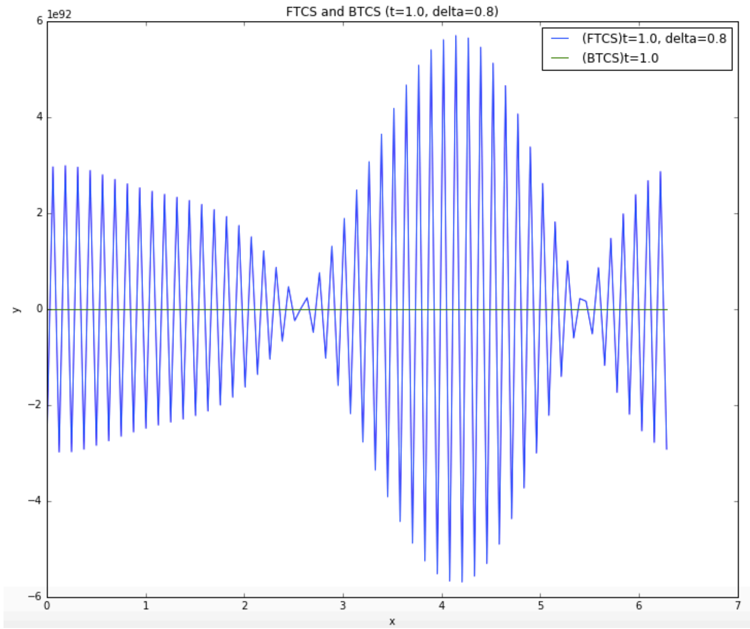
For the second picture, we can compute the numerical solution at t=0.01 by using FTCS scheme and BTCS scheme. For the blue one, it's the FTCS's result when $\delta = 0.5$. For the green one, it's the BTCS's result. By the picture, we can see that there is a little different between the two results. At the bottom and top of the function values, there are some relatively big errors. For BTCS scheme, it has a larger swing than FTCS scheme.



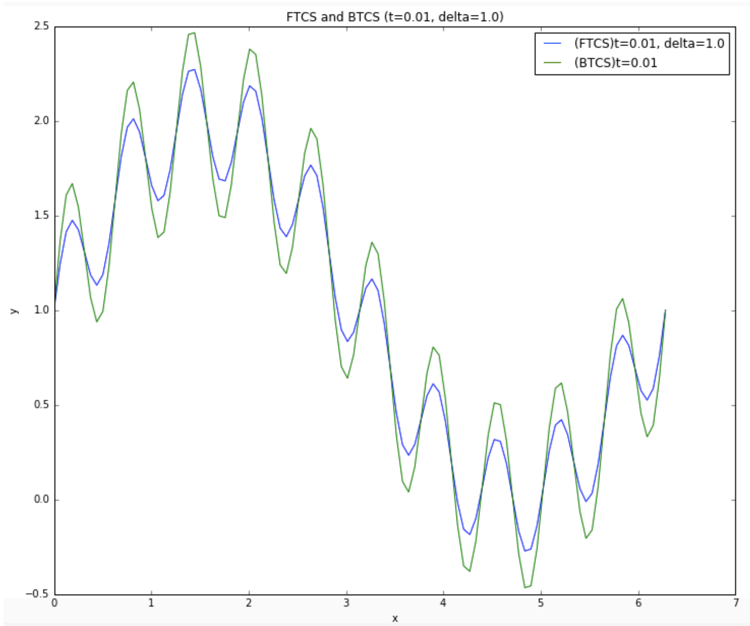
For the third picture, it's the result at t=1.0 for FTCS and BTCS. We can see that both of them converge and have a good result. Again, BTCS scheme is more bias than FTCS. Maybe it's because BTCS is a implict scheme and it's unconditionally stable.



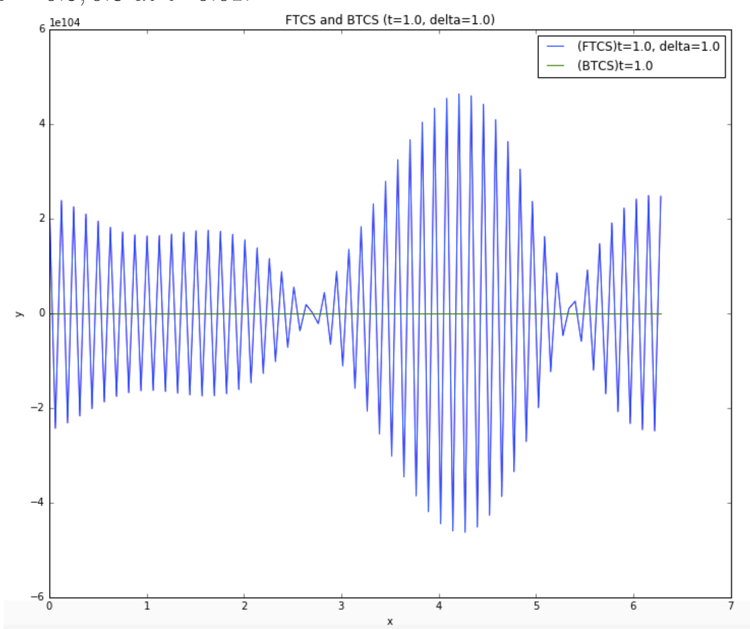
For the forth picture, it's the result at t=0.01 when $\delta = 0.8$, we can see that it's almost the same as the figure 1. So we can say that at t=0.01, they are the same.



From this picture, we can see that at t=1, the FTCS can't converge. We can get that FTCS's y is so large from the picture that it makes BTCS's y almost a line.



From this picture, we get the result at t=0.01 when $\delta = 1.0$, it's also the same as $\delta = 0.5, 0.8$ at t=0.01.



This picture tells us the same result as Figure 5, and FTCS doesn't converge.

Summary Above all, we can see that because of BTCS is unconditionally stable, so it always has a good result. And for FTCS, if $\delta = 0.5$, we can get a good result, but if $\delta > 0.5$, such as 0.8 or 1.0 just like in the example, the result may not converge.