Kalman Filter Algorithm

Content

- Introduction
- Process
- Example
- Practice

Introduction

- **Kalman Filter Algorithm**
  - For statistics and control theory, Kalman filtering, also known as linear quadratic estimation (LQE), is an algorithm that uses a series of measurements observed over time, including statistical noise and other inaccuracies, and produces estimates of unknown variables that tend to be more accurate than those based on a single measurement alone, by estimating a joint probability distribution over the variables for each timeframe.
  - Kalman filtering has numerous technological applications. A common application is for guidance, navigation, and control of vehicles, particularly aircraft, spacecraft and ships positioned dynamically. Furthermore, Kalman filtering is a concept much applied in time series analysis used for topics such as signal processing and econometrics. Kalman filtering is also one of the main topics of robotic motion planning and control and can be used for trajectory optimization.

Process

– The Kalman filter process has two steps: the prediction step, where the next state of the system is predicted given the previous measurements, and the update step, where the current state of the system is estimated given the measurement at that time step.

Process

- The steps translate to equations as follows:
- Prediction:

$$\hat{\boldsymbol{x}}_{n+1,n} = \boldsymbol{F}\hat{\boldsymbol{x}}_{n,n} + \boldsymbol{G}\boldsymbol{u}_n + \boldsymbol{w}_n$$

Where:

$\hat{\boldsymbol{x}}_{n+1,n}$	is a predicted system state vector at time step $n+1$
$\hat{\boldsymbol{x}}_{n,n}$	is an estimated system state vector at time step n
$\boldsymbol{u}_n$	is a control variable
$\boldsymbol{w}_n$	is the process noise
$\boldsymbol{F}$	is the state transition matrix
$\boldsymbol{G}$	is the control matrix

Process

- Prediction:

$$\boldsymbol{P}_{n+1,n} = \boldsymbol{F}\boldsymbol{P}_{n,n}\boldsymbol{F}^{\boldsymbol{T}} + \boldsymbol{Q}$$

Where:

$\boldsymbol{P}_{n,n}$	is the estimate uncertainty (covariance) matrix of the current state
$\boldsymbol{P}_{n+1,n}$	is the predicted estimate uncertainty (covariance) matrix for the next state
$\boldsymbol{F}$	is the state transition matrix
$\boldsymbol{Q}$	is the process noise matrix

Process

- Update:
  - The generalized measurement equation in the matrix form is given by:

$$\boldsymbol{z_n} = \boldsymbol{H}\boldsymbol{x_n} + \boldsymbol{v_n}$$

Where:

$\boldsymbol{z_n}$ is the measurement vector

$\boldsymbol{x_n}$ is the true system state (hidden state)

$\boldsymbol{v_n}$ the random noise vector

$\boldsymbol{H}$ is the observation matrix

Process

- Update:
  - The Kalman Gain in matrix notation is given by:

$$\boldsymbol{K_n} = \boldsymbol{P_{n,n-1}} \boldsymbol{H}^{\boldsymbol{T}} \left(\boldsymbol{H} \boldsymbol{P_{n,n-1}} \boldsymbol{H}^{\boldsymbol{T}} + \boldsymbol{R_n} \right)^{-1}$$

where:

$\boldsymbol{K_n}$ is the Kalman Gain

$\boldsymbol{P_{n,n-1}}$ is a prior estimate uncertainty (covariance) matrix of the current state (predicted at the previous state)

$\boldsymbol{H}$ is the observation matrix

$\boldsymbol{R_n}$ is the Measurement Uncertainty (measurement noise covariance matrix)

Process

- Update:
  - The State Update Equation in matrix form is given by:

$$\hat{\boldsymbol{x}}_{n,n} = \hat{\boldsymbol{x}}_{n,n-1} + \boldsymbol{K}_n(\boldsymbol{z}_n - \boldsymbol{H}\hat{\boldsymbol{x}}_{n,n-1})$$

where:

$\hat{\boldsymbol{x}}_{n,n}$	is the estimated system state vector at time step n
$\hat{\boldsymbol{x}}_{n,n-1}$	is the predicted system state vector at time step $n-1$
$\boldsymbol{K}_n$	is the Kalman Gain
$\boldsymbol{z}_n$	is a measurement
$\boldsymbol{H}$	is the observation matrix

Process

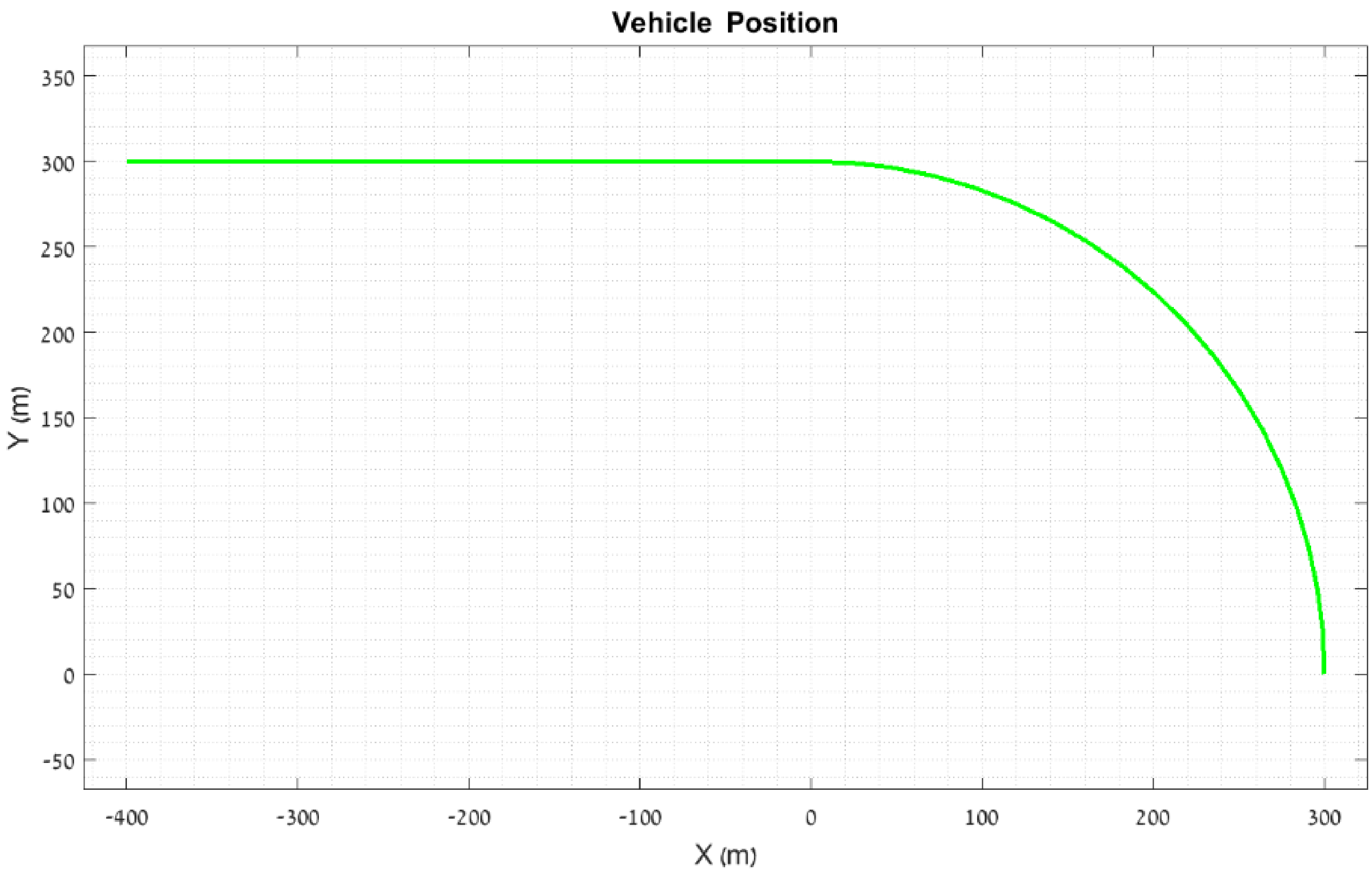
- Update:
  - The Covariance Update Equation is given by:

$$\boldsymbol{P_{n,n}} = (\boldsymbol{I} - \boldsymbol{K_n H})\,\boldsymbol{P_{n,n-1}}(\boldsymbol{I} - \boldsymbol{K_n H})^{\boldsymbol{T}} + \boldsymbol{K_n R_n K_n^T}$$

where:

$\boldsymbol{P_{n,n}}$	is the estimate uncertainty (covariance) matrix of the current state
$\boldsymbol{P_{n,n-1}}$	is the prior estimate uncertainty (covariance) matrix of the current state (predicted at the previous state)
$\boldsymbol{K_n}$	is the Kalman Gain
$\boldsymbol{H}$	is the observation matrix
$\boldsymbol{R_n}$	is the Measurement Uncertainty (measurement noise covariance matrix)

Example



- The measurements period:

$$\Delta t = 1s$$

Example

- THE STATE EXTRAPOLATION EQUATION

  - For our example, the state extrapolation equation can be simplified to(In this example we don't have a control variable u since we don't have control input.):

$$\hat{\boldsymbol{x}}_{n+1,n} = \boldsymbol{F}\hat{\boldsymbol{x}}_{n,n}$$

  - The system state x_n is defined by:

$$\boldsymbol{x}_n = \begin{bmatrix} \hat{x}_n \\ \hat{\dot{x}}_n \\ \hat{\ddot{x}}_n \\ \hat{y}_n \\ \hat{\dot{y}}_n \\ \hat{\ddot{y}}_n \end{bmatrix}$$

Example

- The extrapolated vehicle state for time k can be described by the following system of equations:

$$
\begin{aligned}
\hat{x}_{n+1,n} &= \hat{x}_{n,n} + \hat{\dot{x}}_{n,n}\Delta t + \tfrac{1}{2}\hat{\ddot{x}}_{n,n}\Delta t^2 \\
\hat{\dot{x}}_{n+1,n} &= \hat{\dot{x}}_{n,n} + \hat{\ddot{x}}_{n,n}\Delta t \\
\hat{\ddot{x}}_{n+1,n} &= \hat{\ddot{x}}_{n,n} \\
\hat{y}_{n+1,n} &= \hat{y}_{n,n} + \hat{\dot{y}}_{n,n}\Delta t + \tfrac{1}{2}\hat{\ddot{y}}_{n,n}\Delta t^2 \\
\hat{\dot{y}}_{n+1,n} &= \hat{\dot{y}}_{n,n} + \hat{\ddot{y}}_{n,n}\Delta t \\
\hat{\ddot{y}}_{n+1,n} &= \hat{\ddot{y}}_{n,n}
\end{aligned}
$$

$$
\begin{bmatrix} \hat{x}_{n+1,n} \\ \hat{\dot{x}}_{n+1,n} \\ \hat{\ddot{x}}_{n+1,n} \\ \hat{y}_{n+1,n} \\ \hat{\dot{y}}_{n+1,n} \\ \hat{\ddot{y}}_{n+1,n} \end{bmatrix}
=
\begin{bmatrix}
1 & \Delta t & 0.5\Delta t^2 & 0 & 0 & 0 \\
0 & 1 & \Delta t & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & \Delta t & 0.5\Delta t^2 \\
0 & 0 & 0 & 0 & 1 & \Delta t \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} \hat{x}_{n,n} \\ \hat{\dot{x}}_{n,n} \\ \hat{\ddot{x}}_{n,n} \\ \hat{y}_{n,n} \\ \hat{\dot{y}}_{n,n} \\ \hat{\ddot{y}}_{n,n} \end{bmatrix}
$$

Example

➢ THE COVARIANCE EXTRAPOLATION EQUATION

- The general form of the Covariance Extrapolation Equation is given by:

$$\boldsymbol{P_{n+1,n} = FP_{n,n}F^T + Q}$$

- The estimate uncertainty in matrix form is:

$$\boldsymbol{P} = \begin{bmatrix} p_x & p_{x\dot{x}} & p_{x\ddot{x}} & p_{xy} & p_{x\dot{y}} & p_{x\ddot{y}} \\ p_{\dot{x}x} & p_{\dot{x}} & p_{\dot{x}\ddot{x}} & p_{\dot{x}y} & p_{\dot{x}\dot{y}} & p_{\dot{x}\ddot{y}} \\ p_{\ddot{x}x} & p_{\ddot{x}\dot{x}} & p_{\ddot{x}} & p_{\ddot{x}y} & p_{\ddot{x}\dot{y}} & p_{\ddot{x}\ddot{y}} \\ p_{yx} & p_{y\dot{x}} & p_{y\ddot{x}} & p_y & p_{y\dot{y}} & p_{y\ddot{y}} \\ p_{\dot{y}x} & p_{\dot{y}\dot{x}} & p_{\dot{y}\ddot{x}} & p_{\dot{y}y} & p_{\dot{y}} & p_{\dot{y}\ddot{y}} \\ p_{\ddot{y}x} & p_{\ddot{y}\dot{x}} & p_{\ddot{y}\ddot{x}} & p_{\ddot{y}y} & p_{\ddot{y}\dot{y}} & p_{\ddot{y}} \end{bmatrix}$$

Example

- We will assume that the estimation errors in X and Y axes are not correlated, so the mutual terms can be set to zero:

$$\boldsymbol{P} = \begin{bmatrix} p_x & p_{x\dot{x}} & p_{x\ddot{x}} & 0 & 0 & 0 \\ p_{\dot{x}x} & p_{\dot{x}} & p_{\dot{x}\ddot{x}} & 0 & 0 & 0 \\ p_{\ddot{x}x} & p_{\ddot{x}\dot{x}} & p_{\ddot{x}} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_y & p_{y\dot{y}} & p_{y\ddot{y}} \\ 0 & 0 & 0 & p_{\dot{y}y} & p_{\dot{y}} & p_{\dot{y}\ddot{y}} \\ 0 & 0 & 0 & p_{\ddot{y}y} & p_{\ddot{y}\dot{y}} & p_{\ddot{y}} \end{bmatrix}$$

Example

- THE PROCESS NOISE MATRIX

● We will assume a discrete noise model - the noise is different at each time period, but it is constant between time periods. The process noise matrix for the two-dimensional constant acceleration model looks as following:

$$\boldsymbol{Q} = \begin{bmatrix} \sigma_x^2 & \sigma_{x\dot{x}}^2 & \sigma_{x\ddot{x}}^2 & \sigma_{xy}^2 & \sigma_{x\dot{y}}^2 & \sigma_{x\ddot{y}}^2 \\ \sigma_{\dot{x}x}^2 & \sigma_{\dot{x}}^2 & \sigma_{\dot{x}\ddot{x}}^2 & \sigma_{\dot{x}y}^2 & \sigma_{\dot{x}\dot{y}}^2 & \sigma_{\dot{x}\ddot{y}}^2 \\ \sigma_{\ddot{x}x}^2 & \sigma_{\ddot{x}\dot{x}}^2 & \sigma_{\ddot{x}}^2 & \sigma_{\ddot{x}y}^2 & \sigma_{\ddot{x}\dot{y}}^2 & \sigma_{\ddot{x}\ddot{y}}^2 \\ \sigma_{yx}^2 & \sigma_{y\dot{x}}^2 & \sigma_{y\ddot{x}}^2 & \sigma_y^2 & \sigma_{y\dot{y}}^2 & \sigma_{y\ddot{y}}^2 \\ \sigma_{\dot{y}x}^2 & \sigma_{\dot{y}\dot{x}}^2 & \sigma_{\dot{y}\ddot{x}}^2 & \sigma_{\dot{y}y}^2 & \sigma_{\dot{y}}^2 & \sigma_{\dot{y}\ddot{y}}^2 \\ \sigma_{\ddot{y}x}^2 & \sigma_{\ddot{y}\dot{x}}^2 & \sigma_{\ddot{y}\ddot{x}}^2 & \sigma_{\ddot{y}y}^2 & \sigma_{\ddot{y}\dot{y}}^2 & \sigma_{\ddot{y}}^2 \end{bmatrix}$$

Example

- We will assume that the process noise in X and Y axes is not correlated, so the mutual terms can be set to zero:

$$
\boldsymbol{Q} = \begin{bmatrix}
\sigma_x^2 & \sigma_{x\dot{x}}^2 & \sigma_{x\ddot{x}}^2 & 0 & 0 & 0 \\
\sigma_{\dot{x}x}^2 & \sigma_{\dot{x}}^2 & \sigma_{\dot{x}\ddot{x}}^2 & 0 & 0 & 0 \\
\sigma_{\ddot{x}x}^2 & \sigma_{\ddot{x}\dot{x}}^2 & \sigma_{\ddot{x}}^2 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_y^2 & \sigma_{y\dot{y}}^2 & \sigma_{y\ddot{y}}^2 \\
0 & 0 & 0 & \sigma_{\dot{y}y}^2 & \sigma_{\dot{y}}^2 & \sigma_{\dot{y}\ddot{y}}^2 \\
0 & 0 & 0 & \sigma_{\ddot{y}y}^2 & \sigma_{\ddot{y}\dot{y}}^2 & \sigma_{\ddot{y}}^2
\end{bmatrix}
$$

Example

- We've already derived the Q matrix for the constant acceleration motion model. The Q matrix for our example is:

$$\boldsymbol{Q} = \begin{bmatrix} \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} & \frac{\Delta t^2}{2} & 0 & 0 & 0 \\ \frac{\Delta t^3}{2} & \Delta t^2 & \Delta t & 0 & 0 & 0 \\ \frac{\Delta t^2}{2} & \Delta t & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} & \frac{\Delta t^2}{2} \\ 0 & 0 & 0 & \frac{\Delta t^3}{2} & \Delta t^2 & \Delta t \\ 0 & 0 & 0 & \frac{\Delta t^2}{2} & \Delta t & 1 \end{bmatrix} \sigma_a^2$$

- where:

- Δt is the time between successive measurements
- σ_a^2 is a random variance in acceleration

Example

➢ THE MEASUREMENT UNCERTAINTY

- The measurement covariance matrix is:

$$\boldsymbol{R}_n = \begin{bmatrix} \sigma^2_{x_m} & \sigma^2_{yx_m} \\ \sigma^2_{xy_m} & \sigma^2_{y_m} \end{bmatrix}$$

- The subscript m is for measurement uncertainty.
- Assume that the x and y measurements are uncorrelated, i.e. error in the x coordinate measurement doesn't depend on the error in the y coordinate measurement.

$$\boldsymbol{R}_n = \begin{bmatrix} \sigma^2_{x_m} & 0 \\ 0 & \sigma^2_{y_m} \end{bmatrix}$$

Example

- The state transition matrix A would be(The measurements period: Δt=1s):

$$F = \begin{bmatrix} 1 & \Delta t & 0.5\Delta t^2 & 0 & 0 & 0 \\ 0 & 1 & \Delta t & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \Delta t & 0.5\Delta t^2 \\ 0 & 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0.5 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- The process noise matrix Q would be(The random acceleration standard deviation: $\sigma_a = 0.2\frac{m}{s^2}$):

$$\boldsymbol{Q} = \begin{bmatrix} \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} & \frac{\Delta t^2}{2} & 0 & 0 & 0 \\ \frac{\Delta t^3}{2} & \Delta t^2 & \Delta t & 0 & 0 & 0 \\ \frac{\Delta t^2}{2} & \Delta t & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} & \frac{\Delta t^2}{2} \\ 0 & 0 & 0 & \frac{\Delta t^3}{2} & \Delta t^2 & \Delta t \\ 0 & 0 & 0 & \frac{\Delta t^2}{2} & \Delta t & 1 \end{bmatrix} \sigma_a^2 = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 1 \\ 0 & 0 & 0 & \frac{1}{2} & 1 & 1 \end{bmatrix} 0.2^2$$

Example

- The observation matrix H would be:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

- The measurement uncertainty R would be(The measurement error standard deviation: σ_{xm}=σ_{ym}=3m):

$$\boldsymbol{R_n} = \begin{bmatrix} \sigma_{x_m}^2 & 0 \\ 0 & \sigma_{y_m}^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

- The following table contains the set of 35 noisy measurements:

	1	2...	35
x(m)	-393.66	-375.93...	299.89
y(m)	300.4	301.78...	2.14

Example

➢ INITIALIZATION

- We don't know the vehicle location; we will set initial position, velocity and acceleration to 0.

$$X_{0,0} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

- Since our initial state vector is a guess, we will set a very high estimate uncertainty. The high estimate uncertainty results in a high Kalman Gain, giving a high weight to the measurement:

$$p_{0,0} = \begin{bmatrix} 500 & 0 & 0 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 & 0 & 0 \\ 0 & 0 & 500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 500 & 0 & 0 \\ 0 & 0 & 0 & 0 & 500 & 0 \\ 0 & 0 & 0 & 0 & 0 & 500 \end{bmatrix}$$

Example

- PREDICTION
  - Now we can predict the next state based on the initialization values.

$$\hat{\boldsymbol{x}}_{1,0} = \boldsymbol{F}\hat{\boldsymbol{x}}_{0,0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{P}_{1,0} = \boldsymbol{F}\boldsymbol{P}_{0,0}\boldsymbol{F}^T + \boldsymbol{Q} = \begin{bmatrix} 1125 & 750 & 250 & 0 & 0 & 0 \\ 750 & 1000 & 500 & 0 & 0 & 0 \\ 250 & 500 & 500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1125 & 750 & 250 \\ 0 & 0 & 0 & 750 & 1000 & 500 \\ 0 & 0 & 0 & 250 & 500 & 500 \end{bmatrix}$$

Example

- **FIRST ITERATION**
  - STEP 1 - MEASURE
    - The measurement values:

$$z_1 = \begin{bmatrix} -393.66 \\ 300.4 \end{bmatrix}$$

  - STEP 2 - UPDATE
    - The Kalman Gain calculation:

$$K_1 = P_{1,0}H^T\left(HP_{1,0}H^T + R\right)^{-1} = \begin{bmatrix} 0.9921 & 0 \\ 0.6614 & 0 \\ 0.2205 & 0 \\ 0 & 0.9921 \\ 0 & 0.6614 \\ 0 & 0.2205 \end{bmatrix}$$

As you can see, the Kalman Gain for position is 0.9921, that means that the weight of the first measurement is significantly higher than the weight of the estimation. The weight of the estimation is negligible

Example

- Estimate the current state:

$$\hat{\boldsymbol{x}}_{1,1} = \hat{\boldsymbol{x}}_{1,0} + \boldsymbol{K}_1(\boldsymbol{z}_1 - \boldsymbol{H}\hat{\boldsymbol{x}}_{1,0}) = \begin{bmatrix} -390.54 \\ -260.36 \\ -86.8 \\ 298.02 \\ 198.7 \\ 66.23 \end{bmatrix}$$

- Update the current estimate uncertainty:

$$\boldsymbol{P}_{1,1} = (\boldsymbol{I} - \boldsymbol{K}_1\boldsymbol{H})\boldsymbol{P}_{1,0}(\boldsymbol{I} - \boldsymbol{K}_1\boldsymbol{H})^T + \boldsymbol{K}_1\boldsymbol{R}\boldsymbol{K}_1^T = \begin{bmatrix} 8.93 & 5.95 & 2 & 0 & 0 & 0 \\ 5.95 & 504 & 334.7 & 0 & 0 & 0 \\ 2 & 334.7 & 444.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8.93 & 5.95 & 2 \\ 0 & 0 & 0 & 5.95 & 504 & 334.7 \\ 0 & 0 & 0 & 2 & 334.7 & 444.9 \end{bmatrix}$$

Example

- STEP 3 - PREDICT

$$\hat{\boldsymbol{x}}_{2,1} = \boldsymbol{F}\hat{\boldsymbol{x}}_{1,1} = \begin{bmatrix} -694.3 \\ -347.15 \\ -86.8 \\ 529.8 \\ 264.9 \\ 66.23 \end{bmatrix}$$

$$\boldsymbol{P}_{2,1} = \boldsymbol{F}\boldsymbol{P}_{1,1}\boldsymbol{F}^{T} + \boldsymbol{Q} = \begin{bmatrix} 972 & 1236 & 559 & 0 & 0 & 0 \\ 1236 & 1618 & 780 & 0 & 0 & 0 \\ 559 & 780 & 445 & 0 & 0 & 0 \\ 0 & 0 & 0 & 972 & 1236 & 559 \\ 0 & 0 & 0 & 1236 & 1618 & 780 \\ 0 & 0 & 0 & 559 & 780 & 445 \end{bmatrix}$$

Example

- **THIRTY FIFTH ITERATION**
  - STEP 1 - MEASURE
    - The measurement values:

$$z_{35} = \begin{bmatrix} 299.89 \\ 2.14 \end{bmatrix}$$

  - STEP 2 - UPDATE
    - The Kalman Gain calculation:

$$\boldsymbol{K_{35}} = \boldsymbol{P_{35,34}H^T}\left(\boldsymbol{HP_{35,34}H^T} + \boldsymbol{R}\right)^{-1} = \begin{bmatrix} 0.5556 & 0 \\ 0.2222 & 0 \\ 0.0444 & 0 \\ 0 & 0.5556 \\ 0 & 0.2222 \\ 0 & 0.0444 \end{bmatrix}$$

The Kalman Gain for position has converged to 0.56, which means that the measurement weight and the estimation weight are almost equal.

Example

- Estimate the current state:

$$\hat{\boldsymbol{x}}_{35,35} = \hat{\boldsymbol{x}}_{35,34} + \boldsymbol{K}_{35}(\boldsymbol{z}_{35} - \boldsymbol{H}\hat{\boldsymbol{x}}_{35,34}) = \begin{bmatrix} 299.2 \\ 0.25 \\ -1.9 \\ 3.3 \\ -25.5 \\ -0.64 \end{bmatrix}$$

- Update the current estimate uncertainty:

$$\boldsymbol{P}_{35,35} = (\boldsymbol{I} - \boldsymbol{K}_{35}\boldsymbol{H})\,\boldsymbol{P}_{35,34}(\boldsymbol{I} - \boldsymbol{K}_{35}\boldsymbol{H})^{T} + \boldsymbol{K}_{35}\boldsymbol{R}\boldsymbol{K}_{35}^{T} = \begin{bmatrix} 5 & 2 & 0.4 & 0 & 0 & 0 \\ 2 & 1.4 & 0.4 & 0 & 0 & 0 \\ 0.4 & 0.4 & 0.16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 2 & 0.4 \\ 0 & 0 & 0 & 2 & 1.4 & 0.4 \\ 0 & 0 & 0 & 0.4 & 0.4 & 0.16 \end{bmatrix}$$

Example

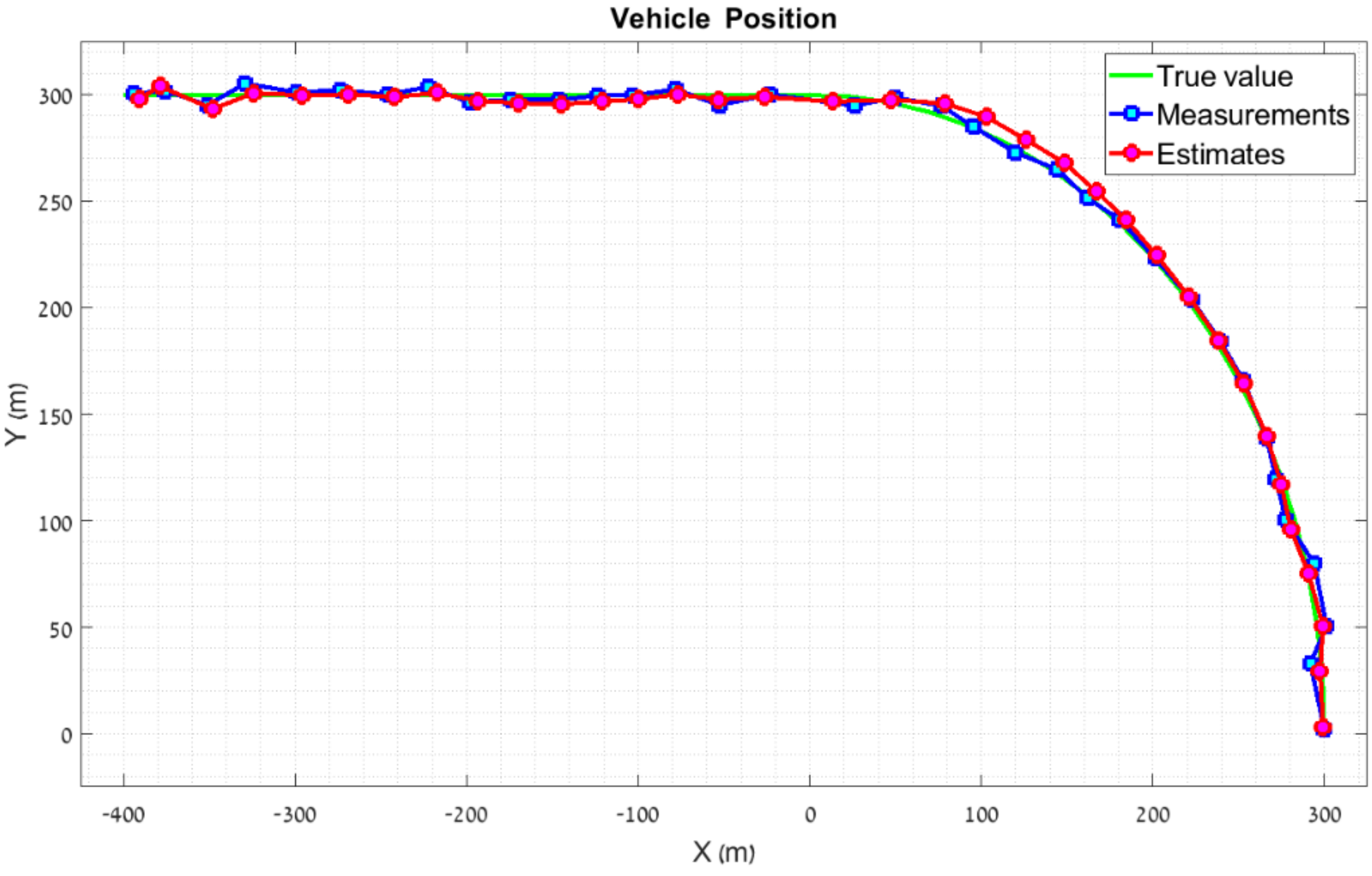
- STEP 3 - PREDICT

$$\boldsymbol{\hat{x}_{36,35}} = \boldsymbol{F\hat{x}_{35,35}} = \begin{bmatrix} 298.5 \\ -1.65 \\ -1.9 \\ -22.5 \\ -26.1 \\ -0.64 \end{bmatrix}$$

$$\boldsymbol{P_{36,35}} = \boldsymbol{FP_{35,35}F^T} + \boldsymbol{Q} = \begin{bmatrix} 11.25 & 4.5 & 0.9 & 0 & 0 & 0 \\ 4.5 & 2.4 & 0.6 & 0 & 0 & 0 \\ 0.9 & 0.6 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 11.25 & 4.5 & 0.9 \\ 0 & 0 & 0 & 4.5 & 2.4 & 0.6 \\ 0 & 0 & 0 & 0.9 & 0.6 & 0.2 \end{bmatrix}$$

Example

- **EXAMPLE SUMMARY**
  - The following chart compares the true value, measured values and estimates of the vehicle position:



Practice

- **Refer to *kalman_filter.py*, please implement the Kalman Filter Algorithm in your code.**

Thank you !